Optimal spin quantization axes for dileptons and quarkonium

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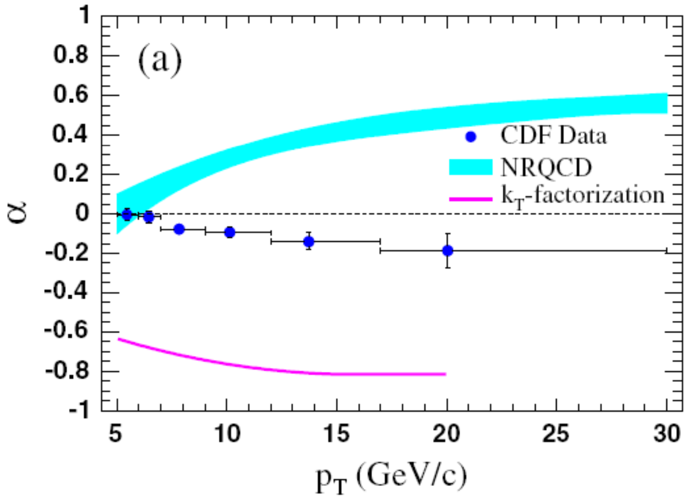
Outline

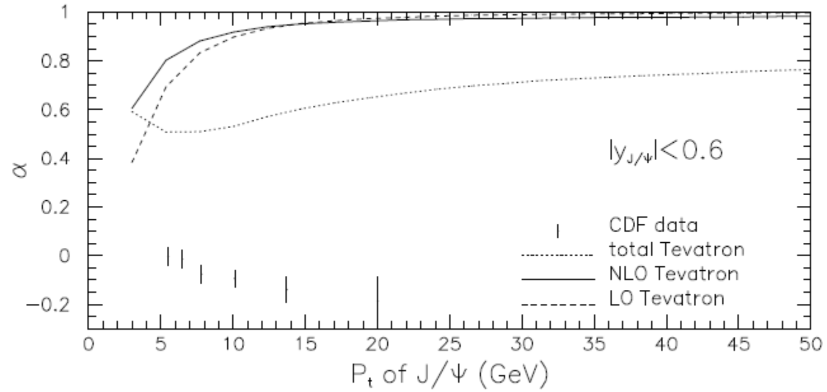
- Motivations
- Spin quantization axis (SQA)
- Optimal SQA's for Drell-Yan process
- Optimal SQA's for dileptons at large Q_T
- Optimal SQA's for quarkonium at large Q_T
- Conclusions

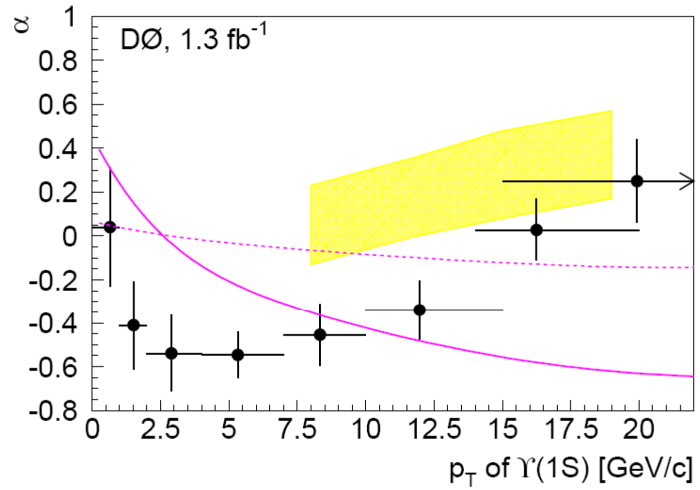
Motivations



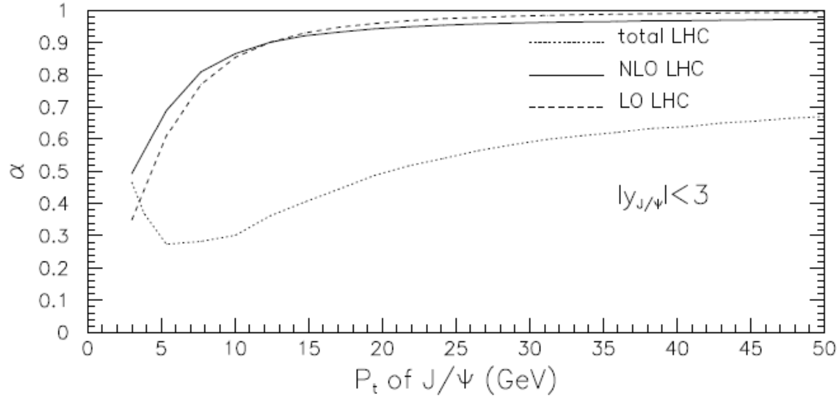
CDF, PRL99, 132001 (2007)

$$\alpha = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}$$





D0, arXiv:0804.2799



Gong, Wang, arXiv:0805.4751

See talks by J.-X. Wang, J.-P. Lansberg, and P. Faccioli.

Motivations

- Spin carries important information about the fundamental interactions.

  - cannot be measured directly.

  - can be inferred from angular distributions of decay products with respect to *spin quantization axis*.

- Accessibility of the information depends on the choice of the spin quantization axis.

- An additional complication in hadron collisions.

  - involve collisions of partons with varying longitudinal momentum.

- In principle, the information can be obtained by measuring the complete angular distributions.

- In practice, not enough data.

Which spin quantization axis will maximize that information?

- Address this question for the simplest process.

Production of a dilepton from decay of a virtual photon.

- Then extend to production of quarkonium.

Spin quantization axis (SQA)

- Most general longitudinal polarization vector ϵ_L

$$\epsilon_L^\mu = \frac{\tilde{X}^\mu}{\sqrt{-\tilde{X}^2}},$$
$$\tilde{X}^\mu = \left(-g^{\mu\nu} + \frac{Q^\mu Q^\nu}{Q^2}\right) X_\nu.$$

$\boxed{Q : \text{dilepton momentum}}$

- The physical interpretation of X is

  - in the rest frame of Q, the SQA is the direction of the 3-vector $-\boldsymbol{X}$.

  - if $X^2 > 0$, the projection of the spin along the SQA is identical to the helicity in the rest frame of X.

Spin quantization axis (SQA)

- SQA in collision plane of colliding hadrons

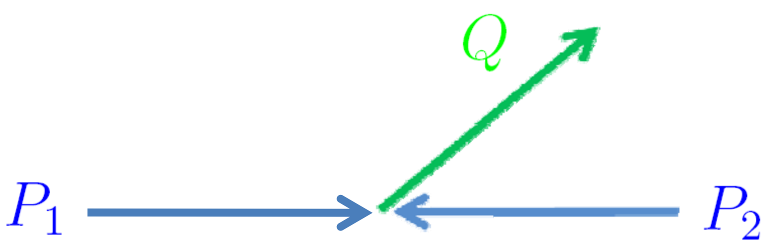
$$X^\mu = aP_1^\mu + bP_2^\mu .$$

a, b : scalar functions
P_i : hadron momentum

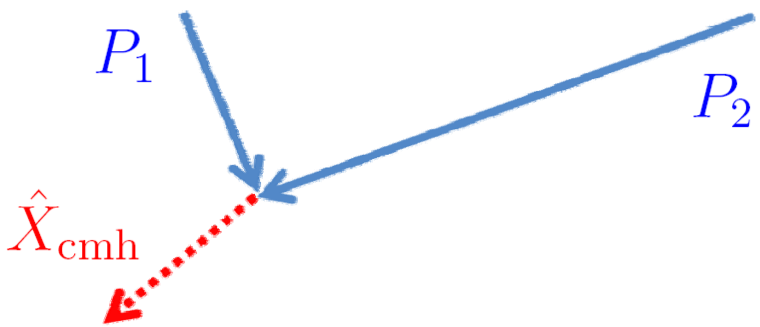
- ϵ_L is determined by the ratio a/b.

- *c.m. helicity axis*

$$X_{\rm cmh}^\mu = P_1^\mu + P_2^\mu .$$



hadron c.m. frame
spin = helicity



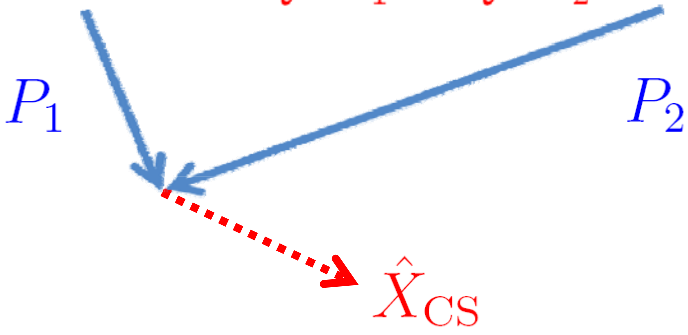
rest frame of Q

Spin quantization axis (SQA)

- *Collins-Soper axis*

Collins, Soper, PRD16, 2219 (1977)

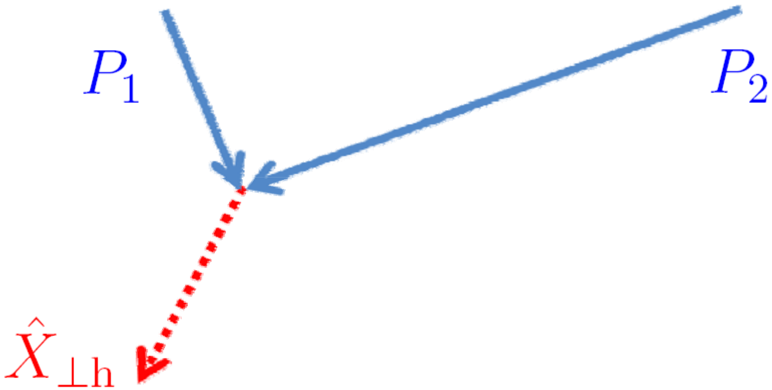
$$X^{\mu}_{\mathrm{CS}} = \frac{P_1^{\mu}}{Q\cdot P_1} - \frac{P_2^{\mu}}{Q\cdot P_2}.$$



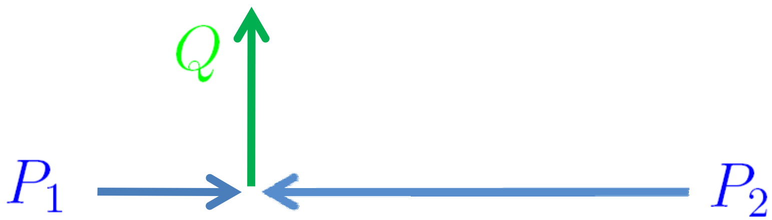
rest frame of Q

- *perpendicular helicity axis*

$$X^{\mu}_{\perp\mathrm{h}} = \frac{P_1^{\mu}}{Q\cdot P_1} + \frac{P_2^{\mu}}{Q\cdot P_2}.$$



rest frame of Q



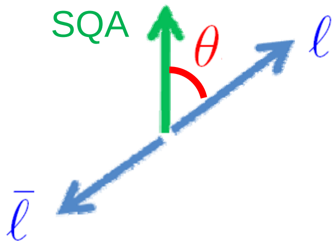
perpendicular frame of Q

spin = helicity

Spin quantization axis (SQA)

- The differential cross section for a dilepton

$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha}{4\pi}\int\frac{dQ^2}{Q^2}\left[\sigma_T\frac{1+\cos^2\theta}{2}+\sigma_L(1-\cos^2\theta)\right].$$



rest frame of Q

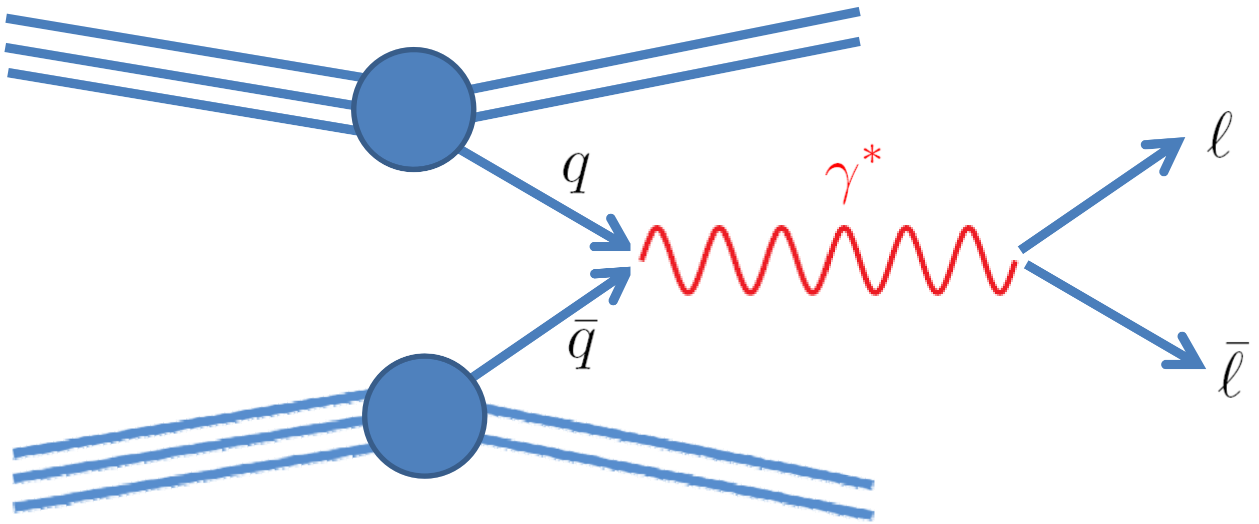
- Our question can be restated:

For which SQA is the virtual photon most strongly polarized?

Drell-Yan process

- Production of a dilepton with large invariant mass.

Drell, Yan, PRL25, 316 (1970)



- Collinear parton model:

$$\hat{\sigma} = \hat{\sigma}_T = \frac{4\pi^2 e_q^2 \alpha}{3} \delta(x_1 x_2 s - Q^2).$$

- virtual photon is transversely polarized for any SQA.

Drell-Yan process

- A rigorous QCD calculation requires resumming the effects of the soft gluon emission from the colliding partons.

Collins, Soper, Sterman, NPB250, 199 (1985)

- gives transverse momenta to the colliding partons.

- Parton model with intrinsic transverse momentum :

$$\hat{\sigma}_L = \frac{8\pi e_q^2 \alpha \langle k_\perp^2 \rangle (a^2 x_2^2 + b^2 x_1^2)}{3Q^2 (a x_2 - b x_1)^2} \delta(x_1 x_2 s - Q^2).$$

- longitudinal cross section depends on the ratio a/b.
- minimized by choosing $\frac{a}{b} = -\frac{x_1}{x_2}$.
- maximized by choosing $\frac{a}{b} = +\frac{x_1}{x_2}$.

Drell-Yan process

- Determine x_1/x_2 from parton kinematics

$$\frac{x_1}{x_2} = \frac{Q \cdot P_2}{Q \cdot P_1}.$$

- at leading order in the intrinsic transverse momentum.

- The maximal and minimal SQA's are

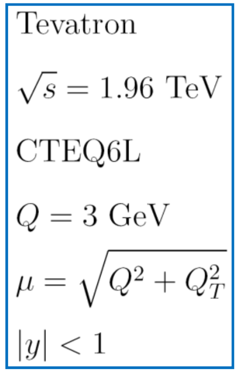
$$X^\mu_{\rm CS} = \frac{P_1^\mu}{Q \cdot P_1} - \frac{P_2^\mu}{Q \cdot P_2},$$
$$X^\mu_{\perp\rm h} = \frac{P_1^\mu}{Q \cdot P_1} + \frac{P_2^\mu}{Q \cdot P_2}.$$

- the Collins-Soper axis maximizes the transverse polarization.

Lam, Tung, PRD18, 2447 (1978)

- the perpendicular helicity axis minimizes the transverse polarization.

Optimal SQA's for dileptons at large Q_T

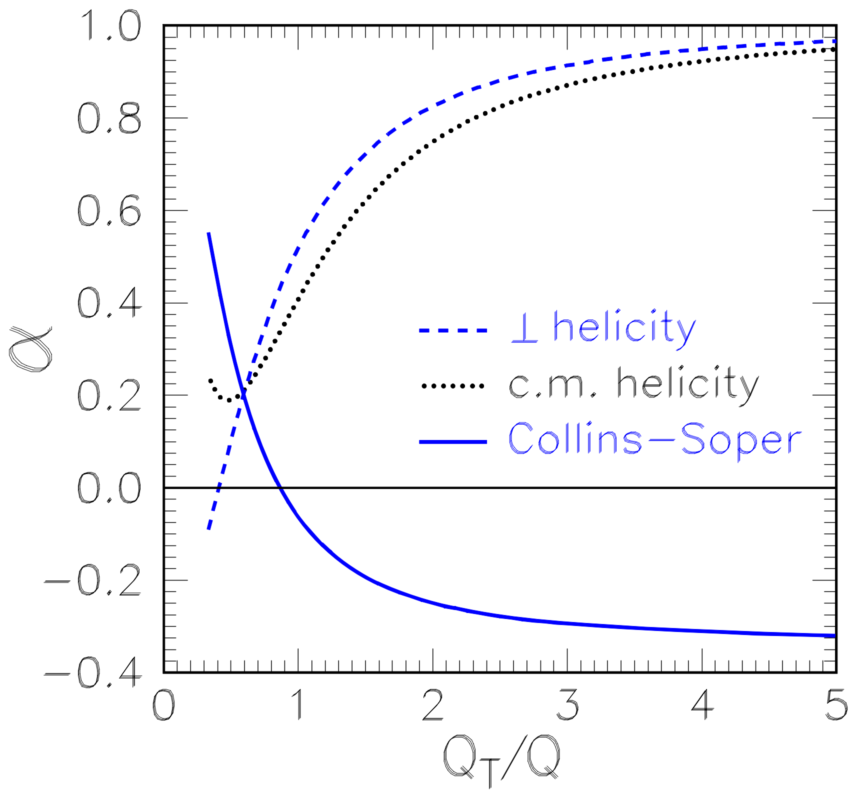


- Production of a dilepton with $Q_T \gg \Lambda_{\rm QCD}$.
- A convenient polarization variable is

$$\alpha = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}$$

- Polarization depends on choice of SQA.

- LO parton processes
  - $q\bar{q} \rightarrow \gamma^* g$
  - $qg \rightarrow \gamma^* q$
  - $\bar{q}g \rightarrow \gamma^* \bar{q}$



Optimal SQA's for dileptons at large Q_T

$\boxed{i, j, k : q, \bar{q}, g}$

- *Optimal* ij *axes*: minimize/maximize $\hat{\sigma}_L$ for $ij \to \gamma^* k$.
- at Tevatron and LHC, dominant parton process is

$$qg \longrightarrow \gamma^* + q.$$

- Longitudinal cross section

$$d\hat{\sigma}_L \propto \frac{(ax_2 - bx_1)^2 + b^2 x_1^2}{(aw_1 + bw_2)^2 - abQ^2 s}.$$

$$\boxed{\begin{aligned} w_1 &= Q \cdot P_1, \\ w_2 &= Q \cdot P_2. \end{aligned}}$$

- minimize/maximize with respect to a/b.

$$\frac{a}{b}\bigg|_{qg\to\gamma^* q} = \frac{2x_1^2 w_1^2 - x_2^2 w_2^2 \pm Z^{1/2}}{x_2(2x_1 w_1^2 + 2x_2 w_1 w_2 - x_2 Q^2 s)},$$

$$Z = (2x_1^2 w_1^2 + x_2^2 w_2^2 + 2x_1 x_2 w_1 w_2 - x_1 x_2 Q^2 s)^2 - x_1^2 x_2^2 Q^2 s(4w_1 w_2 - Q^2 s).$$

- a/b depends on x_1 and x_2.

Optimal SQA's for dileptons at large Q_T

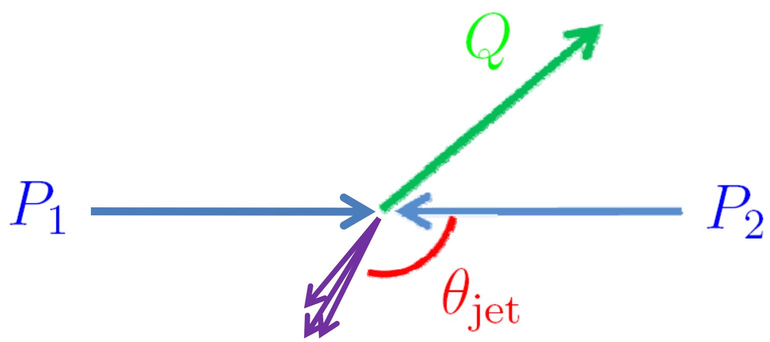
• Optimal qg axes depend on the longitudinal momentum fractions x_1 and x_2.

• Determine x_1 and x_2 from parton kinematics

$$2x_1 P_1 \cdot Q + 2x_2 P_2 \cdot Q = x_1 x_2 s + Q^2$$

and from direction of jet with balancing p_T

$$\frac{x_1}{x_2} = \frac{(Q_0 + Q_L)\sin\theta_{\rm jet} + Q_T(1+\cos\theta_{\rm jet})}{(Q_0 - Q_L)\sin\theta_{\rm jet} + Q_T(1-\cos\theta_{\rm jet})}.$$



hadron c.m. frame

Optimal SQA's for dileptons at large Q_T

- Polarization from LO parton cross sections.

Tevatron

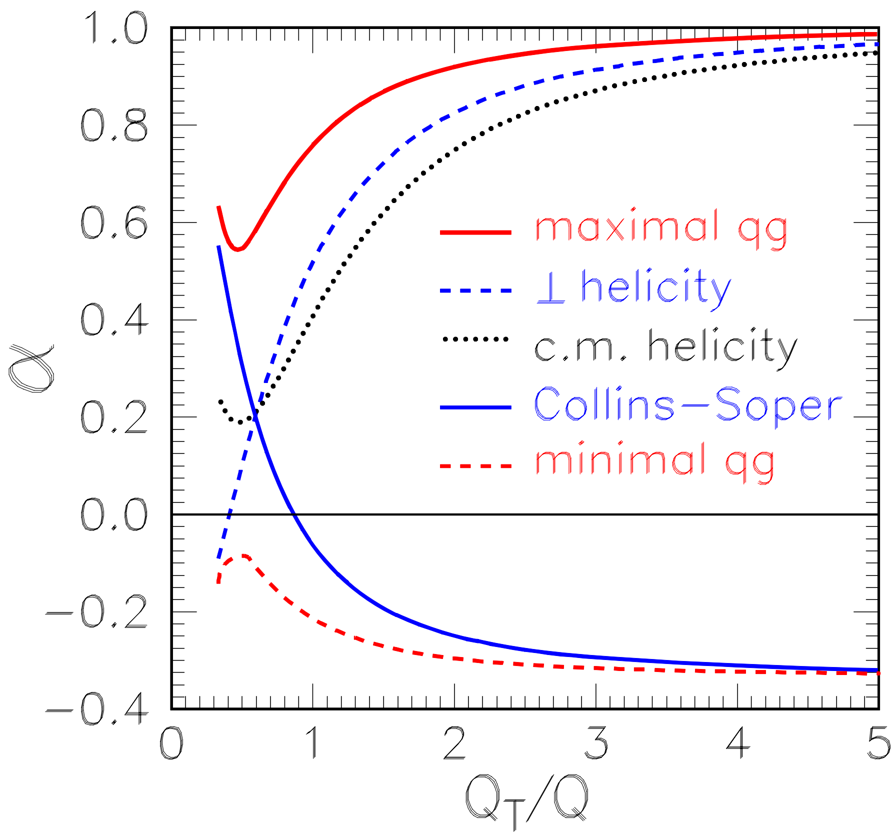
$\sqrt{s} = 1.96$ TeV

CTEQ6L

$Q = 3$ GeV

$\mu = \sqrt{Q^2 + Q_T^2}$

$|y| < 1$



- difference between $\perp$ helicity and C-S axes increases with Q_T.

- difference between maximal and minimal qg axes increases more rapidly.

Optimal SQA's for quarkonium at large Q_T

- Tension between measurements at Tevatron and NRQCD predictions for polarization of quarkonium.
- NRQCD with LO parton cross sections:

  - a direct J/ψ or Υ should be increasingly transversely polarized as Q_T increases.

    - at asymptotically large Q_T, gluon fragmentation $(gg \to gg, g \to c\bar{c})$ dominates.
    - predominantly transversely polarized for almost any choice of SQA.
    - heavy-quark spin symmetry implies binding effects do not change spin states.

- CDF and D0 collaborations:

  - unpolarized or longitudinally polarized at large Q_T.

Optimal SQA's for quarkonium at large Q_T

- Leading order parton processes are

$$q\bar{q} \to (c\bar{c})g, \quad qg \to (c\bar{c})q, \quad \bar{q}g \to (c\bar{c})\bar{q}, \quad gg \to (c\bar{c})q.$$

- Dominant parton process at large Q_T is

$$gg \longrightarrow c\bar{c}_8(^3S_1) + g.$$

$\hat{s}, \hat{t}, \hat{u}$: parton Mandelstam variables

$w_1 = Q \cdot P_1, \; w_2 = Q \cdot P_2$

- Longitudinal cross section

$$\hat{\sigma}_L \propto \frac{a^2 x_2^2 \hat{u}^2 + b^2 x_1^2 \hat{t}^2 + (a x_2 - b x_1)^2 \hat{s}^2}{(a w_1 + b w_2)^2 - ab Q^2 s}.$$

- Maximal and minimal gg axes:

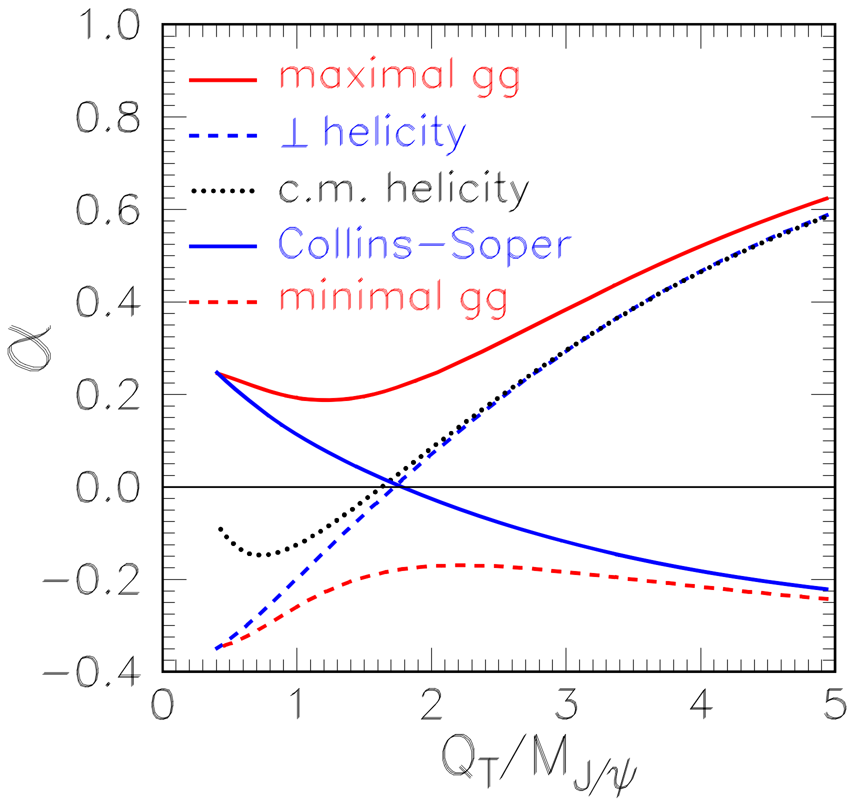
  - minimize/maximize $\hat{\sigma}_L$ with respect to a/b.
  - determine x_1 and x_2 from parton kinematics and

$$\frac{x_1}{x_2} = \frac{(Q_0 + Q_L)\sin\theta_{\text{jet}} + Q_T(1 + \cos\theta_{\text{jet}})}{(Q_0 - Q_L)\sin\theta_{\text{jet}} + Q_T(1 - \cos\theta_{\text{jet}})}.$$

Optimal SQA's for quarkonium at large Q_T

Tevatron
$\sqrt{s} = 1.96$ TeV
CTEQ6L
$Q = 3$ GeV
$\mu = \sqrt{Q^2 + Q_T^2}$
$\lvert y \rvert < 1$

- Polarization from LO parton cross sections.



- difference between $\perp$ helicity and C-S axes increases with Q_T.
- difference between maximal and minimal gg axes increases more rapidly.

Conclusions

- Pairs of SQA's (such as $\perp$ helicity and C-S axes) give more information about spin.
- With optimal SQA's, the dilepton or quarkonium will be significantly more strongly polarized.

  - quantitative predictions require NLO calculations.

- For large Q_T, the optimal SQA's require the measurement of the direction of a recoiling jet.

  - does not dramatically decrease the size of the data sample.

- Similar methods could be used to derive optimal SQA's for the production of heavy SM particles or new particles at LHC.

Thank you!

Back up

Optimal SQA's for dileptons at large Q_T

- *Optimal* $q\bar{q}$ *axis*

$$Q^0 \frac{d\hat{\sigma}_L}{d^3Q} \propto \frac{a^2x_2^2 + b^2x_1^2}{(aw_1 + bw_2)^2 - abQ^2 s}.$$

$$\boxed{\begin{aligned} w_1 &= Q \cdot P_1, \\ w_2 &= Q \cdot P_2. \end{aligned}}$$

- minimize with respect to a/b.

$$\left.\frac{a}{b}\right|_{q\bar{q}\to\gamma^* g} = \frac{x_1^2w_1^2 - x_2^2w_2^2 + Z^{1/2}}{x_2^2(2w_1w_2 - Q^2 s},$$

$$Z = (x_1^2w_1^2 + x_2^2w_2^2)^2 - x_1^2x_2^2Q^2 s(4w_1w_2 - Q^2 s).$$

- *Optimal* qg *axis*

$$d\hat{\sigma}_L \propto \frac{(ax_2 - bx_1)^2 + b^2x_1^2}{(aw_1 + bw_2)^2 - abQ^2 s}.$$

- minimize/maximize with respect to a/b.

$$\left.\frac{a}{b}\right|_{qg\to\gamma^* q} = \frac{2x_1^2w_1^2 - x_2^2w_2^2 + Z^{1/2}}{x_2^2(w_1w_2 2x_2w_1w_2 - x_2Q^2 s},$$

$$Z = (2x_1^2w_1^2 + x_2^2w_2^2 + 2x_1x_2w_1w_2 - x_1x_2Q^2 s)^2 - x_1^2x_2^2Q^2 s(4w_1w_2 - Q^2 s).$$

Optimal SQA's for dileptons at large Q_T

• Fractions of the dilepton cross section from various parton collisions.

Tevatron

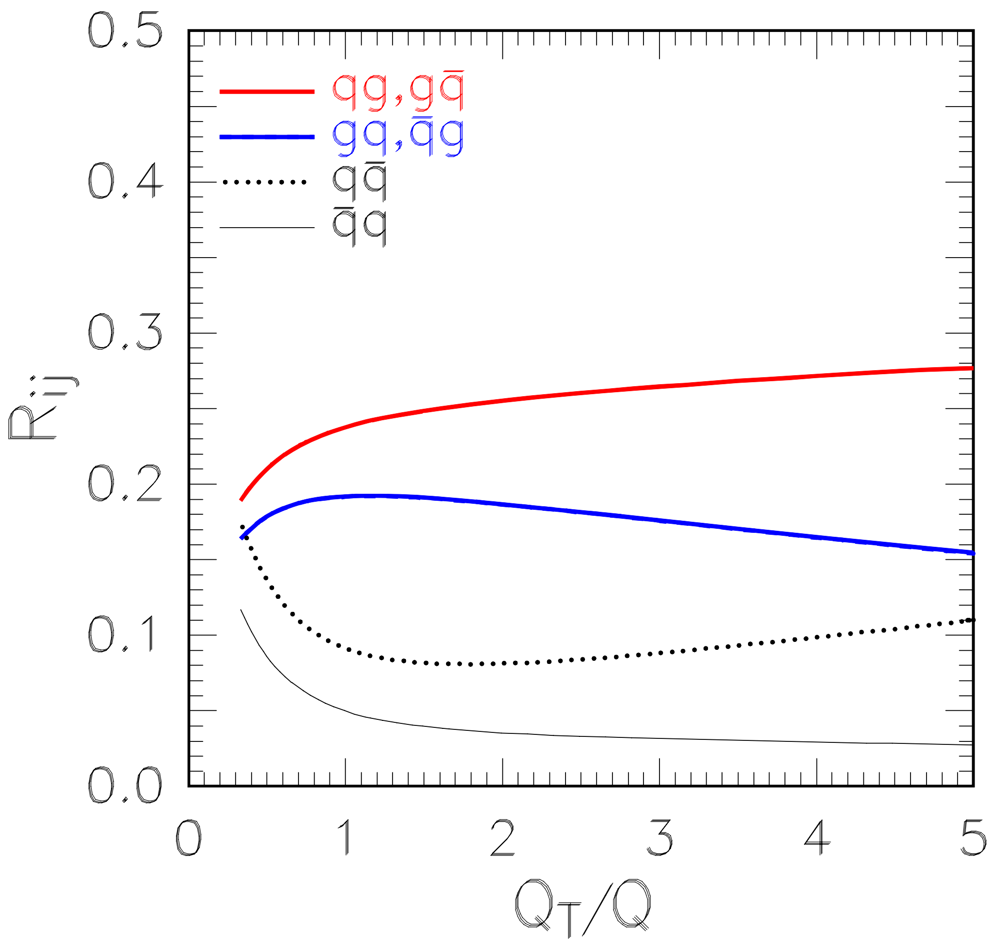
$\sqrt{s} = 1.96$ TeV

CTEQ6L

$Q = 3$ GeV

$\mu = \sqrt{Q^2 + Q_T^2}$

$|y| < 1$



Best optimal axis is expected to be the optimal qg axis.

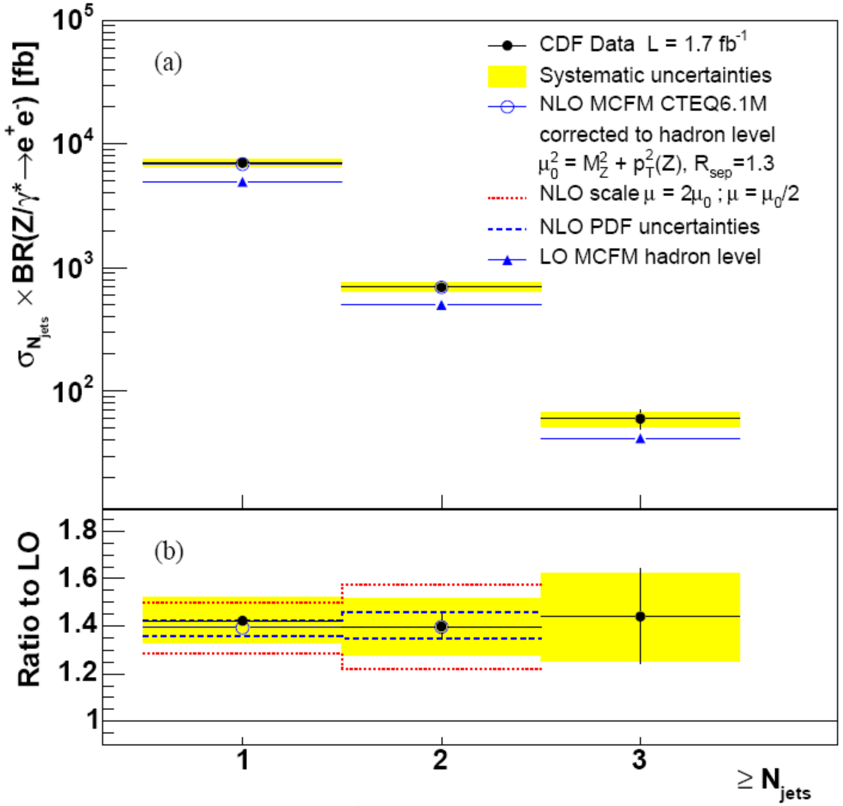
Optimal SQA's for dileptons at large Q_T

• needs calculations to next-to-leading order in α_s.

• Beyond leading order in α_s, there can be more than one parton in the final state.

- multijet.

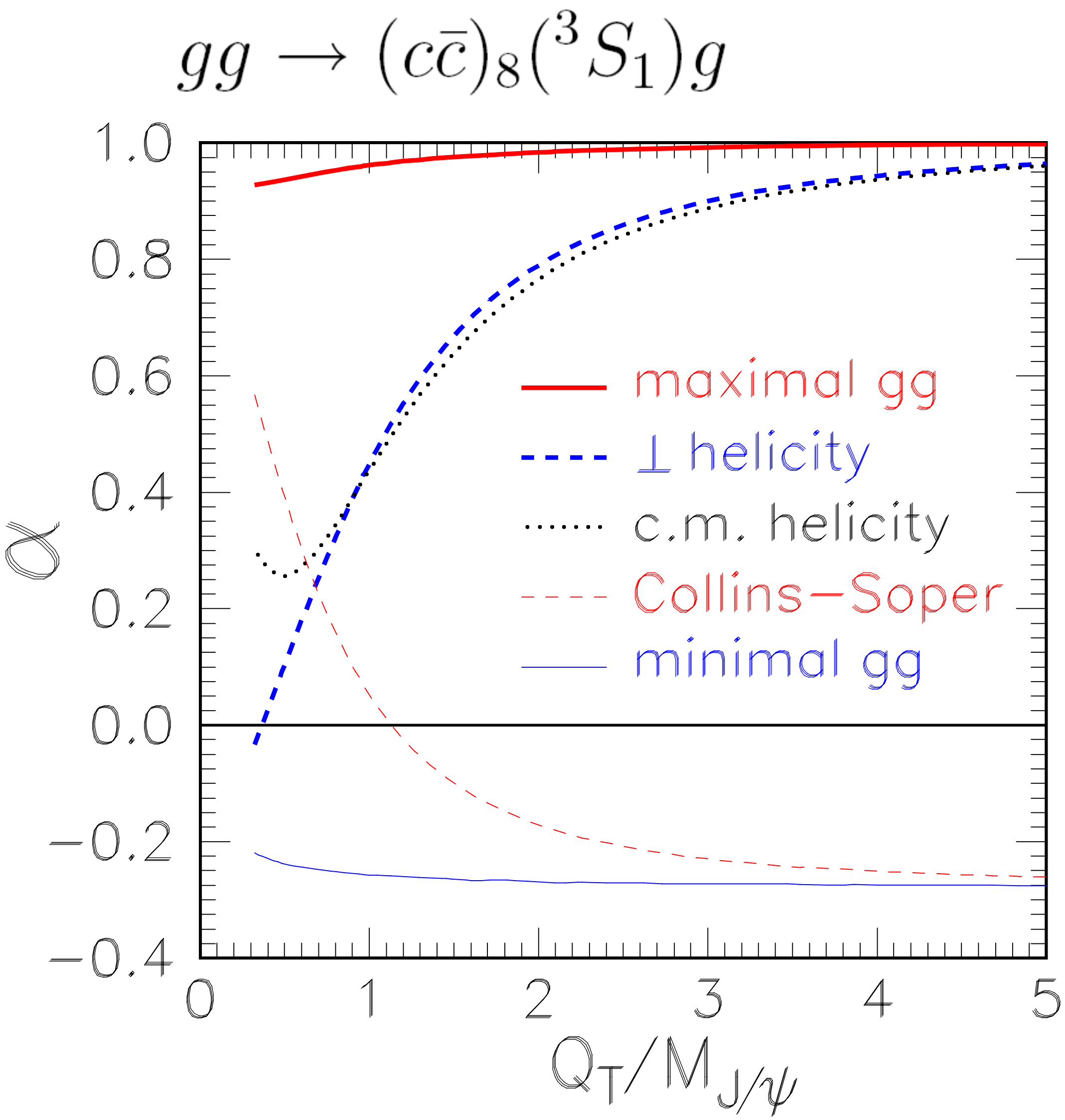
- if reasonably large value is chosen for the cone size, most of events will contain a singlet jet with large transverse momentum.



$66\ \mathrm{GeV} < Q^2 < 116\ \mathrm{GeV}, Q_T > 30\ \mathrm{GeV}.$

CDF, PRL100, 102001 (2008)

Optimal SQA's for quarkonium at large Q_T



Tevatron

$\sqrt{s} = 1.96$ TeV

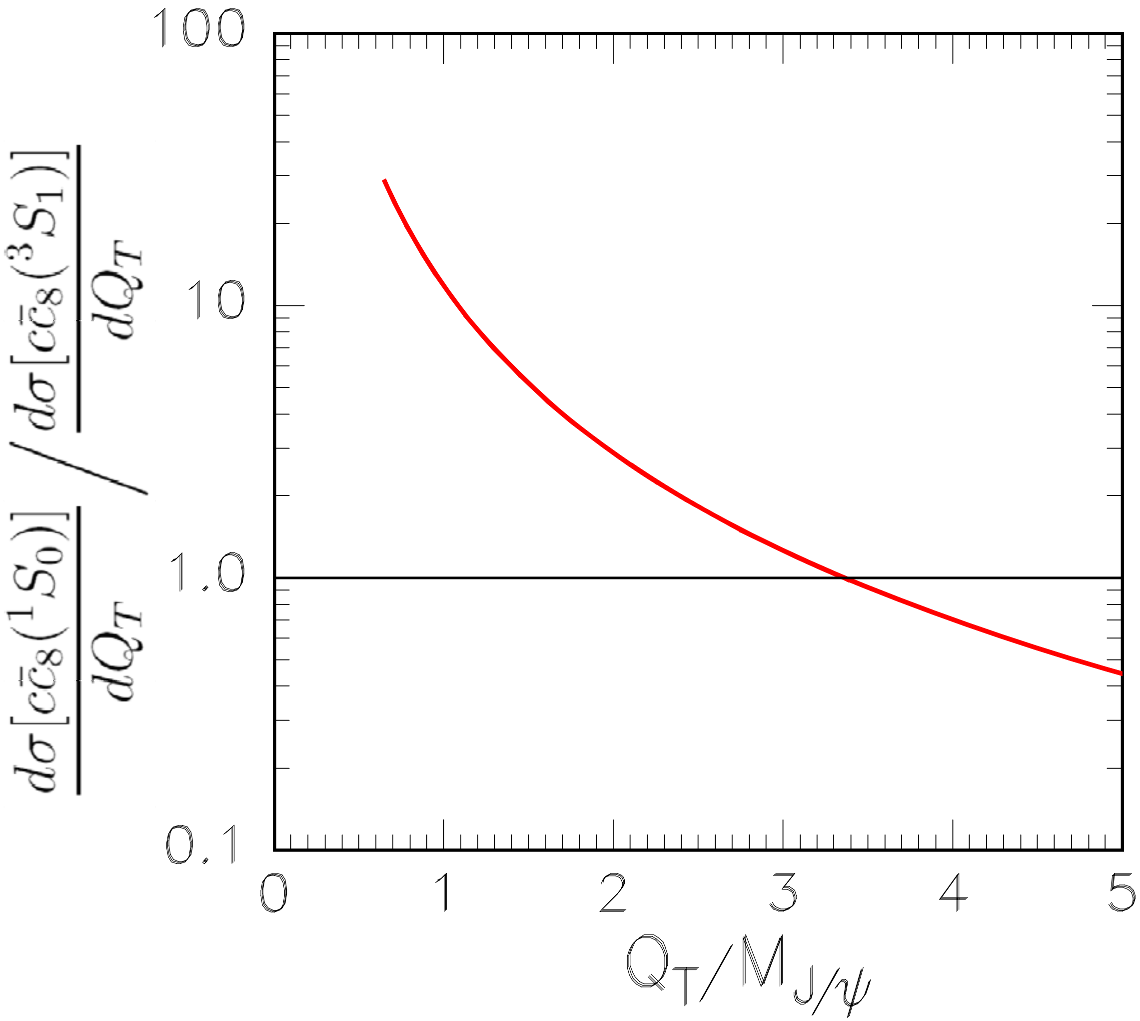
CTEQ6L

$Q = 3$ GeV

$\mu = \sqrt{Q^2 + Q_T^2}$

$|y| < 1$

Optimal SQA's for quarkonium at large Q_T



Tevatron

$\sqrt{s} = 1.96$ TeV

CTEQ6L

$Q = 3$ GeV

$\mu = \sqrt{Q^2 + Q_T^2}$

$|y| < 1$

$\langle O_8(^3S_1)\rangle = 0.0039\ \mathrm{GeV}^3,$

$\langle O_8(^1S_0)\rangle = 0.066\ \mathrm{GeV}^3.$

- dilution from $^1S_0$ octet contributions.

Optimal SQA's for quarkonium at large Q_T

LHC

$\sqrt{s} = 14$ TeV

CTEQ6L

$Q = 9$ GeV

$\mu = \sqrt{Q^2 + Q_T^2}$

$|y| < 3$