QED bound states at finite temperature

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( with Miguel Ángel Escobedo)

Motivation
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We would like to have a QCD-based quantitative understanding of heavy quarkonium systems at finite temperature.
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Can they be used as a thermometer?
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- Which of them survive the deconfinement phase transition?
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  - Which of them survive the deconfinement phase transition?
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Lattice QCD has difficulties with dynamical observables at $T \neq 0$.
Aim
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- Develop (factorization) techniques, which will be relevant to the actual case of QCD, in a simpler theory.
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It has been a fruitful approach in the $T = 0$ case:

Relevant scales
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- $m \neq 0, T = 0$ case:
  - $m$ (hard), electron mass
  - $m\alpha/n$ (soft), inverse Bohr radius, $\alpha = e^2/4\pi$; $e$, electron charge
  - $m\alpha^2/n^2$ (ultrasoft), binding energy
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\( m = 0, T \neq 0 \) case:
- \( T \) (hard), temperature
- \( eT \) (soft), Debye mass
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\[ m \neq 0, T \neq 0 \] case: what is the interplay among the scales above?
EFTs

\[ e \sim 0.3 \ (\alpha \sim 1/137), \text{ the scales are well separated, EFTs are useful:} \]
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- \( m \neq 0, T = 0 \) case:
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- **\( m \neq 0, T \neq 0 \) case:** contributions of energies above \( T \) are exponentially suppressed by Boltzmann factors
The hydrogen atom
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For $T \ll m$, vacuum polarization is suppressed, no HTL!
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Simplest case to test factorization
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Simplest case to test factorization

The $T \ll m\alpha/n$ case was addressed in the early 80’s:

The $T \ll m\alpha/n$ case

\texttt{pNRQED} can be used as a starting point
The $T \ll m \alpha/n$ case

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For $T = \beta^{-1} \ll m\alpha^2/n^2$:

\[\delta E_n = -\frac{4\pi^3\alpha}{45\beta^4} \langle n|\mathbf{x}\frac{\bar{P}_n}{(H_0 - E_n)}\mathbf{x}|n\rangle \left(1 + \mathcal{O}\left(\frac{n^2}{\beta m\alpha}\right)^2\right),\]

\[\delta \Gamma_n = 0.\]
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For $T = \beta^{-1} \gg m\alpha^2/n^2$:

$$\delta E_n = \frac{\alpha\pi}{3m\beta^2} + \frac{2\alpha}{3\pi} \sum_r |\langle n|\mathbf{v}|r\rangle|^2 (E_n - E_r)(\ln(\frac{2\pi}{\beta|E_n - E_r|}) - \gamma)$$

$$\delta \Gamma_n = \frac{4Z^2\alpha^3}{3\beta n^2} \left(1 + \mathcal{O}\left(\frac{\beta m\alpha}{n^2}\right)\right).$$
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The $T \ll m$ case

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**NRQED** can be used as a starting point

- The potentials depend on $T$:

\[
\delta \mathcal{L}(T) = -\frac{\pi \alpha}{6m^3 \beta^2} \nabla \psi^\dagger \nabla \psi + \left( \frac{\alpha \pi}{3m \beta^2} \right) \psi^\dagger \psi - \\
- \frac{4Z \alpha^2}{3m^2} \left( \ln \left( \frac{\beta \mu}{2\pi} \right) + \gamma - \ln 2 + \frac{5}{6} \right) N^\dagger N \psi^\dagger \psi
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- $\mu$, factorization scale arising from IR divergences, which should cancel against the one of ultrasoft contributions in physical observables
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The Boltzmann factors must be expanded in the ultrasoft contributions,

$$\frac{1}{e^{\beta E} - 1} \sim \frac{1}{\beta E} + \cdots$$
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$$\delta E_n = \frac{\alpha \pi}{3m \beta^2} - \frac{\pi \alpha^3}{6m \beta^2 n^2} + \frac{2\alpha}{3\pi} \sum_r |\langle n|v|r\rangle|^2 (E_n - E_r) \left( \ln \left( \frac{2\pi}{\beta|E_n - E_r|} \right) - \gamma \right),$$

$$\delta \Gamma_n = \frac{4Z^2 \alpha^3}{3\beta n^2} + \frac{2\alpha}{3} \sum_r |\langle n|v|r\rangle|^2 |E_n - E_r|$$
The $T \sim m$ case

QED must be the starting point
The $T \sim m$ case

QED must be the starting point

After integrating out the scales $T, m$, we obtain:
The $T \sim m$ case

QED must be the starting point

After integrating out the scales $T$, $m$, we obtain:

- The photon sector (HTL)

$$
\mathcal{L}_{HTL} = e^2 \int \frac{d^3 w}{(2\pi)^3} \frac{2m^2 w^2}{(1 - w^2)^2} \left( \frac{\beta m}{e \sqrt{1 - w^2} + 1} \right) \times 
$$

$$
tr \left( F_{\alpha \beta} \frac{w^\beta w^\gamma}{-(w \cdot \partial)^2} F_{\alpha \gamma} \right)
$$

$$
w_\mu = (1, w), \ w = |w| \in [0, 1)
$$
The $T \sim m$ case, NRQED+HTL
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- The electron sector (NRQED): remains local, with a mass shift

$$\delta m = \frac{\pi \alpha}{3m\beta^2} + \frac{4\alpha m}{\pi} h(m\beta) - \frac{2\alpha g(m\beta)}{\pi m\beta^2}$$

$h(x), g(x)$ go exponentially to zero when $x \to \infty$
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The electron-photon sector (NRQED+HTL): becomes non-local,

$$\delta L = \int d^3 w f(w) w E \frac{1}{w \nabla} \delta (i \partial_0 - i w \nabla) \psi^+ \psi,$$

$$f(w) = \frac{\alpha e m}{\pi^2} \frac{1}{\beta m} \frac{1}{w^2} \frac{1}{(1 - w^2)^{3/2}} \left(1 - \frac{1}{e \sqrt{1-w^2} + 1}\right)$$
The $T \sim m$ case, pNRQED

After integrating out energy scales $\sim k$, the momentum transfer, we obtain pNRQED:

$$V = -\frac{\alpha e^{-m_D r}}{r} + \frac{i16\alpha^2 g(m\beta)}{\pi m_D^2 \beta^3} \phi(m_D r),$$

$$\phi(x) = 2 \int_0^\infty \frac{dzz}{(z^2 + 1)^2} \left[ \frac{\sin(zx)}{zx} \right].$$

$$\delta m = -\frac{\alpha m_D}{2} - i\frac{8\alpha^2 g(m\beta)}{\pi m_D^2 \beta^3},$$

$$m_D = m_D(m\beta) \sim eT$$ goes exponentially to zero when $m\beta \to \infty$.
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Contains an imaginary part!!
The $T \sim m$ case, pNRQED

For $1/r \sim m_D$, $\text{Im}(V) \gg \text{Re}(V)$ !!
The \( T \sim m \) case, \( p\)N\( \)R\( \)Q\( \)E\( \)D

- For \( 1/r \sim m_D \), \( \text{Im}(V) \gg \text{Re}(V) \)

- \( \text{Im}(V) \sim \text{Re}(V) \) for \( 1/r \sim m_d := (16\alpha g(m\beta))^{\frac{1}{3}}T \gg m_D \)

\[
V \sim -\frac{\alpha}{r} + \frac{i2\alpha m_d^3 r^2 \ln m_D r}{3\pi}
\]
The $T \sim m$ case, pNRQED

- For $1/r \sim m_D$, $Im(V) \gg Re(V)$ !!

- $Im(V) \sim Re(V)$ for $1/r \sim m_d := (16\alpha g(m_\beta))^{\frac{1}{3}}T \gg m_D$

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\]

- Imposing $m_d(T_d) = m\alpha/n$ we get,

<table>
<thead>
<tr>
<th>n</th>
<th>$T_d$ (keV)</th>
<th>$m_D$ (keV)</th>
<th>$m_d$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60.4</td>
<td>0.703</td>
<td>3.73</td>
</tr>
<tr>
<td>2</td>
<td>50.1</td>
<td>0.284</td>
<td>1.86</td>
</tr>
<tr>
<td>3</td>
<td>45.6</td>
<td>0.167</td>
<td>1.24</td>
</tr>
<tr>
<td>4</td>
<td>42.9</td>
<td>0.114</td>
<td>0.932</td>
</tr>
</tbody>
</table>
Lessons for Heavy Quarkonium

For $m \to 0$ we recover the abelian limit of M. Laine, O. Philipsen, P. Romatschke, M. Tassler, JHEP 0703: 054, 2007, where the existence of an imaginary part in the potential was pointed out (confirmed by N. Brambilla, J. Ghiglieri, A. Vairo, P. Petreczky, Phys. Rev. D78: 014017, 2008).
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The melting temperature $T_d$ can be parametrically estimated to be $T_d \sim -m_Q \alpha_s^{2/3} / \ln^{1/3} \alpha_s$

\( \Upsilon(1S) \to T_d \sim 500 \text{MeV} \) (compatible with M. Laine, JHEP 0705:028,2007)

\( J/\psi \to T_d \sim 200 \text{MeV} \) (??)
Conclusions

The detailed study of QED bound states at finite temperature is relevant for heavy quarkonium physics:
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- Allows to develop factorization techniques in a simpler theory
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The detailed study of QED bound states at finite temperature is relevant for heavy quarkonium physics:

- Allows to develop factorization techniques in a simpler theory
- Allows to gain physical insight in systems which are free from the difficulties associated to confinement