R_b and R_c in the Threshold Region

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- QCD dynamics near threshold
- Threshold formalism
- The bottom threshold region 10.5 < \sqrt{s} < 11.2
- The \( \psi (3770) \) region
- The charm threshold region 3.8 < \sqrt{s} < 4.5
- Summary and outlook
QCD Dynamics Near Threshold

- QCD dynamics is much richer than present phenomenological models

Lattice QCD - Static Energy

Short distance: Perturbative QCD
- singlet: $-\frac{4}{3} \alpha_s / r$
- octet: $\frac{2}{3} \alpha_s / r$  gluelumps

Large distance: String
- $\sigma \propto r$  NG string behaviour

Graph: $V(r)/V(0)$ vs. $r/r_0$
- $\Sigma_g^+$
- $\Pi_u$
- $m_{ps} + m_s$
- $2 m_{ps}$
- quenched
- $\kappa = 0.1575$

E. Eichten - Fermilab
6th International Workshop on Heavy Quarkonia - Nara, Japan - Dec. 2-5, 2008
Light quark loops and strong decays

Lattice QCD

- light quark loops
  C. T. H. Davies et al.
  [HPQCD, Fermilab Lattice, MILC, and UKQCD Collaborations],

- strong decay

\[
[\mathcal{H}_0 + \mathcal{H}_2 + \mathcal{H}_I] \psi = \omega \psi
\]

\[\mathcal{H}_0, \quad Q\bar{Q} \quad \text{NRQCD (without light quarks)}\]

\[\mathcal{H}_I, \quad Q\bar{Q} \rightarrow Q\bar{q} + q\bar{Q} \quad \text{light quark pair creation}\]

- Cornell model (CCGM)

\[\mathcal{H}_I = \frac{3}{8} \sum_a \int \rho_a(r) V(r - r') \rho_a(r') \, d^3r \, d^3r'\]

- Vacuum Pair Creation model (QPC)

\[\mathcal{H}_I = \gamma \int \bar{\psi} \psi(r) \, d^3r\]

\[\mathcal{H}_2, \quad Q\bar{q} + q\bar{Q} \quad \text{meson pair interactions}\]
Can Lattice QCD provide insight here?
- Yes, But hard to extract information in threshold region

Excited states: \[ C(t) \equiv \langle 0 | \Phi(t) \Phi^\dagger(0) | 0 \rangle = \sum_n \langle 0 | e^{Ht} \Phi(0) e^{-Ht} | n \rangle \langle n | \Phi^\dagger(0) | 0 \rangle \]
\[ = \sum |\langle 0 | \Phi(0) | n \rangle|^2 e^{- (E_n - E_0)t} = \sum A_n e^{- (E_n - E_0)t}, \]

To extract N states in same channel – use an NxN two point correlation function obtained from N independent operators

\[ C_{\alpha \beta}(t) = \langle 0 | \Phi_\alpha(t) \Phi^\dagger_\beta(0) | 0 \rangle \]

Find principal eigenvectors of

\[ C(t_0)^{-1/2} C(t) C(t_0)^{-1/2}, \]

Strong decay channels -- resonances:
At finite volume (V) only discrete momentum values –
Multimeson states have a discrete spectrum –
Use V dependence to disentangle resonances from multibody states.
Threshold Formalism

Physical states are the appropriate basis for analysis.

\[
\begin{pmatrix}
    \mathcal{H}_0 & \mathcal{H}_I^\dagger \\
    \mathcal{H}_I & \mathcal{H}_2
\end{pmatrix}
\begin{pmatrix}
    \psi_1 \\
    \psi_2
\end{pmatrix}
= z
\begin{pmatrix}
    \psi_1 \\
    \psi_2
\end{pmatrix}
\]

\(\psi_1\): one particle states
\(\psi_2\): multi particle states

Eliminating \(\psi_2\):

\[
\left(\mathcal{H}_0 + \mathcal{H}_I^\dagger \frac{1}{z - \mathcal{H}_2} \mathcal{H}_I\right) \psi_1 = z \psi_1
\]

defines \(\Omega(z)\)

General assumptions:

1. For \(\mathcal{H}_2\) assume free meson pairs – No final state interactions or exchange interactions. No one particle bound states (eg. \(X(3872)\), etc.)

2. Solutions of \(\mathcal{H}_0 \psi_1 = z \psi_1\) – A complete basis set is given by the quarkonium states \(|n\rangle\) (e.g. hybrid spectrum is ignored) (Lattice checks on NR limit and corrections)

\[
< n | G(z) | m > = < n | \frac{1}{z - \mathcal{H}_0 - \Omega(z)} | m >
\]

3. For \(R_Q\) generalized VMD is assumed.

\[
R_Q \sim \frac{1}{s} \sum_{nm} \lim_{r \to 0} \psi^*_n(r) \text{Im} G_{nm}(W + i\epsilon) \psi_m(r)
\]
Decay Amplitudes

Cornell Model:

\[ \langle C_1(\vec{\Pi}\lambda_1)\overline{C}_2(\vec{\Pi}'\lambda_2) \mid H_f \mid \psi_n \rangle = -i(2\pi)^{-3/2}\delta^3(\vec{p} + \vec{p}')3^{-1/2}A_{12}(\vec{\Pi}\lambda_1\lambda_2;n),\]

where

\[ A_{12}(\vec{\Pi}\lambda_1\lambda_2;n) = \frac{1}{m_q} \sum_{\{s\}} \int d^3x d^3y [\chi^+(s'_{2})\overline{\chi}(s'_{1})]\frac{dV(\mid \vec{x} \mid)}{d\mid \vec{x} \mid} \phi_1^*(\vec{x}s_1s'_{1})\phi_2^*(\vec{y} - \vec{y}, s_2s'_{2})\psi_n(\bar{\psi}s_1s_2)e^{-i\mu_0\vec{p} \cdot \vec{y}}.\]

\[ dV(x)/dx = 1/a^2 + \kappa/x^2 \rightarrow \text{ignoring } \kappa \text{ term} \]

similar form as vacuum pair creation (QPC) model

Hence

\[ \Omega_{nL, mL^{'}}(W) = \sum_i \int_0^\infty P^2 dP \frac{H^i_{nL, mL^{'}}(P)}{W - E_1(P) - E_2(P) + i0}, \]

where

\[ H^i_{nL, mL^{'}}(P) = f^2 \sum_i C(JLL^{'}, l)I^l_{nL}(P)I^l_{mL^{'}}(P). \]

Statistical factor

Decay amplitudes I(p)
\( H = (Q \bar{q}) \)

\[ H_s = \{ H, H^*, H_s \} \]

\( H_{P}^{1/2} = \{ H(0^+), H(1^+), H_s(0^+), H_s(1^+) \} \)

\( H_{P}^{3/2} = \{ H(1^+), H(2^+), H_s(1^+), H_s(2^+) \} \)

### Table 1: Statistical Factors

<table>
<thead>
<tr>
<th>Initial state</th>
<th>Final state ([j_i^P])</th>
<th>outgoing wave</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>([\frac{1}{2}^-] [\frac{1}{2}^-])</td>
<td>P-wave</td>
</tr>
<tr>
<td>( H H )</td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>( H \bar{H}^*)</td>
<td>2/3</td>
<td></td>
</tr>
<tr>
<td>( H^* \bar{H}^*)</td>
<td>7/3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>([\frac{1}{2}^-] [\frac{1}{2}^-])</td>
<td>P-wave</td>
</tr>
<tr>
<td>( H H )</td>
<td>2/3</td>
<td>0</td>
</tr>
<tr>
<td>( H \bar{H}^*)</td>
<td>1/3</td>
<td>0</td>
</tr>
<tr>
<td>( H^* \bar{H}^*)</td>
<td>4/15</td>
<td>12/5</td>
</tr>
<tr>
<td></td>
<td>([\frac{1}{2}^-] [\frac{1}{2}^+])</td>
<td>S-wave</td>
</tr>
<tr>
<td>( H H (0^+))</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( H \bar{H} (1^+))</td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>( H^* \bar{H} (0^+))</td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>( H^* \bar{H} (1^+))</td>
<td>2/3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>([\frac{1}{2}^-] [\frac{3}{2}^+])</td>
<td>D-wave</td>
</tr>
<tr>
<td>( H H (1^+))</td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>( H \bar{H} (2^+))</td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>( H^* \bar{H} (1^+))</td>
<td>2/3</td>
<td></td>
</tr>
<tr>
<td>( H^* \bar{H} (2^+))</td>
<td>4/3</td>
<td></td>
</tr>
</tbody>
</table>
\[ I_{nL}^l(P) = \int_0^\infty dt \, \Phi(t) R_{nL}(t \beta^{-1/2}) j_i(\mu \beta^{-1/2} P t) \]

**Key point:** The only part of \( I(p) \) that depends on the pair production model is the function \( \Phi(t) \):

For the CCCM (\( \kappa = 0 \)):

\[ (t = y \sqrt{\beta_S}) \]

\[ \Phi(t) = t e^{-t^2} + (\pi/2)^{1/2}(t^2 - 1) e^{-t^2/2} \text{erf}(t/\sqrt{2}) \]

Using HQET this function \( \Phi(t) \) is the same for all final states in a \( j_l^P \) multiplet.

Apart from overall light quark mass factors, \( \Phi(t) \) is approximately SU(3) invariant. So independent of light quark flavor (u,d,s).

One universal function, \( \Phi(t) \), determines \( R_Q \) in the threshold region.
Lattice effort to directly compute the strong decay couplings.

G.S. Bali, H. Neff, T. Dussel, T. Lippert and K. Schilling [SESAM Collaboration],

\[
C(t) = \begin{pmatrix}
C_{QQ}(t) & C_{QB}(t) \\
C_{BQ}(t) & C_{BB}(t)
\end{pmatrix}
\begin{pmatrix}
\sqrt{n_f} \\
\sqrt{n_f} & -n_f
\end{pmatrix}
\begin{pmatrix}
\sqrt{n_f} \\
\sqrt{n_f} & -n_f
\end{pmatrix} + \begin{pmatrix}
\sqrt{n_f} \\
\sqrt{n_f} & -n_f
\end{pmatrix},
\]

transition amplitude is difficult to extract accurately

\[
g = \left. \frac{dC_{QB}(t)}{dt} \right|_{t=0} \frac{1}{\sqrt{C_{BB}(0)C_{QQ}(0)}}.
\]

However, this is exactly the function \(\Phi(r)\) we need!

FIG. 13: The two energy levels, as a function of \(\tau\), normalized with respect to \(2m_B\) (horizontal line). The curve corresponds to the three parameter fit to \(E_i(\tau)\), Eqs. (80)–(82), for \(0.2 \text{ fm} \leq \tau \leq 0.9 \text{ fm} < r_c\).

FIG. 18: The transition rate \(g\) between \(|B\rangle\) and \(|Q\rangle\) states, as a function of \(\tau\).
• Threshold behavior: $R_b$ and $R_c$

$$ R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} $$

Consider the contribution to $R$ from heavy quark ($Q = c, b$ or $t$) near pair production threshold:

$$ i \int d^4x \ e^{iqx} \langle 0 | T j_Q^\mu(x) j_Q^\nu(0) | 0 \rangle = $$

hence $R_Q$ is given by:

$$ R_Q = 12\pi e^2 Q Im \Pi(s + i\epsilon) $$

and the electromagnetic current can be expressed in NR form:

$$ j_Q^i = s_1 \psi^\dagger \sigma^i \chi + \frac{s_2}{m_Q^2} \psi^\dagger \sigma^i D^2 \chi $$

$$ + \frac{d_2}{m_Q^2} \psi^\dagger \sigma^j \left[ \frac{1}{2} (D^i D^j + D^j D^i) - \frac{1}{3} \delta^{ij} D^2 \right] \chi + ... $$

$s_1, s_2, d_2$ calculable in perturbative QCD
Recent detailed scans of $R_b$ and $R_c$ in the threshold region:

$R_b$

**BaBar:** arXiv:0809.4120 ($R_b$ scan)

**Belle:** hep-ex/0610003 ($\Upsilon(5S) \to B_s(*) B_s(*)$)

**CLEO:** hep-ex/0601044;0511034;0508047 ($\Upsilon(5S) \to B_s(*) B_s(*)$)

$R_c$

**BaBar:** arXiv:0710.1371; arXiv:0708.0032; arXiv:0608.0018 ($R_c$ exclusive channels)


**Belle:** arXiv:0708.0032; arXiv:0608.0018 ($R_c$ exclusive channels)

**CLEO:** arXiv:0708.3313 ($\psi(4415) \to D D_2*$)

Both inclusive and exclusive final states have been studied. Allows comparison with coupled channel models of these nonperturbative QCD effects. Help disentangle the physics of the near threshold region.
Updated model

- Physical masses for heavy flavor mesons
- Measured masses for quarkonium states

Added features

- Include relativistic corrections – Tensor interaction
- Include EM current couplings to $^3D_1$ states

Some tuning

- Fit the leptonic width of $1S$ (cc, bb) and $1D$ (cc) states
- Allow some adjustment of resonance masses above threshold.
The Bottom Threshold Region

- **Structure in $R_b$:** $10.5 < \sqrt{s} < 11.2$

B. Aubert et al. [BaBar Collaboration] [arXiv:0809.4120]

\[ \gamma(4S) \]

$M = 10,579.4 \pm 1.2$

$\Gamma = 20.5 \pm 2.5$

\[ \gamma(5S), \gamma(6S) \]

$M = 10,876 \pm 2, M = 10,996 \pm 2$

$\Gamma = 43 \pm 4, \Gamma = 37 \pm 2$
• CCC Model

The bottom threshold region is simple compared to the charm region:

Can ignore D states

• Direct coupling of EM current to n^3D\(_1\) states is small.
• Negligible mixing between ^3S\(_1\) and ^3D\(_1\) states.

Only the ground state B mesons
are needed (B, B*, B\(_s\), B\(_s^*\))

Analysis includes the lowest seven ^3S\(_1\) (bb) states and
nine final heavy-light pair states.

Mass differences between B\(_u\) and
B\(_d\) states can be ignored.

\[ m(B^0) - m(B^+) = 0.37 \pm 0.24 \text{ MeV} \]
**Bottom Meson Pair Thresholds**

\[ L=0 \quad b \bar{q} \left[ j_i^P = \frac{1}{2}^- \right] \]

<table>
<thead>
<tr>
<th>Meson</th>
<th>Mass (MeV/c^2)</th>
<th>Width (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^- )</td>
<td>5279.15 ± 0.31</td>
<td>(4.02 ± 0.03) \times 10^{-4}</td>
</tr>
<tr>
<td>( \bar{B}^0 )</td>
<td>5279.53 ± 0.33</td>
<td>(4.30 ± 0.03) \times 10^{-4}</td>
</tr>
<tr>
<td>( \bar{B}^0_s )</td>
<td>5366.3 ± 0.6</td>
<td>(4.48 ± 0.08) \times 10^{-4}</td>
</tr>
<tr>
<td>( \bar{B}^{*0} )</td>
<td>5325.1 ± 0.5</td>
<td>780 [21]</td>
</tr>
<tr>
<td>( \bar{B}^0_s )</td>
<td>5412.8 ± 1.3</td>
<td>150 [21]</td>
</tr>
</tbody>
</table>

\[ L=1 \quad b \bar{q} \left[ j_i^P = \frac{1}{2}^+ \right] \]

<table>
<thead>
<tr>
<th>Meson ((J^P))</th>
<th>Mass (MeV/c^2)</th>
<th>Width (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^{*-}(0^+) )</td>
<td>5730 (a)</td>
<td>270 (a)</td>
</tr>
<tr>
<td>( \bar{B}^{*0}(0^+) )</td>
<td>5730 (a)</td>
<td>270 (a)</td>
</tr>
<tr>
<td>( \bar{B}^{0}(1^+) )</td>
<td>5740 (a)</td>
<td>270 (a)</td>
</tr>
<tr>
<td>( \bar{B}^0_s(1^+) )</td>
<td>5763</td>
<td>0.118 [21]</td>
</tr>
</tbody>
</table>
| \( b \bar{q} \left[ j_i^P = \frac{3}{2}^+ \right] \)
<table>
<thead>
<tr>
<th>Meson ((J^P))</th>
<th>Mass (MeV/c^2)</th>
<th>Width (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^-(1^+) )</td>
<td>5725.3</td>
<td>15 (a)</td>
</tr>
<tr>
<td>( \bar{B}^0(1^+) )</td>
<td>5725.3 ± 2.1 [22]</td>
<td>15 (a)</td>
</tr>
<tr>
<td>( \bar{B}^0_s(1^+) )</td>
<td>5829.4 ± 0.7</td>
<td>0.002 (a)</td>
</tr>
<tr>
<td>( B^{*-}(2^+) )</td>
<td>5740.2</td>
<td>22.7</td>
</tr>
<tr>
<td>( B^{*0}(2^+) )</td>
<td>5740.2 ± 1.9 [22]</td>
<td>22.7 ± 5.0 [10.7] [22]</td>
</tr>
<tr>
<td>( \bar{B}^0_s(2^+) )</td>
<td>5839.7 ± 0.6</td>
<td>1.1(a)</td>
</tr>
</tbody>
</table>

**Narrow Thresholds**

\[ \begin{align*}
B\bar{B} & \quad 10,558 \\
B\bar{B}^* + B^*\bar{B} & \quad 10,606 \\
B^*\bar{B} & \quad 10,650 \\
B_s\bar{B}_s & \quad 10,733 \\
B_s\bar{B}_s^* + \bar{B}_sB_s^* & \quad 10,779 \\
B_s^*\bar{B}_s^* & \quad 10,825 \\
B\bar{B}(1^+) + B(1^+)\bar{B} & \quad 11,004 \\
B\bar{B}(2^+) + B(2^+)\bar{B} & \quad 11,019 \\
B^*\bar{B}(1^+) + B(1^+)\bar{B} & \quad 11,050 \\
B^*\bar{B}(2^+) + B(2^+)\bar{B} & \quad 11,065 \\
B_s\bar{B}_s(1^+) + B_s(1^+)\bar{B}_s & \quad 11,129 \\
B_s^*\bar{B}_s(0^+) + B_s(0^+)\bar{B}_s^* & \quad 11,129 \\
B_s^*\bar{B}_s(1^+) + B_s(1^+)\bar{B}_s^* & \quad 11,176 \\
B_s\bar{B}_s(1^+) + B_s(1^+)\bar{B}_s & \quad 11,196 \\
B_s\bar{B}_s(2^+) + B_s(2^+)\bar{B}_s & \quad 11,206 \\
B_s^*\bar{B}_s(1^+) + B_s(1^+)\bar{B}_s^* & \quad 11,232 \\
B_s^*\bar{B}_s(2^+) + B_s(2^+)\bar{B}_s^* & \quad 11,253 \\
\end{align*} \]

**P-wave**

**D-wave**

**S-wave**

**D-wave**

**S-wave**

}\[ \text{wide } B^*B(0^+), B^*(^*)B'(1^+), \ldots \]
\[ \Upsilon(4S) \quad \text{shift mass to fit } R_b \text{ peak} \]

model width = 25 MeV

\[ |\Upsilon(4S)\rangle = (+0.201 - i 0.114)|3S\rangle > \]
\[ (+0.790 + i 0.000)|4S\rangle > \]
\[ (-0.158 + i 0.082)|5S\rangle > \]
\[ (-0.037 - i 0.024)|2D\rangle > \]
\[ (-0.029 - i 0.035)|3D\rangle + ... \]

\[ Z_{b\bar{b}} = 0.714 \]

pole in complex plane

\[ M + i \Gamma = 10.591 + i 0.038 \]

branching fractions:
- \[ B^+ B^- \]
- \[ B^0 \bar{B}^0 \]
- \[ B^{*+} B^- + B^+ B^{*-} \]
- \[ B^{*0} \bar{B}^0 + B^0 \bar{B}^{*0} \]
**Y(5S) and Y(6S)**

Qualitative features in good agreement.

Influence of the $B_S B_{ar{S}}$ channels important in the $Y(6S)$ region. Likely explanation of small $Y(5S) - Y(6S)$ mass splitting.

**$Y(5S)$ branching fractions:**

$B_{S}^{*} \bar{B}_{S}^{*}$

**CCCM** 12%

**PDG** 19.3 ± 2.9 %
Compared with BaBar results

FIG. 1: (Left) Measured resonances resulting from the fit described in the text. The corresponding world averages [17] are also reported. (Right) A zoom of the same plot with the result of the fit described in the text superimposed. The large statistics and the small energy steps of this scan make it possible to observe clear structures corresponding to the opening thresholds of the different final- or initial-state radiation processes.

TABLE I: Contributions to the relative correlated systematic errors on data represent the statistical and the uncorrelated systematic errors added in quadrature. The number of states is, a priori, unknown as are their mass differences in our fit, a simple modification is to replace the flat component representing the interfering continuum states not interfering with resonance thresholds outlined earlier, would be likely to modify the energy dependencies. Therefore, a proper coupled channel approach [15, 16] including the e
er substantially from the PDG values. In the particular the error on resonant depolarization result [13]. We correct for this bias, that comes from the (strongly) non linear impact of the momentum resolution in the invariant mass, and verify on simulated events that it does not depend on

\[ R = \sum_{i} \sqrt{\text{stat}_i^2 + \text{sys}_i^2} \]

where \( \text{stat}_i \) and \( \text{sys}_i \) are the statistical and uncorrelated systematic errors, respectively. The numerical results are summarized in Table I. The large impact on the determination of the different world averages [17] are also reported.

TABLE II: Fit results for the different world averages [17] are also reported. The numerical results are summarized in Table I. The large impact on the determination of the different world averages [17] are also reported.
The $\psi (3770)$ Region


The $\psi (3770)$ resonance exists in the energy region between 3.700 and 3.872 GeV. If there are no other effects distorting the pure D-wave Breit-Weigner shape of the cross sections, we find that it does not lead to the significant effects distorting the $\psi (3770)$ resonance. However, if there are some dynamics effects which distort the pure D-wave Breit-Weigner shape of the cross sections, we obtain almost the same results as these shown in Solution 2 in Table I instead of the large non-statistical significances distorting the $\psi (3770)$ decay. This fit gives $\sigma_{\text{had}}$ of the $\psi (3770)$ resonance there at 7.6, which does not lead to the significant effects distorting the $\psi (3770)$ decay.

\[ M = 3,772.92 \pm 0.35 \]
\[ \Gamma = 27.3 \pm 1.0 \]
TABLE I: Contributions to the systematic error in the cross sections, [%].
pdg 2007

**Mass** $m = 3772.4 \pm 1.1$ MeV, \hspace{1em} \( S = 1.8 \) \hspace{1em} **Full width** $\Gamma = 25.2 \pm 1.8$ MeV

Decay width in good agreement with theory

Production in $e^+e^-$ due to relativistic terms:

(a) Expansion of EM current

$$ j_c^i = s_1 \psi^\dagger \sigma^i \chi + \frac{s_2}{m_c^2} \psi^\dagger \sigma^i D^2 \chi $$

$$ + \frac{d_2}{m_c^2} \psi^\dagger \sigma^j \left[ \frac{1}{2} (D^i D^j + D^j D^i) - \frac{1}{3} \delta^{ij} D^2 \right] \chi + ... $$

(b) S-D mixing terms - short range: $\sim$5 MeV

(c) Induced mixing from $D^*-D$ mass difference - long range

$$ \psi(3772) = 0.10 \left| 2S \right \rangle + 0.01 e^{+0.22i\pi} \left| 3S \right \rangle + ... $$

$$ + 0.69 e^{-0.59i\pi} \left| 1D \right \rangle + 0.10 e^{+0.27i\pi} \left| 2D \right \rangle + ... $$
Induced 2S–1D tensor mixing term

At $\psi(3773)$: $-23$ MeV (total)

Cancellations between ($DD$, $DD^*$, and $D^*D^*$) contributions
Decays into open charm

The ratio, $R^{0/+}$, of $D^0\bar{D}^0$ to $D^+D^-$ production deviates from one due to isospin violating terms:

(a) up–down mass difference

(b) EM interactions

$\rightarrow m(D^+) - m(D^0) = 4.78 \pm 0.10 \text{ MeV}$

$\rightarrow$ different final state interactions

\[
R^{0/+} = \frac{1}{1.25} \times \frac{1.28 \pm 0.14}{1.47} \times 1.36
\]

The shape of the resonance differs from the usual Breit-Wigner:

(1) width $\Gamma(p)$ not pure $p$ wave

(2) interference with 2S state.

\[
\Gamma(p) \sim A \frac{p^3}{\Lambda^2} \exp\left(-\frac{p^2}{\Lambda^2}\right)
\]

$A = 0.18 \quad \Lambda = 0.57 \text{ GeV}$

$p_0 = 283 \text{ MeV} \quad p_+ = 250 \text{ MeV}$
Compare to BES data

- $D^+D^-$
- $D^0\bar{D}^0$
- $D\bar{D}$
The Charm Threshold Region

- Structure in $R_c : 3.8 < \sqrt{s} < 4.5$

J. Libby [for CLEO Collaboration]
[arXiv:0807.1220]

$\psi(3S) \quad \psi(2D)$

- $M = 4,039 \pm 1$
- $\Gamma = 80 \pm 10$

- $M = 4,153 \pm 3$
- $\Gamma = 103 \pm 8$

$\psi(4S)$

- $M = 4,421 \pm 4$
- $\Gamma = 62 \pm 20$
Figure 1: The fit to the $R$ values for the high mass charmonia structure. The dots with error bars are the updated $R$ values. The solid curve shows the best fit, and the other curves show the contributions from each resonance $R_{BW}$, the interference $R_{int}$, the summation of the four resonances $R_{res} = R_{BW} + R_{int}$, and the continuum background $R_{con}$ respectively.

Figure 2: (I) The comparison of $R$ values between the values published in Ref. [14] (triangles: $R_{old}$) and the updated values in this work (points: $R_{now}$). (II) The relative differences between the two sets of $R$ values.

$\chi^2$/d.o.f=1.05

M. Ablikim et al. [BES Collaboration] [arXiv:0705.4500]
## Charm Meson Pair Thresholds

### L=0

\[ c\bar{q} \left[ j_{1}^{P} = \frac{1}{2}^{-} \right] \]

<table>
<thead>
<tr>
<th>Meson</th>
<th>Mass (MeV/$c^2$)</th>
<th>Width (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0$</td>
<td>1864.84 ± 0.17</td>
<td>(1.60 ± 0.01) \times 10^{-3}</td>
</tr>
<tr>
<td>$D^+$</td>
<td>1869.62 ± 0.20</td>
<td>(6.33 ± 0.04) \times 10^{-4}</td>
</tr>
<tr>
<td>$D_s^+$</td>
<td>1968.49 ± 0.34</td>
<td>(1.32 ± 0.02) \times 10^{-3}</td>
</tr>
<tr>
<td>$D_s^{*0}$</td>
<td>2006.97 ± 0.19</td>
<td>77 \times 10^{3} [21]</td>
</tr>
<tr>
<td>$D_s^{*+}$</td>
<td>2010.27 ± 0.17</td>
<td>(96 ± 4 ± 22) \times 10^{3}</td>
</tr>
<tr>
<td>$D_s^{*+}$</td>
<td>2112.3 ± 0.5</td>
<td>440 [21]</td>
</tr>
</tbody>
</table>

### L=1

\[ c\bar{q} \left[ j_{1}^{P} = \frac{1}{2}^{+} \right] \]

<table>
<thead>
<tr>
<th>Meson</th>
<th>Mass (MeV/$c^2$)</th>
<th>Width (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^{*0}(0^+)$</td>
<td>2352 ± 50</td>
<td>261 ± 50</td>
</tr>
<tr>
<td>$D^{*+}(0^+)$</td>
<td>2403 ± 38</td>
<td>283 ± 42</td>
</tr>
<tr>
<td>$D_s^{*+}(0^+)$</td>
<td>2317.8 ± 0.6</td>
<td>0.023 [21]</td>
</tr>
<tr>
<td>$D^{0}(1^+)$</td>
<td>2427 ± 35</td>
<td>384 ± 130/105</td>
</tr>
<tr>
<td>$D^+(1^+)$</td>
<td>2427 (a)</td>
<td>384 (a)</td>
</tr>
<tr>
<td>$D_s^+(1^+)$</td>
<td>2459.6 ± 0.6</td>
<td>0.038 [21]</td>
</tr>
</tbody>
</table>

\[ c\bar{q} \left[ j_{1}^{P} = \frac{3}{2}^{+} \right] \]

<table>
<thead>
<tr>
<th>Meson</th>
<th>Mass (MeV/$c^2$)</th>
<th>Width (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^{0}(1^+)$</td>
<td>2422.3 ± 1.3</td>
<td>20.4 ± 1.7</td>
</tr>
<tr>
<td>$D^+(1^+)$</td>
<td>2423.4 ± 3.1</td>
<td>25 ± 6</td>
</tr>
<tr>
<td>$D_s^+(1^+)$</td>
<td>2535.35 ± 0.6</td>
<td>0.29 (a)</td>
</tr>
<tr>
<td>$D^{*0}(2^+)$</td>
<td>2461.1 ± 1.6</td>
<td>42 ± 4</td>
</tr>
<tr>
<td>$D^{*+}(2^+)$</td>
<td>2460.1 ± 2.6/3.5</td>
<td>37 ± 6</td>
</tr>
<tr>
<td>$D_s^{*+}(2^+)$</td>
<td>2572.6 ± 0.9</td>
<td>20 ± 5</td>
</tr>
</tbody>
</table>

### Narrow Thresholds

- $D\bar{D}$
  - 3729.7(+9.56)
- $D\bar{D}^* + D^*\bar{D}$
  - 3,871.8(+8.08)
- $D_s\bar{D}_s$
  - 3,937.0
- $D_s^*\bar{D}_s^*$
  - 4,013.9(+6.6)
- $D_s\bar{D}_s^* + \bar{D}_sD_s^*$
  - 4,080.8
- $D_s^*\bar{D}_s^*$
  - 4,224.6

### P-wave

- $D\bar{D}(1^+) + D(1^+)\bar{D}$
  - 4,287.1(+5.9)
- $D\bar{D}(2^+) + D(2^+)\bar{D}$
  - 4,325.9(+3.8)
- $D^*\bar{D}(1^+) + D(1^+)\bar{D}^*$
  - 4,429.3(+4.4)
- $D^*\bar{D}(2^+) + D(2^+)\bar{D}^*$
  - 4,468.1(+2.3)
- $D_s\bar{D}_s(1^+) + D_s(1^+)\bar{D}_s$
  - 4,428.1
- $D_s^*\bar{D}_s(0^+) + D_s(0^+)\bar{D}_s^*$
  - 4,430.1
- $D_s^*\bar{D}_s(1^+) + D_s(1^+)\bar{D}_s^*$
  - 4,571.9
- $D_s\bar{D}_s(1^+) + D_s(1^+)\bar{D}_s$
  - 4,540.9
- $D_s\bar{D}_s(2^+) + D_s(2^+)\bar{D}_s$
  - 4,541.1
- $D_s^*\bar{D}_s(1^+) + D_s(1^+)\bar{D}_s^*$
  - 4,647.7
- $D_s^*\bar{D}_s(2^+) + D_s(2^+)\bar{D}_s^*$
  - 4,684.9
- ... 

### S-wave

- $D^*D(0^+), D^*(1^+), ...$

### D-wave

- $D_sD(1^+), D_s(1^+)\bar{D}_s$, etc.

### S-wave

- $D_s\bar{D}_s(1^+) + D_s(1^+)\bar{D}_s$, etc.
Charm Threshold Behaviour

- Significantly differs from light quark expectations
  - Kinematics - $m_Q$ large $\Rightarrow$ $\Delta E/\Delta p$ small
  - Many Narrow Two Body Channels Opening
  - Radially Excited Charmonium Resonances

- Result:
  - Complicated Threshold Behaviour Expected
  - Need Model to Search for New States
Summary and Outlook

Present status

- Detailed experimental measurements of $R_b$ and $R_c$ in the threshold region provide a wealth of information. Particularly useful are scans of exclusive heavy flavor meson pair channels.

- Much of the rich structure in $R_c$ and $R_b$ in the threshold region arises simply from the behavior (i.e. nodes) of decay amplitudes for radially excited quarkonium resonances.

- The peaks in $R$ for individual final states do not coincide.

- Determining the number and properties of resonances in the threshold region is difficult without a detailed decay model.

- Simple phenomenological models work reasonably well but a sounder theoretical footing is needed.

- Above the opening of the $H_S H_P$ decay channels the structure of individual resonances disappears. Many new channels with excited heavy flavor mesons become available. A dual picture from perturbative QCD is more appropriate.
Improving the theory

- Calculate the EM current coupling to L=2 quarkonium states to higher order. (Done?)
- Use the \( \psi(3770) \) region to test the validity of the VMD assumption. How large is the direct \( H_s^+ H_s^- \) coupling to the EM current?
- Make a better model of the pair production function. Here the lattice can help.
- Include the \( H_s H_p \) thresholds (including the wide \( H_p \) states) in the threshold analysis.

More data

- Exclusive channel scans for \( R_b \) in the \( \Upsilon(5S) \) and \( \Upsilon(6S) \) region.
- Exclusive channel scans for \( R_c \) in the \( \psi(4S) \) region. Particularly the \( H_s H_p \) channels.
- More detailed studies of \( R_c \) in the region around the \( \psi(3770) \).
Backup Slides
Charmonium family

$\eta_c(1S)$

$\psi(2S)$

$\psi(1^3D_1)$

$\eta_c(2S)$

$\eta_c(3S)$

$\psi(2D)$

$\psi(3S)$

$\psi(4S)$ or hybrid

$\chi_{c1}(2P)$

$\chi_{c2}(2P)$

$\chi_{c1}(1P)$

$\chi_{c2}(1P)$

$\chi_{c0}(1P)$

$J/\psi$

$\gamma_{M1}$

$\gamma_{E1}$

$\pi^0$

$h_c(1P)$

$\eta$

$\omega$

$\rho$

$\pi^0$

$\eta_c(1P)$

$\chi_{c0}(1P)$

$\chi_{c1}(1P)$

$\chi_{c2}(1P)$

$\gamma_{E1}$

$\gamma_{M1}$

Mass (GeV/c$^2$)

$J^{PC} = 0^{--}$

$L = 0$

$1^{--}$

$1^{++}$

$0^{++}$

$1^{++}$

$2^{++}$

2 M(D)

$D^0 \bar{D}^0 \pi^0$

$D\bar{D}$

$\pi^+\pi^- J/\psi$

$\eta_c(3S)$

$\psi(3S)$

$\psi(2S)$

$\eta_c(2S)$

$\eta_c(1S)$

$\gamma_{M1}$

$\gamma_{E1}$

$\pi^0$

$h_c(1P)$

$\eta$

$\omega$

$\rho$

$\pi^0$

$\eta_c(1P)$

$\chi_{c0}(1P)$

$\chi_{c1}(1P)$

$\chi_{c2}(1P)$

$\gamma_{E1}$

$\gamma_{M1}$

Mass (GeV/c$^2$)

$J^{PC} = 0^{--}$

$L = 0$

$1^{--}$

$1^{++}$

$0^{++}$

$1^{++}$

$2^{++}$

2 M(D)

$D^0 \bar{D}^0 \pi^0$

$D\bar{D}$

$\pi^+\pi^- J/\psi$
Phase shifts for DD scattering

FIG. 1: The $R_{uds(c)+\psi(3770)}(s)$ versus the c.m. energy (see text).

FIG. 9. Argand plot of the $D\bar{D} S$ matrix in the $1^{--}$ state. The rather narrow elastic $^{3}D_{1}$ resonance $\psi(3772)$ is clearly in evidence, as is an inelastic resonance at $\sim 4.15$ GeV due to the $3^{3}S c\bar{s}$ state. The parameters are the same as in Figs. 7 and 8.
Non DD decays of the $\psi(3770)$

- $\mathbf{XJ/\psi}$

Theory expectation for $\pi^+\pi^-J/\psi$: 0.1–0.7%

- $\mathbf{\gamma X_{cJ}}$

Good agreement with theory expectations including relativistic effects

- $\mathbf{\text{light hadrons}}$

No evidence for direct decays to light hadrons seen yet.

Puzzle of missing decays

$$\sigma_{\psi(3770)} = 6.38 \pm 0.08 \pm 0.41 \pm 0.30 \text{ nb}$$

$$\sigma_{\psi(3770)} = 0.01 \pm 0.08 \pm 0.41 \pm 0.30 \text{ nb}$$

$$\sigma_{\psi(3770)} = 7.25 \pm 0.27 \pm 0.34 \text{ nb}$$

BES

CLEO

<table>
<thead>
<tr>
<th>Mode</th>
<th>$E_\gamma$ (MeV)</th>
<th>Predicted (keV)</th>
<th>CLEO (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma X_{c2}$</td>
<td>208.8</td>
<td>3.2</td>
<td>24 ± 4</td>
</tr>
<tr>
<td>$\gamma X_{c1}$</td>
<td>251.4</td>
<td>3.2</td>
<td>3.3</td>
</tr>
<tr>
<td>$\gamma X_{c0}$</td>
<td>339.5</td>
<td>3.2</td>
<td>3.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay Mode</th>
<th>$\sigma_{\psi(3770)}$</th>
<th>$\sigma_{\psi(3770)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi \pi^0$</td>
<td>&lt; 3.5 $^{+3.5}_{-1.5}$</td>
<td>&lt; 3.5 $^{+3.5}_{-1.5}$</td>
</tr>
<tr>
<td>$\phi \eta$</td>
<td>&lt; 12.6 $^{+12.6}_{-6.5}$</td>
<td>&lt; 12.6 $^{+12.6}_{-6.5}$</td>
</tr>
<tr>
<td>$\phi K^+ K^-$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
</tr>
<tr>
<td>$\phi K^0 K^-$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
</tr>
<tr>
<td>$\phi \eta$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
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</tr>
<tr>
<td>$\phi K^+ K^-$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
</tr>
<tr>
<td>$\phi \eta$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
</tr>
<tr>
<td>$\phi K^+ K^-$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
</tr>
<tr>
<td>$\phi \eta$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
</tr>
<tr>
<td>$\phi K^+ K^-$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
<td>&lt; 11.1 $^{+11.1}_{-5.6}$</td>
</tr>
</tbody>
</table>

BES [hep-ex/0705.2276]
Model - Cornell Coupled Channel
Model - Cornell Coupled Channel