NLO QCD corrections to J/ψ production in e^+e^- annihilation at B factories

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NLO QCD corrections to J/ψ production in e^+e^- annihilation at B factories (*Outline*)

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- $e^+e^- \rightarrow J/\psi + \chi_{c0}$
  - ***Leading Order***
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1 Introduction

Nonrelativistic QCD (NRQCD) (Bodwin-Braaten-Lepage, 1995)[*], new factorization formulism for heavy quarkonium production and annihilation.

- Successes of NRQCD in heavy quarkonium production
  - ⋆ Quarkonium Production at the Tevatron and color-octet mechanism (? [**]);
  - ⋆ $\gamma\gamma \to J/\psi$ at LEP;
- Challenging Problems in NRQCD Factorization Approach
  - ⋆ Polarizations of quarkonia at the Tevatron [**];
  - ⋆ J/ψ production in e^+e^- annihilation at B Factories.

[*] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995) [Erratum-ibid. D **55**, 5853 (1997)]

[**] For new developments, P. Artoisenet *et al.*, hep-ph/0703129, arXiv:0806.3282; J. Campbell *et al.*, hep-ph/0703113; B. Gong *et al.*, arXiv:0805.4751, arXiv:0805.2469, arXiv:0802.3727.

Problems in J/ψ production in e^+e^- annihilation at B Factories

⋆ Large rates for double charmonia: $\sigma[e^+e^- \rightarrow J/\psi + \eta_c(\eta_c(2S), \chi_{c0})$, a factor of 10 larger than LO results.

⋆ Large rates for J/ψ associated with open charm pair: $\sigma[e^+e^- \rightarrow J/\psi + c\bar{c} + X]$, a factor of 5-10 larger than LO results.

⋆ Large ratio $R_{c\bar{c}}$: much larger than theoretical predictions.

$$R_{c\bar{c}} = \frac{\sigma[e^+e^- \rightarrow J/\psi + c\bar{c} + X]}{\sigma[e^+e^- \rightarrow J/\psi + X]} \tag{1}$$

The experiment data [Belle, PRL 89 142001]:

$$R_{c\bar{c}} = 0.59^{+0.15}_{-0.13} \pm 0.12 \tag{2}$$

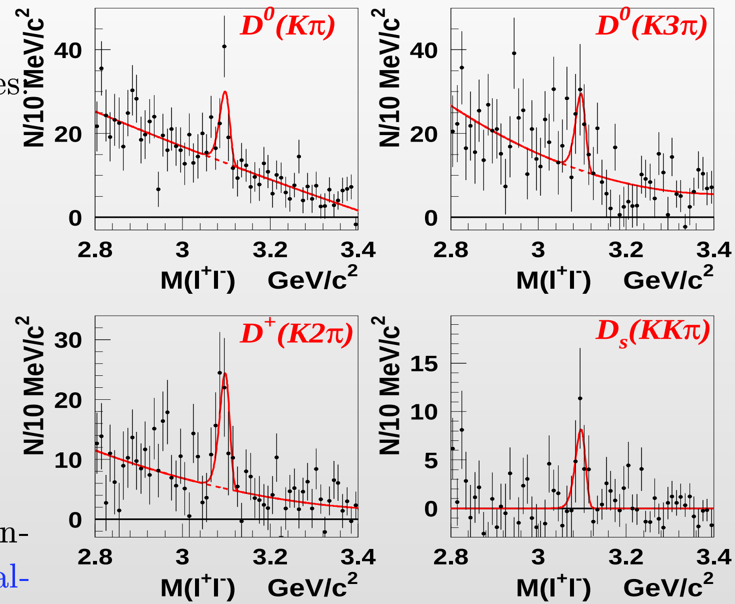
LO theoretical prediction (including color-octet contribution) ≈ 0.1 [Cho et al, PRD 54 6690, Yuan et al, PRD56, 321, ...].

In EPS'2003 Belle[Uglov, EPJC 33 S235]:

$$R_{c\bar{c}} = 0.82 \pm 0.15 \pm 0.14 \tag{3}$$

Improved measurement of $\sigma(e^+e^- \to J/\psi\, c\bar{c})$

- $\mathcal{L} = 101\,\mathrm{fb}^{-1}$
- Reconstruct all charm ground states: D^0, D^+, D_s^+, Λ_c
- Fit $D(\Lambda_c)$ signals in $M_{\ell^+\ell^-}$ bins
- Fit J/ψ to yields distributions:
  $N_{D^0\to K\pi} = 49.6 \pm 13.3\ (3.7\sigma)$
  $N_{D^0\to K3\pi} = 53.0 \pm 21.2\ (2.5\sigma)$
  $N_{D^+\to K2\pi} = 56.2 \pm 15.4\ (3.6\sigma)$
  $N_{D_s^+\to KK\pi} = 23.8 \pm 9.4\ (2.6\sigma)$
  $N_{\Lambda_c\to Kp\pi} = 3.0 \pm 4.2$
- All $c\bar{c}$ final states except for Ξ_c reconstructed $\Rightarrow$ Do not need model to calculate $2(c\bar{c})$ X-section!



$$\left.\frac{\sigma(e^+e^-\to J/\psi\, c\bar{c})}{\sigma(e^+e^-\to J/\psi\, X)}\right|_{P_{J/\psi}>2.0\ \mathrm{GeV}/c} = \frac{0.5(N_{D^0}+N_{D^+}+N_{D_s^+}+N_{\Lambda_c})+N_{(c\bar{c})res}}{N_{J/\psi}} = 0.82 \pm 0.15 \pm 0.14$$

Belle result on $R_{c\bar{c}}$ in EPS 2003.

⋆ In NRQCD, $\sigma[J/\psi + X]$ include color-singlet contributions $\sigma[J/\psi(^3S_1^{[1]}) + c\bar{c}]$ and $\sigma[J/\psi(^3S_1^{[1]}) + gg]$ and color-octet contributions $\sigma[J/\psi(^3P_J^{[8]}, ^1S_0^{[8]}) + g]$. Contributions of other Fock states suppressed by α_s or v^2.

⋆ $\sigma[J/\psi(^3P_J^{[8]}, ^1S_0^{[8]}) + g]$ calculated at LO [*] and NLO [**] in α_s. But the color-octet matrix elements have large uncertainties and may be smaller than previously expected.

⋆ Large logarithms related to $\alpha_s log(1 - (E_{c\bar{c}}/E_{c\bar{c}}^{max}))$ at the end point should be resummed in SCET [***].

⋆ If neglect the color octet contribution, $R_{c\bar{c}} = \sigma[J/\psi + c\bar{c}]/(\sigma[J/\psi + c\bar{c}] + \sigma[J/\psi + gg])$.

⋆ $\sigma[J/\psi + c\bar{c}]$ and $\sigma[J/\psi + gg]$ sensitive to input parameters, α_s, $|R_s(0)|^2$, $\sqrt{s}$, and m_c. But $R_{c\bar{c}}$ at LO in α_s only depends on $m_c/\sqrt{s}$.

Comparing experiment with theoretical results of $R_{c\bar{c}}$, important to test the production mechanism in NRQCD.

Theoretical predictions of $R_{c\bar{c}}$ at LO in α_s are 0.36 within NRQCD[****] or $0.09 \sim 0.17$ within pQCD methods[*****].

[*] E. Braaten and Y. Q. Chen, Phys. Rev. Lett. **76**, 730 (1996) [arXiv:hep-ph/9508373].
[**] Y. J. Zhang, Y. Q. Ma and K. T. Chao, To be submitted.
[***]S. Fleming， A.K. Leibovich, and T. Mehen, Phys. Rev. D68, 094011 (2003).
[****] K. Hagiwara, E. Kou, Z. H. Lin, C. F. Qiao and G. H. Zhu, Phys. Rev. D **70**, 034013 (2004)
[*****] A. V. Berezhnoy and A. K. Likhoded, Phys. Atom. Nucl. **67**, 757 (2004) [arXiv:hep-ph/0303145].

Cross sections e.g. of $e^+e^- \to J/\psi(\psi(2S))\chi_{c0}$ measured by Belle[*]

$$\begin{aligned} \sigma[J/\psi + \chi_{c0}] \times B^{\chi_{c0}}[>2] &= (16 \pm 5 \pm 4)\ \text{fb}, \\ \sigma[\psi(2S) + \chi_{c0}] \times B^{\chi_{c0}}[>2] &= (17 \pm 8 \pm 7)\ \text{fb}, \end{aligned} \tag{4}$$

larger than LO NRQCD predictions by about an order of magnitude (or at least a factor of 5). ($B^{\chi_{c0}}[>2]$ is the branching fraction for the χ_{c0} decay into more than 2 charged tracks)

LO in α_s and v [**][***] cross-section of $e^+e^- \to J/\psi(\psi(2S))\chi_{c0}$ at $\sqrt{s} = 10.6\text{GeV}$ is only $1.0 \sim 4.4(2.4 \sim 6.7)$fb(depending on input parameters, e.g., the long-distance matrix elements, m_c and α_s).

[*] T.V. Uglov, Eur. Phys. J. C **33**, S235 (2004).
[**]E. Braaten and J. Lee, Phys. Rev. D **67**, 054007 (2003)
[***]K. Y. Liu, Z. G. He and K. T. Chao, Phys. Lett. B **557**, 45 (2003)

Summary of double charmonium production cross-sections

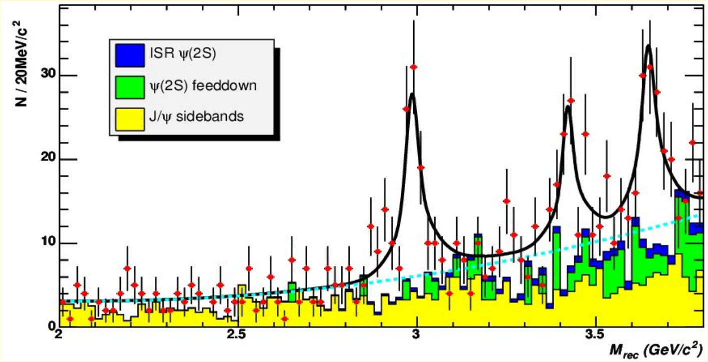
PRELIMINARY!

$$\sigma\left(e^+e^- \to (\bar{c}c)_1\,(c\bar{c})_2\right) \times \mathcal{B}\left((c\bar{c})_2 \to > 2\ \ charged\right) \qquad \textbf{(fb)}$$

		RECONSTRUCTED CHARMONIUM			
		J/ψ	χ_{c1}	χ_{c2}	$\psi(2S)$
RECOIL CHARMONIUM	η_c	$46 \pm 6^{+7}_{-9}\,(2.3)$	$< 21\ \ (1.3 \cdot 10^{-3})$	$< 38\,(0.5 \cdot 10^{-3})$	$18 \pm 8 \pm 7\,(0.9)$
	J/ψ	$< 8\ (8.7)$	< 21	< 38	$< 64\ (7.2)$
	χ_{c0}	$16 \pm 5 \pm 4$	< 21	< 38	$17 \pm 8 \pm 7$
	χ_{c1}	< 8	< 21	< 38	< 24
	χ_{c2}	< 8	< 21	< 38	< 24
	$\eta_c(2S)$	$25 \pm 6 \pm 6\ (0.9)$	$< 21(0.5 \pm \cdot 10^{-3})$	$< 38(0.2 \cdot 10^{-3})$	$31 \pm 9 \pm 10\ (0.4)$
	$\psi(2S)$	$< 16(7.2)$	< 21	< 38	$< 18(1.5)$

Belle result of double charmonium in EPS'2003.

Double $c\bar{c}$ production



124 fb^{-1}, preliminary, hep-ex in preparation.

	$J/\psi\ c\bar{c}$	η_c	χ_{c0}	$\eta_c(2S)$
Expt	$\sigma \times \mathcal{B}_{>2}$	$17.6 \pm 2.8 \pm 2.1$	$10.3 \pm 2.5 \pm 1.8$	$16.4 \pm 3.7 \pm 3.0$
	$\sigma \times \mathcal{B}_{>2}$ BELLE	$25.6 \pm 2.8 \pm 3.4$	$6.4 \pm 1.7 \pm 1.0$	$16.5 \pm 3.0 \pm 2.4$
Th.	Braaten Lee PRD 67 054007(2003)	2.31 ± 1.09	2.28 ± 1.03	0.96 ± 0.45
	Liu, He, Chao hep-ph/0408141	5.5	6.9	3.7

Applicability of NRQCD : Bondar, Chernyak, hep-ph/0412335

BaBar result of double charmonium production

Theoretical studies were suggested in order to resolve the large discrepancies.

To the $R_{c\bar{c}}$,

- Liu, He, and Chao, the color-octet contribution and $J/\psi + c\bar{c}$ production via two photons within NRQCD, but can not make up the large discrepancy[*].
- Kaidalov, the nonperturbative quark-gluon-string model [***].

[*] K. Y. Liu, Z. G. He and K. T. Chao, arXiv:hep-ph/0301218, arXiv:hep-ph/0305084.
[***] A. B. Kaidalov, JETP Lett. **77**, 349 (2003) [arXiv:hep-ph/0301246].

- Kang, Lee, and Lee, $R_{c\bar{c}} = 0.049$ in color-evaporation-model[*].
- Berezhnoy calculated $\sigma[J/\psi + c\bar{c}]$ with the light cone wave function for massive charm quark, finding the effect can be neglected [**].
- Berezhnoy and Likhoded calculate $R_{c\bar{c}}$ with two pQCD methods: J/ψ wave function and quark-hadron duality. Their result is $R_{c\bar{c}} = 0.09 \sim 0.17$ [***].

[*] D. Kang, *et al.*, Phys. Rev. D **71**, 094019 (2005) [arXiv:hep-ph/0412381];
[**] A. V. Berezhnoy, arXiv:hep-ph/0703143.
[***] A. V. Berezhnoy and A. K. Likhoded, Phys. Atom. Nucl. **67**, 757 (2004) [arXiv:hep-ph/0303145].

Factorization in NRQCD involving P-wave quarkonium

- Differing from S-wave states, the χ_{c0} is a P-wave charmonium, it is unclear whether QCD factorization can still hold at NLO for $e^+e^- \to J/\psi(\psi(2S))\chi_{c0}$.
- In processes where P-wave states are involved, there often appear nonfactorizable infrared (IR) divergences due to the non-vanishing relative momentum between the heavy quark and antiquark in quarkonium.
- One distinct example is the B meson exclusive decay to P-wave charmonium $B \to \chi_{c0}K$, IR divergence does exist in the vertex corrections and breaks down factorization[*].
- The color transfer has been noticed in associated heavy-quarkonium production, e.g., $e^+e^- \to J/\psi c\bar{c}$, where IR divergence appears due to soft interactions between the associated c (or $\bar{c}$) quark and the $c\bar{c}$ pair of charmonium, and hence breaks down factorization at NNLO[**].
- Recently, factorization theorems for exclusive heavy-quarkonium production were studied by Bodwin, Tormo, and Lee[***]

[*] Z.Z. Song and K.T. Chao, Phys. Lett. B **568**, 127 (2003); Z.Z. Song, C. Meng, Y.J. Gao and K.T. Chao, Phys. Rev. D **69**, 054009 (2004).
[**] G.C. Nayak, J.W. Qiu and G. Sterman, Phys. Rev. Lett. 99, 212001 (2007); arXiv:0711.3476 [hep-ph].
[***] G. T. Bodwin, X. G. i. Tormo and J. Lee, arXiv:0805.3876 [hep-ph].

A number of recent calculations[*,**] show that the NLO QCD corrections to heavy quarkonia production may be very large. $R_{c\bar{c}}$ and $\sigma[J/\psi + \chi_{c0}]$ should be calculated to NLO in α_s.

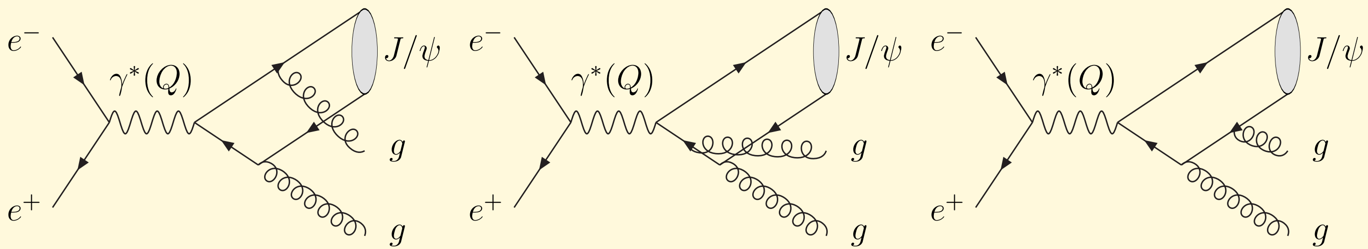
- We calculated NLO correction to $\sigma[J/\psi + c\bar{c}]$ in α_s, which significantly enhances the cross section[**].
- To resolve the problem of $R_{c\bar{c}}$, we calculated NLO QCD correction to $e^+e^- \rightarrow J/\psi + gg + X$.
- $\sigma[J/\psi + \eta_c]$ has been calculated at NLO in α_s, crucially reducing the large discrepancy between theory and experimental data [***]. QCD corrections may also be important in $\sigma[J/\psi + \chi_{c0}]$.
- Factorization of $e^+e^- \rightarrow J/\psi + \chi_{c0}$ should be clarified.

[*] Y. J. Zhang and K. T. Chao, arXiv:0808.2985 [hep-ph]; arXiv:0802.3655 [hep-ph]; R. Li and J. X. Wang, arXiv:0811.0963 [hep-ph]; B. Gong, Y. Jia and J. X. Wang, arXiv:0808.1034 [hep-ph]; B. Gong, X. Q. Li and J. X. Wang, arXiv:0805.4751 [hep-ph]; B. Gong and J. X. Wang, arXiv:0805.2469 [hep-ph], Phys. Rev. Lett. **100**, 232001 (2008), Phys. Rev. Lett. **100**, 181803 (2008), J. Campbell, F. Maltoni, F. Tramontano, Phys. Rev. Lett. 98, 252002(2007); P. Artoisenet, J.P. Lansberg, F. Maltoni, Phys. Lett. B653, 60 (2007);
[**] Y. J. Zhang and K. T. Chao, Phys. Rev. Lett. **98**, 092003 (2007).

[***] Y. J. Zhang, Y. J. Gao and K. T. Chao, Phys. Rev. Lett. **96**, 092001 (2006); B. Gong and J. X. Wang, Phys. Rev. D **77**, 054028 (2008).

2 $e^+e^- \to J/\psi + gg$

2.1. Leading Order of $e^+e^- \to J/\psi + gg$

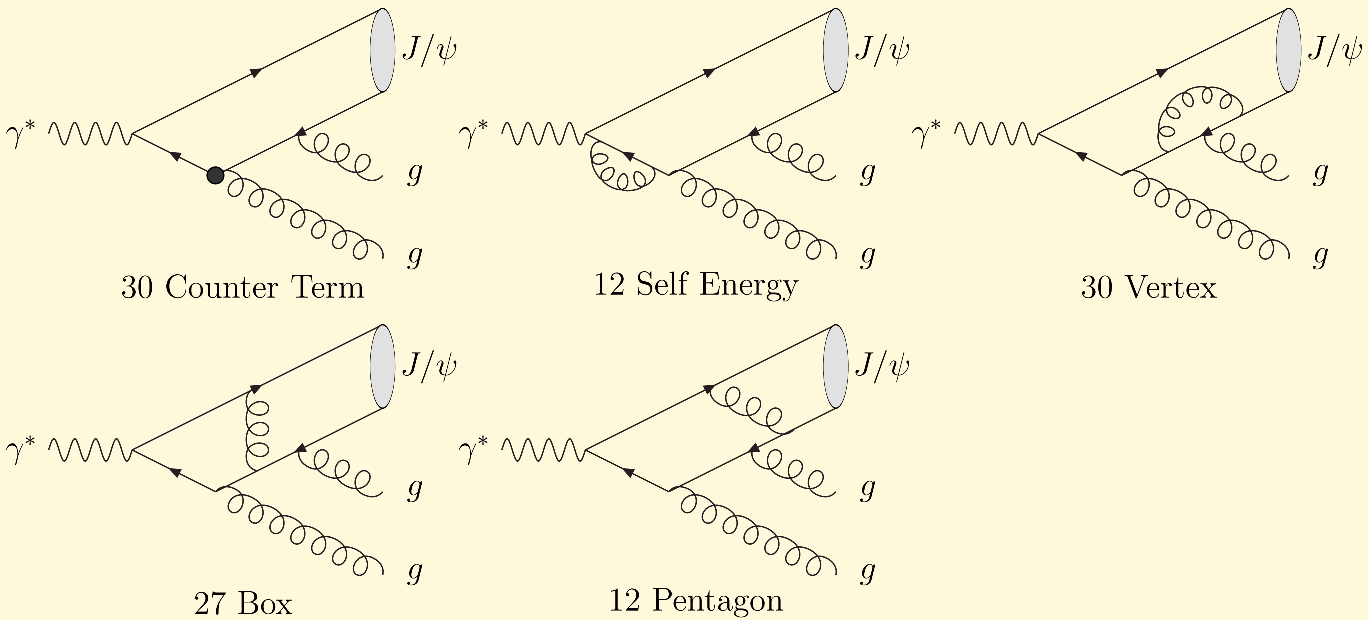


Half of leading order Feynman diagrams for $e^-(k_1)e^+(k_2) \to J/\psi(2p_1) + g(k_3) + g(k_4)$**.**

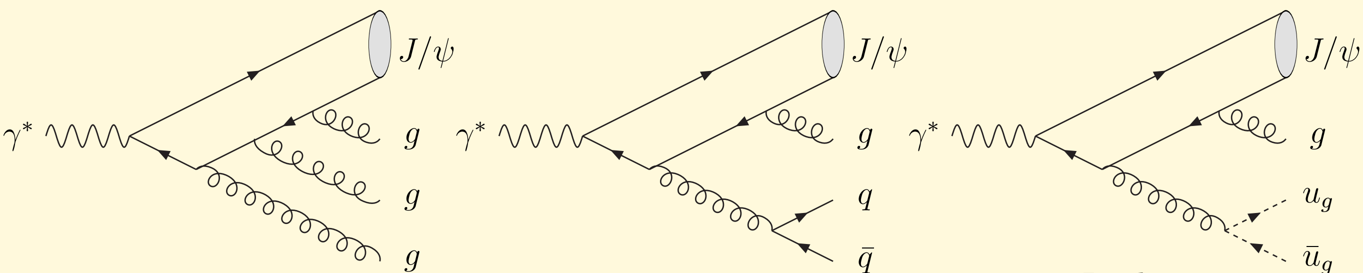
The production amplitude:

$$\begin{aligned}
&\mathcal{A}(a+b \to Q\bar{Q}(^{2S_\psi+1}L_{J_\psi})(2p_1) + g(k_3) + g(k_4) \\
&\quad = \sqrt{C_{L_\psi}} \sum_{L_{\psi z}S_{\psi z}} \sum_{s_1,s_2} \sum_{jk,il} \\
&\qquad \times \langle s_1; s_2 | S_\psi S_{\psi z}\rangle \langle L_\psi L_{\psi z}; S_\psi S_{\psi z} | J_\psi J_{\psi z}\rangle \langle 3j; \bar{3}k | 1\rangle \\
&\qquad \times \mathcal{A}(a+b \to Q_j(p_1) + \bar{Q}_k(p_1) + g(k_3) + g(k_4)) \qquad (5)
\end{aligned}$$

2.2. Next-to-Leading Order of $e^+e^- \to J/\psi + gg$



Virtual correction diagrams for $e^-e^+ \to J/\psi gg$**.**



42 Real ggg 6 Real $gq\bar{q}$ 6 Real $gu_g\bar{u}_g$

Real correction diagrams for $e^-e^+ \to J/\psi gg$.

2.3. Numerical Result of $e^+e^- \longrightarrow J/\psi + gg$

Input parameters: $|R_{J/\psi}(0)|^2 = 1.01\ \text{GeV}^3$, $m = 1.4$ GeV, $m_{J/\psi} = 2m$, $\Lambda_{\overline{MS}}^{(4)} = 338$ MeV. $\alpha_s(\mu) = 0.267$ for $\mu = 2m$, and the cross section at NLO in α_s is

$$\sigma(e^+e^- \to J/\psi gg) = 0.498\ \text{pb}, \tag{6}$$

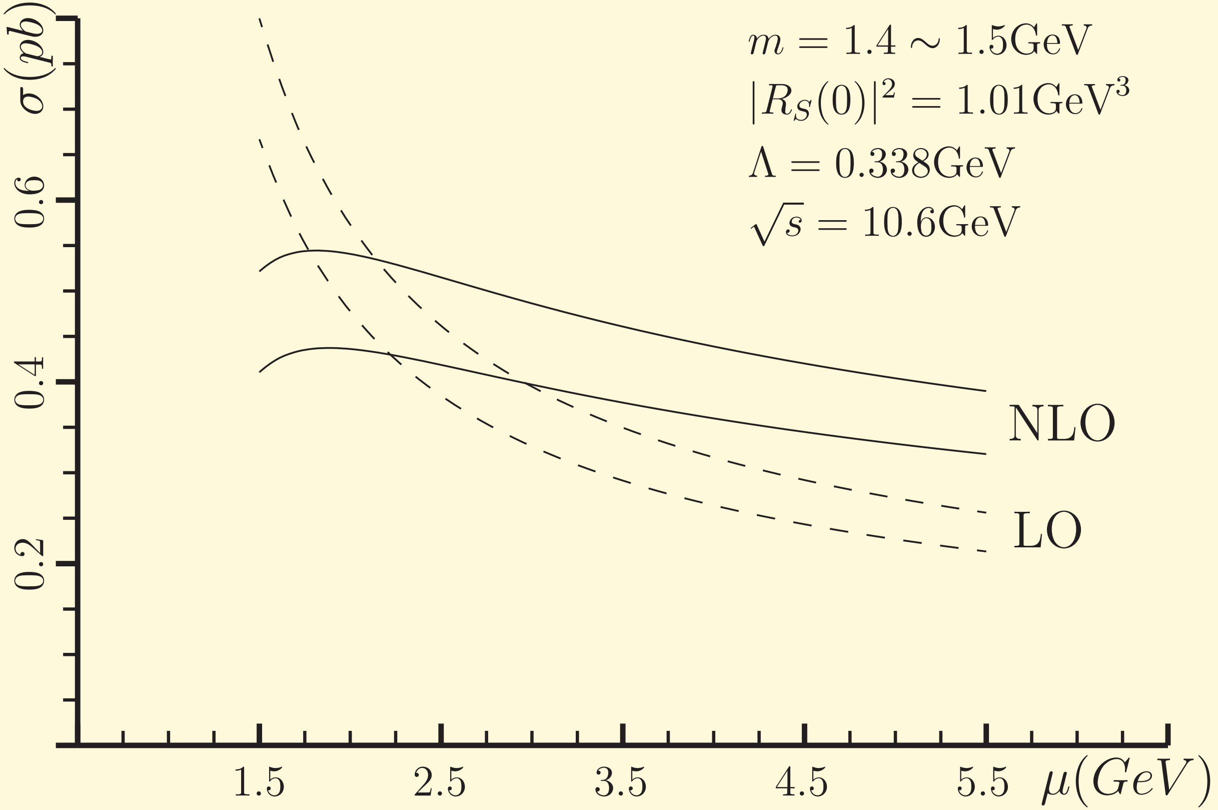
which is a factor of 1.19 larger then the LO cross section 0.418 pb.

In contrast to $\sigma(e^+e^- \to J/\psi c\bar{c})$ at NLO in α_s [*], where correction is much larger (K factor=1.8 for $m = 1.4$ GeV and $\mu = 2m$).

$\Longrightarrow R_{c\bar{c}} \approx$ 0.491 at NLO and 0.397 at LO.

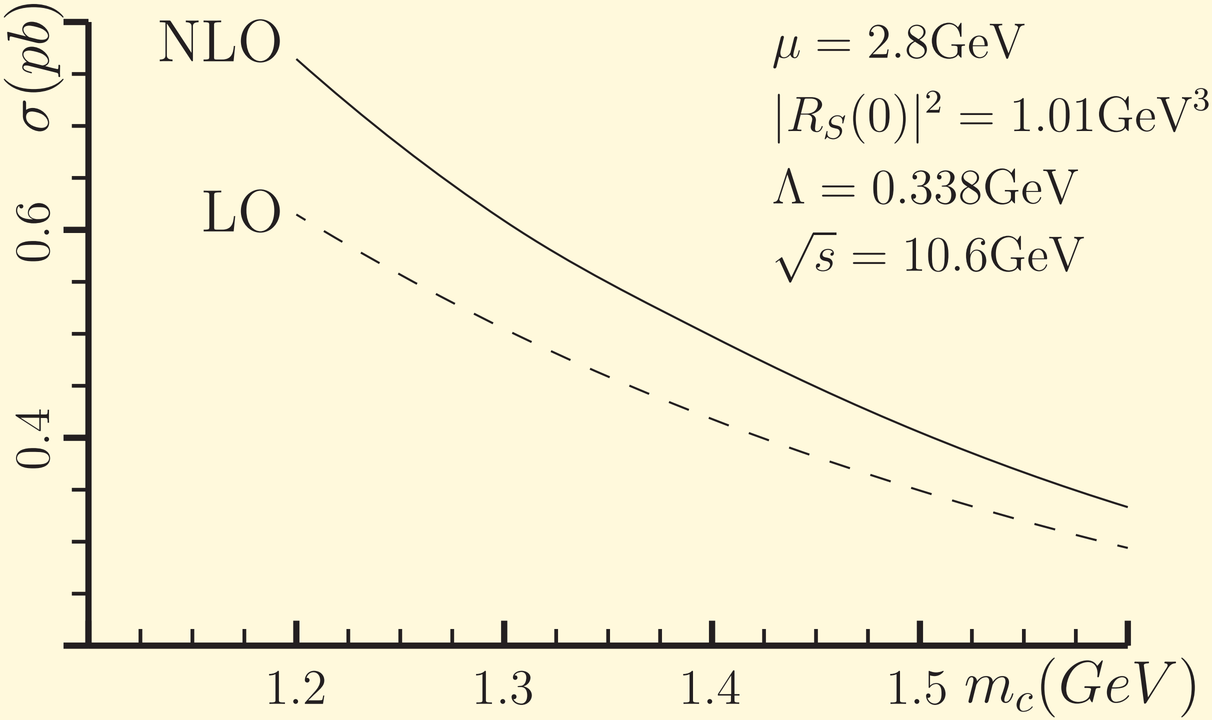
[*] Y. J. Zhang and K. T. Chao, Phys. Rev. Lett. **98**, 092003 (2007).

We see the NLO QCD correction improve the renormalization scale μ dependence of the cross sections substantially.



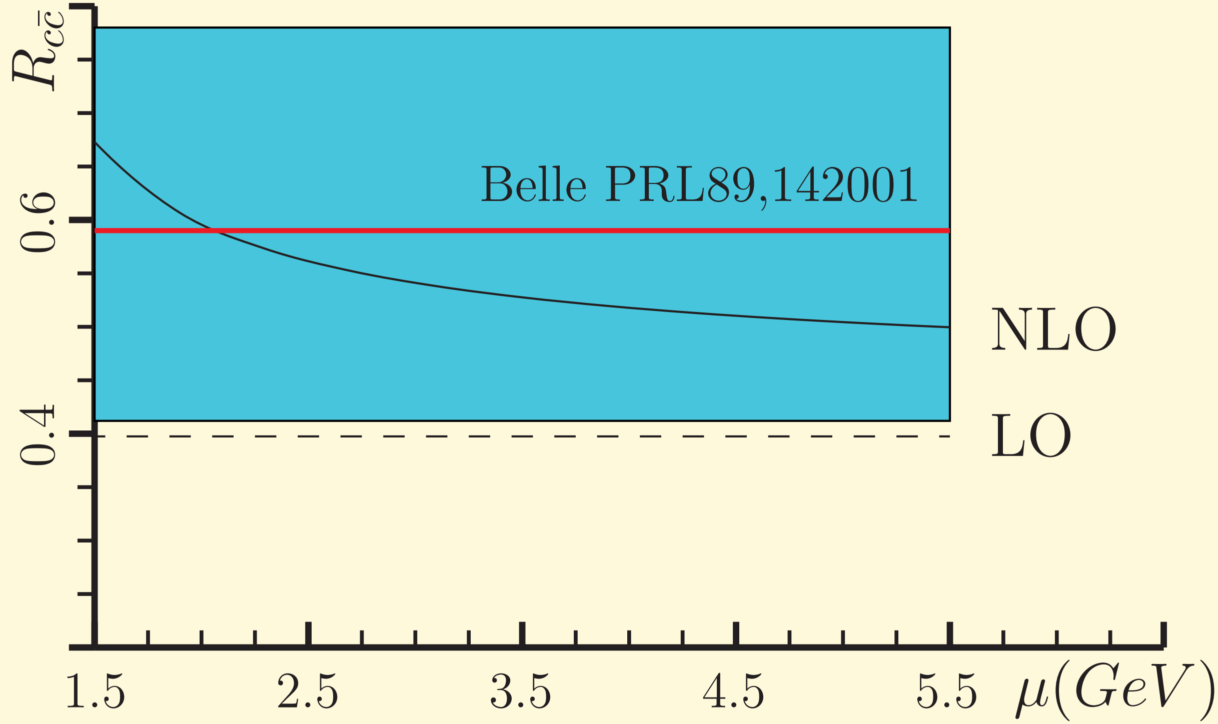
$\sigma[e^+e^- \to J/\psi gg]$ **as functions of the renormalization scale μ at LO and NLO in α_s.**

We see the NLO QCD correction enhances the cross sections by about a factor of 1.2 with existing a little theoretical uncertainty.



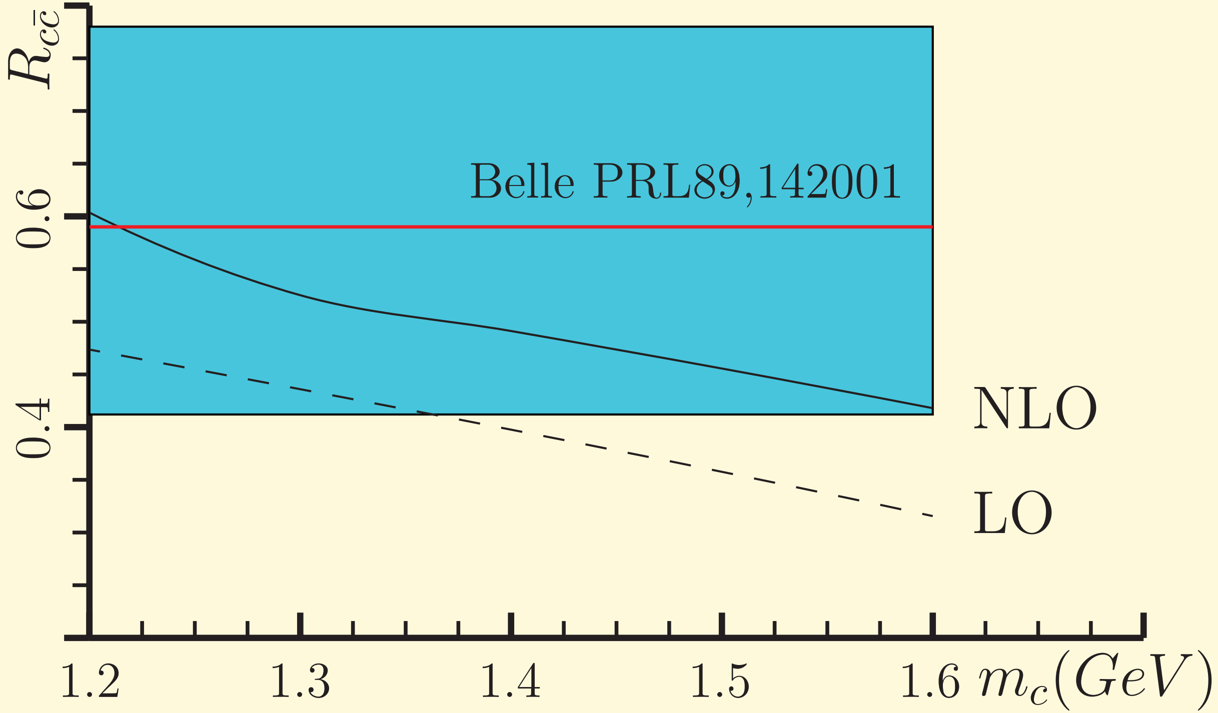
$\sigma[e^+e^- \to J/\psi gg]$ **as functions of the charm quark mass** m_c **at LO and NLO in** α_s**. Here** $\mu = 2.8$ **GeV.**

The LO $R_{c\bar{c}}$ is fix at 0.397 and much lower than the experiment data[?]. The NLO QCD corrections can enhance $R_{c\bar{c}}$ to the band of the experiment data.



$R_{c\bar{c}}$ **as functions of the renormalization scale** μ **at LO and NLO in** α_s**. Here** $m_c = 1.4$ **GeV.**

The NLO QCD corrections can enhance $R_{c\bar{c}}$ to the band of the experiment data.



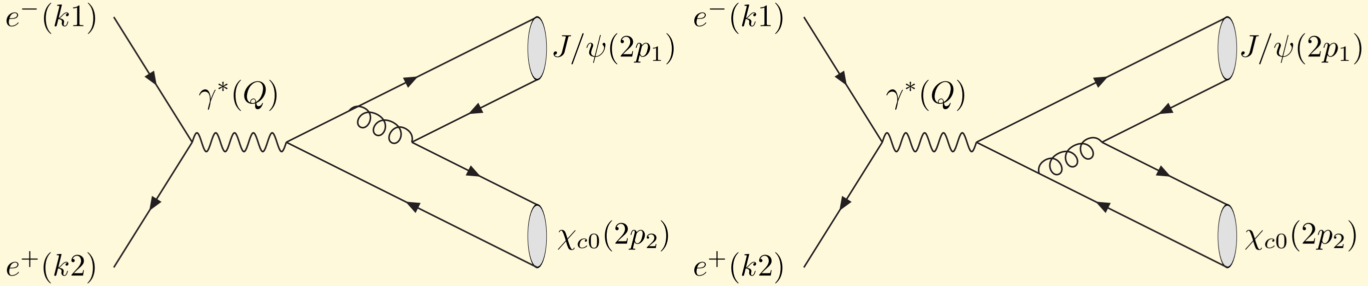
$R_{c\bar{c}}$ **as functions of the charm quark mass** m_c **at LO and NLO in** α_s**. Here** $\mu = 2.8$ **GeV.**

- To the prompt production cross section, the contribution of $\psi(2S)$ decay into J/ψ should be included. It enhance the cross section by a factor 0.355[*].
- If we select $m = 1.4$ GeV and $\mu = 2m$, the prompt production cross section of $\sigma(e^+e^- \to J/\psi gg)$ is 0.68 pb at NLO in α_s and 0.57 pb at LO.
- The prompt production cross section of $\sigma(e^+e^- \to J/\psi c\bar{c})$ is given in Ref.[*], which is 0.70 pb at NLO and 0.43 pb at LO (color octet contributions is excluded).
- Then we give $R_{c\bar{c}} = 0.51$ at NLO and $R_{c\bar{c}} = 0.43$ at LO.

[*] Y. J. Zhang and K. T. Chao, Phys. Rev. Lett. **98**, 092003 (2007).

3 $e^+e^- \to J/\psi + \chi_{c0}$

3.1. Leading Order of $e^+e^- \to J/\psi + \chi_{c0}$

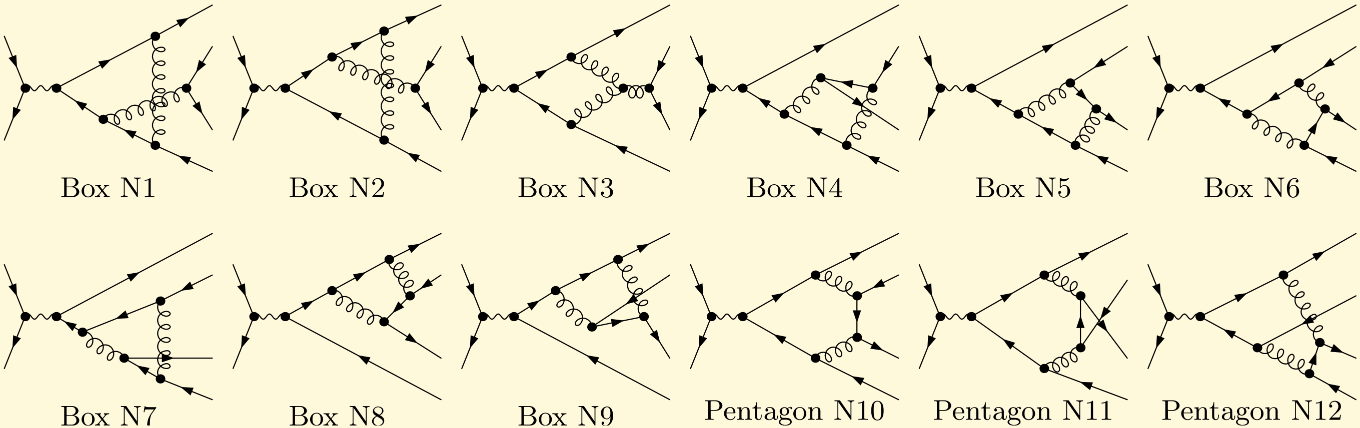


Half of leading order Feynman diagrams for $e^-(k_1)e^+(k_2) \to J/\psi(2p_1)\chi_{c0}(2p_2)$**.**

Using the NRQCD factorization formalism, we can write down the scattering amplitude :

$$
\begin{aligned}
&\mathcal{A}(a+b \to Q\bar{Q}(^{2S_\psi+1}L_{J_\psi})(2p_1) + Q\bar{Q}(^{2S_{\chi_{c0}}+1}L_{J_{\chi_{c0}}})(2p_2)) \\
&= \sqrt{C_{L_\psi} C_{L_{\chi_{c0}}}} \sum_{L_{\psi z} S_{\psi z}} \sum_{L_{\chi_{c0} z} S_{\chi_{c0} z}} \sum_{s_1 s_2, s_3 s_4} \sum_{jk,il} \\
&\quad \times \langle s_1; s_2 | S_\psi S_{\psi z}\rangle \langle L_\psi L_{\psi z}; S_\psi S_{\psi z} | J_\psi J_{\psi z}\rangle \langle 3j; \bar{3}k | 1\rangle \\
&\quad \times \langle s_3; s_4 | S_{\chi_{c0}} S_{\chi_{c0} z}\rangle \langle L_{\chi_{c0}} L_{\chi_{c0} z}; S_{\chi_{c0}} S_{\chi_{c0} z} | J_{\chi_{c0}} J_{\chi_{c0} z}\rangle \langle 3l; \bar{3}i | 1\rangle \\
&\quad \times \varepsilon^{*\alpha} \frac{\partial}{\partial q^\alpha} \mathcal{A}(a+b \to Q_j(p_1) + \bar{Q}_k(p_1) + Q_l(p_2+q) + \bar{Q}_i(p_2-q))
\end{aligned}
\tag{7}
$$

3.2. Next-to-Leading Order of $e^+e^- \to J/\psi + \chi_{c0}$



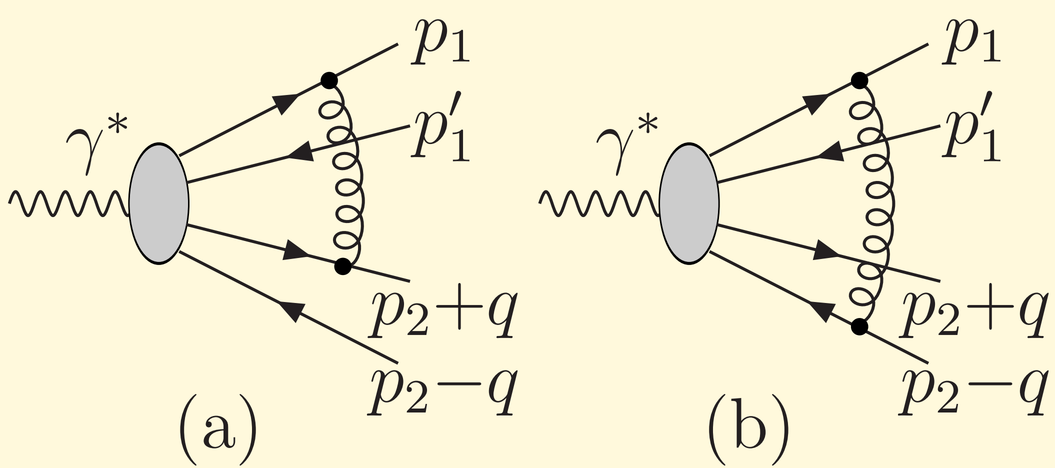
Half of box diagrams for $e^-(k_1)e^+(k_2) \to J/\psi(2p_1)\chi_{c0}(2p_2)$**.**

The IR-divergence terms can be calculated like $J/\psi+\eta_c$. Then all the IR-divergence terms become $C_0[p_1,-p_2,0,m,m]$, $C_0[p_{1c},-p_{1\bar{c}},0,m,m]$, and $\frac{\partial C_0[p_1,q-p_2,0,m,m]}{\partial q}|_{q=0}$. Then we find that

- $\mathrm{Box\,N3, N6, N7}$ and $\mathrm{PentagonN12}$ are IR-finite respectively
- $\mathrm{Box\,N1+N4}$ and $\mathrm{Box\,N2+N9}$ is IR-finite
- IR-divergence term of $\mathrm{Pentagon\,N11}$ is canceled by $\mathrm{VertexN7+N9}$
- IR-divergence and Coulomb-divergence terms of $\mathrm{Box\,N5+N8}$ and $\mathrm{Pentagon\,N10}$ are canceled by counter terms
- Coulomb singularity is canceled by next-to-leading order of matrix element.

- UV term is canceled by counter terms.
- Then the result is UV-, IR-, and Coulomb-finite.

3.3. Fictorization of $e^+e^- \to J/\psi + \chi_{c0}$



Half of the diagrams for one-loop virtual IR corrections with two charm quark pairs $c(p_1)\bar{c}(p_1')$ and $c(p_2+q)\bar{c}(p_2-q)$. The other diagrams can be obtained by replacing $c(p_1)$ with $\bar{c}(p_1')$.

The IR term for $c(p_1)$ and $c(p_2+q)$ shown in(a) is proportional to $p_1 \cdot (p_2+q)C_0[-p_1, p_2+q_1, 0, m, m]$ [*]. It is proportional to $-p_1 \cdot (p_2-q)C_0[-p_1, p_2-q, 0, m, m]$ for $c(p_1)$ and $\bar{c}(p_2-q)$ as shown in (b).

[*] G.C. Nayak, J.W. Qiu and G. Sterman, Phys. Rev. Lett. 99, 212001 (2007); arXiv:0711.3476 [hep-ph]; S. Dittmaier, Nucl. Phys. B **675**, 447 (2003).

Then for the charm quark $c(p_1)$ associated with a colorless charm quark pair $c(p_2+q)\bar{c}(p_2-q)$, the IR term becomes

$$
\begin{aligned}
& \mathcal{M}^{IR}_{NLO}[c(p_1)+c(p_2+q)\bar{c}(p_2-q)] \\
\propto\quad & (p_2+q)\cdot p_1 C_0[p_2+q,-p_1,0,m,m]-(p_2-q)\cdot p_1 C_0[p_2-q,-p_1,0,m,m] \\
=\quad & 0+2q^\alpha\left[p_{1\alpha}C_0[p_2,-p_1,0,m,m]+p_2\cdot p_1\frac{\partial C_0[p_2+q,-p_1,0,m,m]}{\partial q^\alpha}\Big|_{q=0}\right]+\mathcal{O}(q^2), \qquad (8)
\end{aligned}
$$

where it is expanded in powers of the relative momentum q at $q=0$.

The IR terms between $c(p_2+q)$ and $\bar{c}(p_2-q)$ or between $c(p_1)$ and $\bar{c}(p_1')$ are ignored since these $c\bar{c}$ pairs should evolve to bound states at large distances.

$$\mathcal{M}^{IR}_{NLO}[c(p_1)+c(p_2+q)\bar{c}(p_2-q)]$$
$$\propto 0+2q^{\alpha}\Big[p_{1\alpha}C_0[p_2,-p_1,0,m,m]+p_2\cdot p_1\frac{\partial C_0[p_2+q,-p_1,0,m,m]}{\partial q^{\alpha}}\Big|_{q=0}\Big]+\mathcal{O}(q^2),$$

- It is IR finite when an individual charm quark in χ_{c0} connects with both legs of the J/ψ, which is a S-wave state and can set $q=0$ at LO in v, and this corresponds to the zeroth order i.e. the $\mathcal{O}(q^0)$ term in Eq.(8).
- But it becomes IR divergent when a charm quark connects individually with both legs of χ_{c0}, since the relative momentum in the P-wave state has to be retained, corresponding to the first order i.e. the $\mathcal{O}(q^1)$ term in Eq.(8).It is just the reason that $B\to\chi_{c0}K$ is nonfactorizable [*].
- So, the cancelation of IR divergencies depends on the existence of an associated S-wave $c\bar{c}$ state.

[*] Z.Z. Song, C. Meng, Y.J. Gao and K.T. Chao, Phys. Rev. D **69**, 054009 (2004); Z.Z. Song and K.T. Chao, Phys. Lett. B **568**, 127 (2003).

3.4. Numerical Result of $e^+e^- \to J/\psi + \chi_{c0}$

To be consistent with the NLO result the values of wave functions squared at the origin should be extracted from the leptonic width of $J/\psi(\psi(2S))$ and the two-photon width of χ_{c0} at NLO in α_s:

$$
\begin{aligned}
|R_S(0)|^2 &= \frac{9m^2_{J/\psi}}{16\alpha^2(1-4C_F\alpha_s/\pi)}\Gamma(J/\psi \to e^+e^-) \\
|R'_P(0)|^2 &= \frac{3m^4_{\chi_{c0}}}{256\alpha^2(1+(3\pi^2-28)\alpha_s/(9\pi))}\Gamma(\chi_{c0} \to \gamma\gamma) \qquad (9)
\end{aligned}
$$

Using PDG 2006 data[*], we obtain $|R_{1S}(0)|^2 = 1.01\mathrm{GeV}^3$, $|R_{2S}(0)|^2 = 0.639\mathrm{GeV}^3$, $|R'_{1P}(0)|^2 = 0.0575\mathrm{GeV}^5$.

[*] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).

Taking $\Lambda_{\overline{\mathrm{MS}}}^{(4)} = 338\mathrm{MeV}$, $m_{J/\psi} = m_{\chi_{c0}} = 2m$ (in the nonrelativistic limit),

- If we set $m = 1.5\ GeV$ and $\mu = 2m$, the cross section at next-to-leading order of α_s is

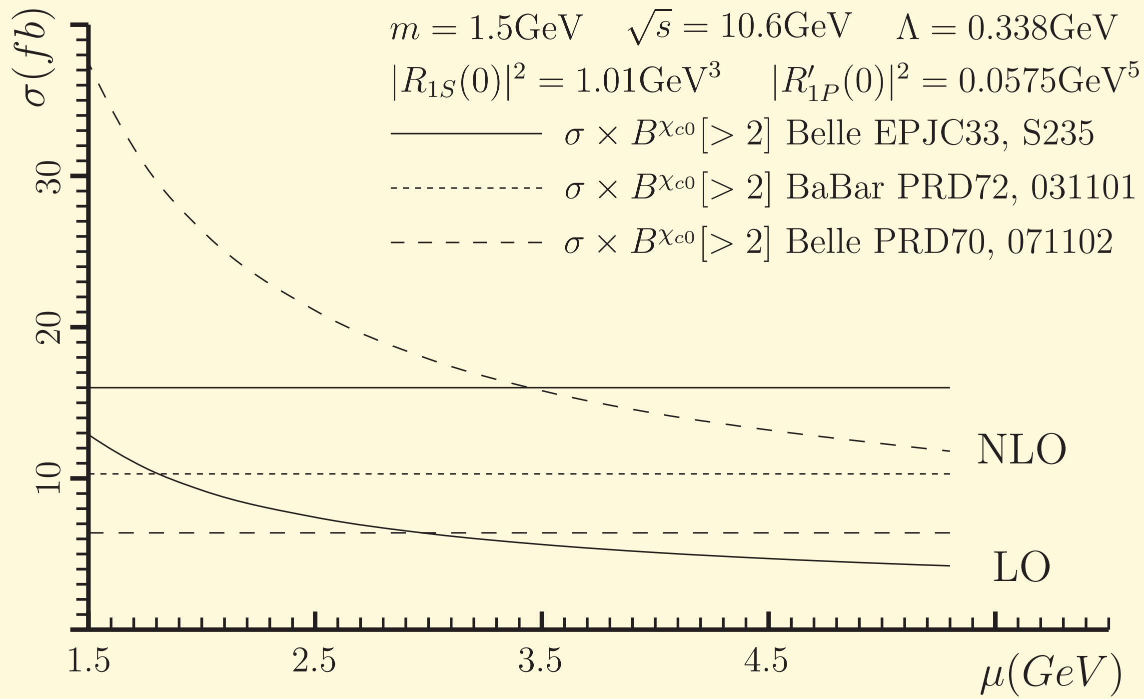
$$\begin{aligned} \sigma(e^+ + e^- \to J/\psi + \chi_{c0}) &= 17.9\mathrm{fb}, \\ \sigma(e^+ + e^- \to \psi(2S) + \chi_{c0}) &= 11.3\mathrm{fb}, \end{aligned} \tag{10}$$

  which are a factor of 2.8 larger than the LO cross sections $6.35(4.02)$ fb for $J/\psi(\psi(2S))$.

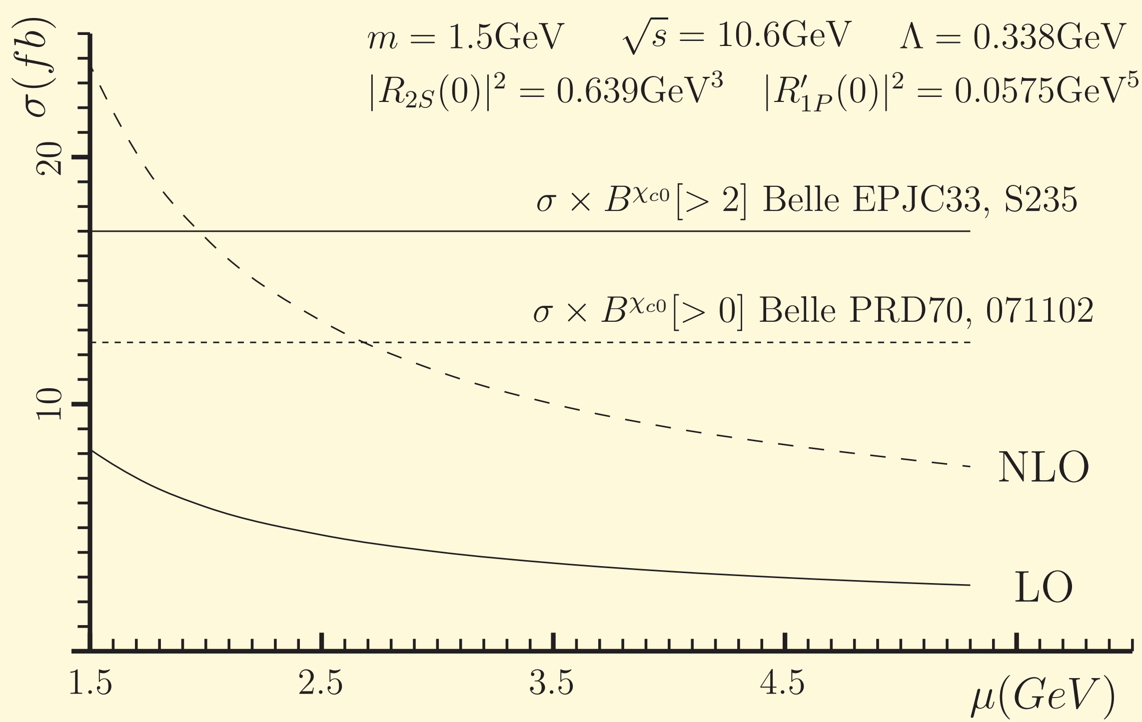
- If we use the BLM scale[*], we get $\mu_{BLM} = e^{-5/6}\sqrt{s}/2 = 2.30\mathrm{GeV}$, $\alpha_s = 0.291$, and the corresponding cross section is

$$\begin{aligned} \sigma(e^+ + e^- \to J/\psi + \chi_{c0}) &= 22.8\mathrm{fb}, \\ \sigma(e^+ + e^- \to \psi(2S) + \chi_{c0}) &= 14.4\mathrm{fb}, \end{aligned} \tag{11}$$

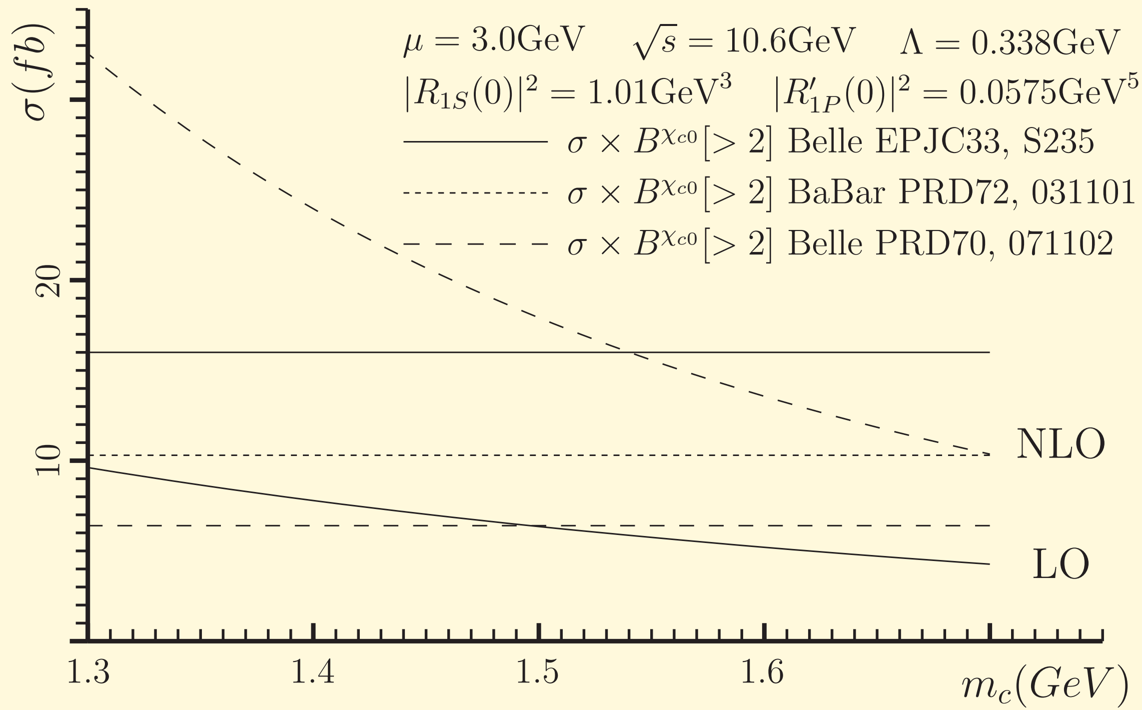
  which are a factor of 2.8 larger than the LO cross sections $8.02(5.08)$ fb for $J/\psi(\psi(2S))$.



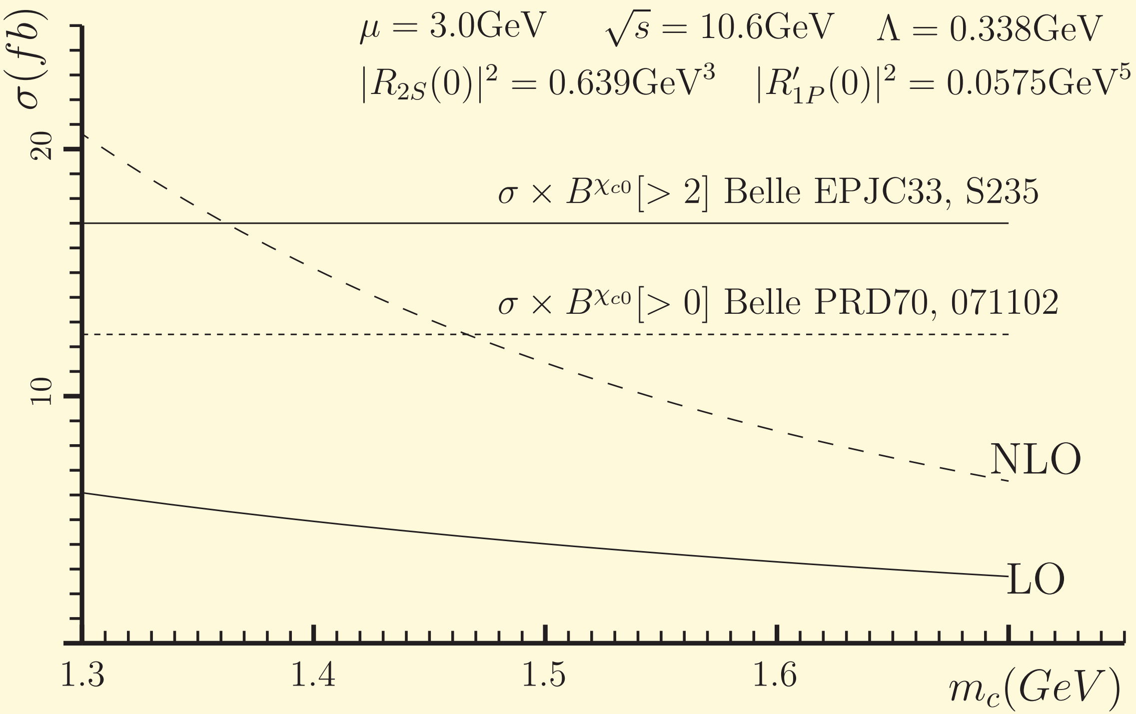
Cross section of $e^+e^- \to J/\psi + \chi_{c0}$ as functions of the renormalization scale μ.



Cross section of $e^+e^- \to \psi(2S) + \chi_{c0}$ as functions of the renormalization scale μ.



Cross section of $e^+e^- \to J/\psi + \chi_{c0}$ **as functions of the charm quark mass**



Cross section of $e^+e^- \to \psi(2S) + \chi_{c0}$ **as functions of the charm quark mass.**

Table: Experimental and theoretical cross sections of $e^+e^- \to J/\psi(\psi(2S)) + \chi_{c0}$ at B factories in units of fb. We use $|R_{1S}(0)|^2 = 1.01\text{GeV}^3$, $|R_{2S}(0)|^2 = 0.639\text{GeV}^3$, $|R'_{1P}(0)|^2 = 0.0575\text{GeV}^5$, $\Lambda = 0.338\text{GeV}$, $\sqrt{s} = 10.6\text{GeV}$, $m_c = 1.5\text{GeV}$, and $\mu = 2m_c$. The experimental data are the cross sections times the branching fraction for χ_{c0} decay into more than 2 charged tracks. But the Belle's data of $\psi(2S) + \chi_{c0}$ in Ref.[**] corresponds to χ_{c0} decay into at least 1 charged tracks.

	$J/\psi + \chi_{c0}$	$\psi(2S) + \chi_{c0}$
Belle $\sigma \times B^{\chi_{c0}}[>2]$[*]	$16 \pm 5 \pm 4$	$17 \pm 8 \pm 7$
Belle $\sigma \times B^{\chi_{c0}}[>2(0)]$[**]	$6.4 \pm 1.7 \pm 1.0$	$12.5 \pm 3.8 \pm 3.1$
BaBar $\sigma \times B^{\chi_{c0}}[>2]$[***]	$10.3 \pm 2.5^{+1.4}_{-1.8}$	-
Braaten and Lee [****]	2.4	1.0
Liu He and Chao [*****]	6.7	4.4
Braguta et al. [******]	14.4	7.8
Our LO result	6.35	4.02
Our NLO result	17.9	11.3

[*] T.V. Uglov, Eur. Phys. J. C **33**, S235 (2004).

[**] K. Abe *et al.*,(Belle Collaboration), Phys.Rev. D70 (2004) 071102.K. Abe *et al.*, arXiv:hep-ex/0507019.

[***]B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **72**, 031101 (2005)

[****]E. Braaten and J. Lee, Phys. Rev. D **67**, 054007 (2003)

[*****]K. Y. Liu, Z. G. He and K. T. Chao, Phys. Lett. B **557**, 45 (2003).

[******] V. V. Braguta, A. K. Likhoded and A. V. Luchinsky, Phys. Lett. B **635**, 299 (2006)

4 Conclusions

We get the cross section of $e^+e^- \to J/\psi gg$ and $e^+e^- \to J/\psi + \chi_{c0}$ at $\sqrt{s} = 10.6$ GeV at the NLO of α_s and LO in v^2.

- If we select $m = 1.4$ GeV and $\mu = 2m$, the prompt production cross section of $\sigma(e^+e^- \to J/\psi gg)$ is 0.68 pb at NLO in α_s and 0.57 pb at LO.
- At NLO in α_s with $m = 1.4$ GeV and $\mu = 2m$, the sum of $\sigma(e^+e^- \to J/\psi gg)$ and $\sigma(e^+e^- \to J/\psi c\bar{c})$ is about 1.4 pb, close to the central value of 1.47 pb for the J/ψ inclusive production cross section observed by Belle. However, we must pay attention to the large uncertainties, especially the μ dependence.
- We give $R_{c\bar{c}} = 0.51$ at NLO and $R_{c\bar{c}} = 0.43$ at LO. The NLO value for R is also close to the data.

- NRQCD factorization holds for the double charmonium exclusive production $e^+e^- \rightarrow J/\psi(\psi(2S))\chi_{c0}$ at NLO in α_s and LO in v, where the P-wave state χ_{c0} is associated with an S-wave state J/ψ or $\psi(2S)$.
- The NLO QCD corrections can substantially enhance the cross sections with a K factor (the ratio of NLO to LO) of about 2.8. With $m = 1.5\text{GeV}$ and $\mu = 2m$, the NLO cross sections of $e^+e^- \rightarrow J/\psi(\psi(2S))\chi_{c0}$ are estimated to be 17.9(11.3) fb, which reach the lower bounds of experiment.

Thanks!