NEW RESULTS ON $\alpha_s$ FROM THE LATTICE AND HADRONIC $\tau$ DECAYS

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QWG08, Nara

OUTLINE

- Context/tension between lattice and $\tau$ decay determinations
- Recent update(s) of the UKQCD/HPQCD lattice approach
- New results on the hadronic $\tau$ decay determination
HPQCD/UKQCD, PRL95 (2005) 052002: perturbative analysis of UV-sensitive lattice observables

\[ \left[ \alpha_s(M_Z) \right]_{\text{latt}} = 0.1170(12) \]

(dominates PDG08 \( \alpha_s(M_Z) = 0.1176(20) \) assessment)

ALEPH, OPAL [e.g., EPJC56 (2008) 305]: “(k,m) spectral weight” hadronic \( \tau \) decay determination

\[ \left[ \alpha_s(M_Z) \right]_{\tau} = 0.1212(11) \]
• c.f. other recent (non-τ, non-lattice) determinations

<table>
<thead>
<tr>
<th>Source</th>
<th>$\alpha_s(M_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global EW fit</td>
<td>0.1193(28)</td>
</tr>
<tr>
<td>H1+ZEUS NLO inclusive jets</td>
<td>0.1198(32)</td>
</tr>
<tr>
<td>H1 high-$Q^2$ NLO jets</td>
<td>0.1182(45)</td>
</tr>
<tr>
<td>NNLO LEP event shapes</td>
<td>0.1240(33)</td>
</tr>
<tr>
<td>NNNLL ALEPH+OPAL thrust distributions</td>
<td>0.1172(22)</td>
</tr>
<tr>
<td>$\sigma[e^+e^- \rightarrow \text{hadrons}]$ (2-10.6 GeV)</td>
<td>0.1190(110)</td>
</tr>
<tr>
<td>$\Gamma[\gamma(1s)\rightarrow\gamma X] / \Gamma[\gamma(1s)\rightarrow X]$</td>
<td>0.1190(60)</td>
</tr>
</tbody>
</table>

• individual errors large c.f. nominal lattice, τ

• Non-τ, non-lattice weighted average: $\alpha_s(M_Z) = 0.1193(13)$
UPDATE(S) OF THE LATTICE DETERMINATION

- Based on perturbative analyses of observables, $O_k$, measured on MILC $n_f = 2 + 1$ ensembles

- $O(\alpha_s^3)$ $D = 0$ expansion

  $$[O_k]_{D=0} = D_k \alpha_T(Q_k) \left[ 1 + c_1^{(k)} \alpha_T(Q_k) + c_2^{(k)} \alpha^2_T(Q_k) + \cdots \right]$$

  with $Q_k = d_k/a$ the BLM scale for $O_k$

- $D_k, c_1^{(k)}, c_2^{(k)}, d_k$: Q. Mason et al. 3-loop lattice PT
• Original HPQCD/UKQCD analysis [PRL 95 (2005) 052002]: $a \sim 0.18, 0.12, 0.09$ fm ensembles

• HPQCD [arXiv:0807.1687], CSSM [arXiv:0807.2020]: expanded $a \sim 0.18, 0.15, 0.12, 0.09, 0.06$ fm set

• linear $m_q$ extrapolation/subtraction for $m_q$-dependent NP contributions

• estimated $m_q$-independent NP subtraction via LO $\langle aG^2 \rangle$ (supplemented by fitted $D > 4$ in 2008 HPQCD v2)
Some relevant details

- $D = 0$ to $O(\alpha_s^3)$ insufficient to account for observed scale dependence $\Rightarrow$ must fit additional HO term(s)

- 2008 HPQCD, CSSM employ different $D = 0$ expansion parameters $\Rightarrow$ different (complementary) handling of residual HO perturbative uncertainties

- Fewer $am_\ell/am_s$ combinations to test linearity of $m_q$-dependent NP subtraction for finer lattices, but VERY linear where testable in detail
• Original 2005 HPQCD/UKQCD, 2008 HPQCD:

  - $r_1, \frac{r_1}{a}, \langle aG^2 \rangle$: independent fit w/ priors for each $O_k$

  - $r_1, \frac{r_1}{a}$: small (measured) prior widths $\Rightarrow$ possible unphysical observable-dependence effects small

  - Relation of expansion parameter, $\alpha_V$, to $\alpha_{s\overline{MS}}$ unknown beyond 4th order

  - $O_k$ with potentially sizeable $m_q$-independent NP subtractions included in analysis

  - (2008 update): better agreement of $\langle aG^2 \rangle$ from different $O_k$ when $D > 4$ forms included, with fitted coefficients, for more NP observables [HPQCD private communications] but full details not available
• 2008 CSSM re-analysis:

  – measured $r_1$, $\frac{r_1}{a}$, charmonium sum-rule $\langle aG^2 \rangle$ (with errors): common, external input for all $O_k$

  – LO $D = 4 \langle aG^2 \rangle$ estimate of $m_q$-independent NP contribution/subtraction

  – Expansion parameter defined so relation to $\alpha_s^{MS}$ exactly specified

  – focus on $O_k$ where estimated $D = 4$ NP $\langle aG^2 \rangle$ subtraction small, hence $D > 4$ presumably even smaller
• More on the two $D = 0$ expansion parameters choices

− $D = 0$ expansion parameter $\alpha_T$, $\beta$ function $\beta^T$ to 4-loops from $\beta^{\overline{MS}} \Rightarrow \beta^T_{4,5,...}$ incompletely known

− Expand $\alpha_T$ in $\alpha_0 = \alpha_T(Q_k^{\text{max}})$, $t_k = \log[(Q_k/Q_k^{\text{max}})^2]$

\[
\frac{O_k}{D_k} = \cdots + \alpha_0^4 (c_3^{(k)} + \cdots) + \alpha_0^5 \left( c_4^{(k)} - 2.87 c_3^{(k)} t_k + \cdots \right) \\
+ \alpha_0^6 \left( c_5^{(k)} - 0.0033 \beta^T_4 t_k - 3.58 c_4^{(k)} t_k \right) \\
+ [5.13t_k^2 - 1.62t_k]c_3^{(k)} + \cdots + \alpha_0^7 \left( c_6^{(k)} - 0.0010 \beta^T_5 t_k + [0.0094t_k^2 - 0.0065c_1^{(k)} t_k] \beta^T_4 \right. \\
- 4.30 c_5^{(k)} t_k + [7.69t_k^2 - 2.03t_k]c_4^{(k)} \\
+ [-7.35t_k^3 + 6.39t_k^2 - 4.38t_k]c_3^{(k)} + \cdots \right) + \cdots
\]

− Incompletely known $\beta^T_{4,5,...}$ distorts fit parameters
- HPQCD approach

* $\alpha_T \rightarrow \alpha_V$ defined such that $\beta_4^V = \beta_5^V = \cdots = 0$

* $\Rightarrow$ no distortion of fit parameters

* expansion for $\alpha_V$ in terms of $\alpha_{s(MS)}$ in principle well-defined

* (however) expansion coefficients beyond 4\(th\) order depend on $\beta_{4,5,\ldots}^{MS}$, hence not known

* impact of HO (after fitting $c_{3,4,\ldots}^{(k)}$) localized to conversion/running to $\alpha_s(M_Z)$
CSSM approach

* $\alpha_T$ defined as 3-order-truncated expansion of $\alpha_V^p$

* $\Rightarrow$ conversion to $\alpha_{\overline{MS}}^s$ exact but $\beta_{4,5}^T$... depend on $\beta_{4,5}^{\overline{MS}}$, hence incompletely known

* Fit parameter distortions reducible by hand:
  - focus on highest intrinsic scale $O_k$
  - restrict $t_k$ (subset of finest lattices)
  - stability c.f. expanding subset as test
HPQCD RESULTS

Large NP subtractions for 3 outliers

Ave (all): 0.1184(9)
Ave w/ out: 0.1186(9)
CSSM RESULTS

\[\begin{array}{c|c|c}
\hline
O_k & \alpha_s(M_Z) \\
\hline
\log(W_{11}) & 0.1190(11) \\
\log(W_{12}) & 0.1191(11) \\
\log \left( \frac{W_{12}}{u_0^6} \right) & 0.1191(11) \\
\hline
\end{array}\]
THE HADRONIC $\tau$ DETERMINATION

- Based on FESRs for $\Pi_{T;ud}^{(0+1)}$, $T = V, A, V + A$

$$\int_0^{s_0} w(s) \rho_{T;ud}^{(0+1)}(s) \, ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s) \Pi_{T;ud}^{(0+1)}(s) \, ds$$

- valid for any $s_0$, analytic $w(s)$

- LHS: data; RHS: OPE (hence $\alpha_s$) for $s_0 >> \Lambda_{QCD}^2$
• The spectral integrals

\[ \mathcal{V}, \mathcal{A}_{ij} = ud, us, (J) = (0+1), (0) \text{ spectral functions} \]

from experimental differential decay distributions

\[
\frac{dR_{\mathcal{V}/\mathcal{A};ij}}{ds} = \frac{12\pi^2 |V_{ij}|^2 S_{EW}}{m_\tau^2} \left[ w_T(y_\tau) \rho_{\mathcal{V}/\mathcal{A};ij}^{(0+1)}(s) + w_L(y_\tau) \rho_{\mathcal{V}/\mathcal{A};ij}^{(0)}(s) \right]
\]

with \( R_{\mathcal{V}/\mathcal{A};ij} \equiv \frac{\Gamma[\tau \rightarrow \nu_\tau \text{ hadrons}_{\mathcal{V}/\mathcal{A};ij}(\gamma)]}{\Gamma[\tau^\rightarrow \nu_\tau e^{-\nu_e}(\gamma)]}, \ y_\tau = s/m_\tau^2, \)

\( w_T(y), w_L(y) \) known polynomials
- accurately known $ud$, $J = 0$ contributions

\[
\rho_{A;ud}^{(0)}(s) = 2f_\pi^2 \delta(s - m_\pi^2) + O[(m_d + m_u)^2]
\]
\[
\rho_{V;ud}^{(0)}(s) = O[(m_d - m_u)^2]
\]

\[\Rightarrow \rho_{T;ud}^{(0+1)}(s) \text{ from } |V_{ud}|, \text{ experimental } dR_V + A;ud/ds\]

- \(\Rightarrow\) experimental access to generic \((J) = (0 + 1); w(s)\)-weighted, \(0 < s \leq s_0 \leq m_T^2\) spectral integrals

\[
I_{spec;T}^w(s_0) = \int_0^{s_0} ds w(s) \rho_{T;ud}^{(0+1)}(s)
\]
- **The OPE side**

  - $w(s_0) = 0$ ("pinching"), $s_0 \gtrsim 2$ GeV$^2$ to avoid OPE breakdown

  - $D = 0$: fixed by $\alpha_s$ (known to 5 loops); strongly dominant for $s_0 \gtrsim 2$ GeV$^2$

  - $D = 2$: $O\left[(m_d + m_u)^2, \alpha_s^2 m_s^2\right]$, hence tiny

  - $D = 4$: fixed by $\langle aG^2 \rangle$, $\langle m_\ell \bar{\ell} \ell \rangle$, $\langle m_s \bar{s}s \rangle$

  - $D = 6, 8, \cdots$ not known phenomenologically, hence fitted to data
More on fitting the $D > 4$ contributions

* $\sim 1\%$ for $\alpha_s(M_Z) \leftrightarrow \sim 3\%$ for $\alpha_s(m_\tau) \Rightarrow$ need integrated $D > 4$ to $\lesssim 0.5\%$ of $D = 0$

* integrated $D = 2k + 2 \geq 2$ contribution for $y = s/s_0$, $w(y) = \sum_{m=0} b_m y^m \Leftrightarrow b_k \neq 0$ (up to $O[\alpha_s^2(s_0)]$ corrections) $\Rightarrow$ contributions up to $D_{\text{max}} = 2N + 2$ for degree $N$ $w(y)$

* different $s_0$-dependences for different $D$

\[
-\frac{1}{2\pi i} \oint_{|s|=s_0} ds \, w(y) \frac{C_D}{Q_D} = \sum_{m=2} \frac{(-1)^m b_m C_{2m+2}}{s_0^m}
\]
THE ALEPH, OPAL ANALYSES

\( w_{(00)}(y) = 1 - 3y^2 + 2y^3 \Rightarrow \) OPE up to \( D = 6, 8 \)

\( \Gamma[\tau \rightarrow \text{hadrons}_{ud} \nu_{\tau}] \) alone (\( \leftrightarrow \) \( I^{w(00)}_{spec;V+A}(m_{\tau}^2) \)) insufficient to fix \( \alpha_s, C_6, C_8 \)

ALEPH, OPAL approach

- add \( s_0 = m_{\tau}^2, (km) = (10), (11), (12), (13) \) “spectral weight” FESRs \( [w(y) \rightarrow y^m (1 - y)^k w_{(00)}(y)] \)

- neglect (in ppl present) \( D = 10, \cdots, 16 \) contribs

- \( \alpha_s, \langle aG^2 \rangle, C_6, C_8 \) fitted to 5 integral set
• Potential problem: single $s_0 (\equiv m_T^2) \Rightarrow D > 8$ (if non-negligible) distort $D = 0, 4, 6, 8$ fit parameters

• Test for possible symptoms (systematic $s_0$-dependence problems) using “fit qualities”

$$F^w_T(s_0) \equiv \left[ I_{spec;T}(s_0) - I_{OPE;T}(s_0) \right] / \delta I_{spec;T}(s_0)$$

• **FIGURE:** $F^w_V(s_0)$ for ALEPH data, OPE fit, and

  - 3 $w_{(k,m)}$ used in fit

  - 3 other degree 3 $w(y)$ (independent $C_{6,8}$ tests)
Mismatch ⇒ either a $D > 8$ problem or OPE breakdown: either way a problem for extracted $\alpha_s$
A MODIFIED ANALYSIS

- V, A and V+A, \( w_N(y) \equiv 1 - \frac{N}{N-1}y + \frac{1}{N-1}y^N \) FESRs

- single unsuppressed \( D = 2N + 2 > 4 \) contrib \((N \geq 2)\)

\[
\frac{-1}{2\pi i} \oint_{|s|=s_0} ds \ w_N(y) \frac{C_D}{Q^D} = (-1)^N \frac{C_{2N+2}}{(N-1)s_0^N}
\]

- scales as \( 1/s_0^{N+1} \) c.f. leading \( D = 0 \) term
- \( 1/(N-1) \) coefficient suppression with increasing \( N \)
- MUCH better \( D = 0 \) emphasis than \( w_{(k,m)} \) set
RESULTS

- For the CIPT $D = 0$ treatment

  - OPAL-based V+A results

<table>
<thead>
<tr>
<th>$w(y)$</th>
<th>$\alpha_s(m_T^2)$</th>
<th>$C_{2N+2}/m_T^{2N+2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_2$</td>
<td>0.322(7)(12)</td>
<td>$-0.000233(59)(114)$</td>
</tr>
<tr>
<td>$w_3$</td>
<td>0.322(7)(12)</td>
<td>0.000205(74)(120)</td>
</tr>
<tr>
<td>$w_4$</td>
<td>0.322(7)(12)</td>
<td>$-0.000162(76)(105)$</td>
</tr>
<tr>
<td>$w_5$</td>
<td>0.322(7)(12)</td>
<td>0.000122(70)(86)</td>
</tr>
<tr>
<td>$w_6$</td>
<td>0.322(8)(12)</td>
<td>$-0.000091(60)(67)$</td>
</tr>
</tbody>
</table>
- ALEPH-based V, A and V+A results

<table>
<thead>
<tr>
<th>Channel</th>
<th>$w(y)$</th>
<th>$\alpha_s\left(m^2_\tau\right)$</th>
<th>$C_{2N+2}/m^{2N+2}_\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>$w_2$</td>
<td>0.321(7)(12)</td>
<td>−0.000187(29)(56)</td>
</tr>
<tr>
<td></td>
<td>$w_3$</td>
<td>0.321(7)(12)</td>
<td>0.000060(36)(60)</td>
</tr>
<tr>
<td></td>
<td>$w_4$</td>
<td>0.321(7)(12)</td>
<td>0.000015(36)(53)</td>
</tr>
<tr>
<td></td>
<td>$w_5$</td>
<td>0.321(7)(12)</td>
<td>−0.000043(33)(44)</td>
</tr>
<tr>
<td></td>
<td>$w_6$</td>
<td>0.321(7)(12)</td>
<td>0.000046(27)(35)</td>
</tr>
<tr>
<td>A</td>
<td>$w_2$</td>
<td>0.319(6)(12)</td>
<td>−0.000072(24)(60)</td>
</tr>
<tr>
<td></td>
<td>$w_3$</td>
<td>0.319(6)(12)</td>
<td>0.000182(28)(71)</td>
</tr>
<tr>
<td></td>
<td>$w_4$</td>
<td>0.319(6)(12)</td>
<td>−0.000216(27)(70)</td>
</tr>
<tr>
<td></td>
<td>$w_5$</td>
<td>0.319(6)(12)</td>
<td>0.000201(23)(66)</td>
</tr>
<tr>
<td></td>
<td>$w_6$</td>
<td>0.319(6)(12)</td>
<td>−0.000166(19)(59)</td>
</tr>
<tr>
<td>V+A</td>
<td>$w_2$</td>
<td>0.320(5)(12)</td>
<td>−0.000261(35)(114)</td>
</tr>
<tr>
<td></td>
<td>$w_3$</td>
<td>0.320(5)(12)</td>
<td>0.000247(45)(125)</td>
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<tr>
<td></td>
<td>$w_4$</td>
<td>0.320(5)(12)</td>
<td>−0.000208(44)(111)</td>
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<tr>
<td></td>
<td>$w_5$</td>
<td>0.320(5)(12)</td>
<td>0.000166(39)(97)</td>
</tr>
<tr>
<td></td>
<td>$w_6$</td>
<td>0.320(5)(12)</td>
<td>−0.000126(34)(88)</td>
</tr>
</tbody>
</table>
• Much improved $F^{w}_{V}(s_0)$ for $w = w_{N}$ c.f. $w = w_{(k,m)}$
- CIPT $w_2, \ldots, w_6$ fit values consistent to $\pm 0.0001$

- Averaging ALEPH and OPAL based results with non-normalization component of error $\Rightarrow$

\[
\alpha_{s}^{(n_f=3)}(m_\tau) = 0.3209(46)_{\text{exp}}(118)_{\text{th}}
\]

- Standard self-consistent combination of 4-loop running, 3-loop matching at flavor thresholds $\Rightarrow$

\[
\alpha_{s}^{(n_f=5)}(M_Z) = 0.1187(3)_{\text{evol}}(6)_{\text{exp}}(15)_{\text{th}}
\]
• Good agreement with (updated) lattice

• Theory error dominant (~ 2.5 times expt’l)

• $D = 0$ truncation dominant theory error source (for $|FOPT-CIPT| \oplus O(a^5)$ estimate $\sim 0.010$ of 0.012 total) $\Rightarrow$ main bottleneck for future improvements

• Beneke-Jamin-like exploration (taking into account divergent nature of $D = 0$ series) crucial to any possibility of significant future error reduction
CONCLUSIONS/SUMMARY

• Lattice and $\tau$ determinations

$$[\alpha_s(M_Z)]_{latt} = 0.1184(9), \ 0.1190(11)$$
$$[\alpha_s(M_Z)]_{\tau} = 0.1187(16)$$

in good agreement and currently the two most accurate determinations

• $a \sim 0.045$ fm MILC data will allow some improvement of lattice determination

• significant improvement for $\tau$ only with better understanding of $D = 0$ truncation uncertainty
SUPPLEMENTARY MATERIAL

- More on consistency of V+A fit results
- Some observations on the Beneke-Jamin calculation
• More on the consistency of the V+A fit results

V+A fit results for $\alpha_s(m_\tau)$

<table>
<thead>
<tr>
<th>$w(y)$</th>
<th>CIPT full fit</th>
<th>$s_0 = m_\tau^2$ CIPT $D &gt; 4 \rightarrow 0$</th>
<th>FOPT full fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_2$</td>
<td>0.320</td>
<td>0.310</td>
<td>0.320</td>
</tr>
<tr>
<td>$w_3$</td>
<td>0.320</td>
<td>0.316</td>
<td>0.315</td>
</tr>
<tr>
<td>$w_4$</td>
<td>0.320</td>
<td>0.319</td>
<td>0.313</td>
</tr>
<tr>
<td>$w_5$</td>
<td>0.320</td>
<td>0.321</td>
<td>0.312</td>
</tr>
<tr>
<td>$w_6$</td>
<td>0.320</td>
<td>0.322</td>
<td>0.312</td>
</tr>
</tbody>
</table>
• Some observations on the Beneke-Jamin calculation

  – As for the spectral weight analysis, control of $D > 4$ contributions essential for precision $\alpha_s$ (independent of choice of FOPT or CIPT for $D = 0$ contributions)

  – Can test BJ input assumptions for $C_{6,8}$ for consistency with output FOPT fit $\alpha_s$ using $F^w_{V+A}(s_0)$ for various degree $\leq 3$ $w(y)$ (FIGURE)

  – Find problems for combination of assumed $D = 6,8$ and FOPT fitted $\alpha_s$
The plot shows the function $F_{V+A}^{s_0}(s_0)$ as a function of $s_0$ (GeV$^2$). The graph includes several curves labeled with different notations, which likely represent different terms or components of the function, such as $w_{00}$, $w_2$, $w_3$, and $y(1-y)^2$. The exact interpretation of these terms would require additional context from the surrounding text or the specific field of study.
Exercise to test implications of (minimal, 5-parameter) BJ model for the resummed $D = 0$ series

* Features of the minimal model:
  
  - good approximation to full model sum using FOPT for a range of $w(y)$ (FIGURES)
  
  - CIPT approximation inferior to FOPT most strongly so for $w_{(0,0)}$ (FIGURES)
  
  - $\Rightarrow$ expect consistency of various FOPT fits, reduced consistency for CIPT fits

* FIGURE: FOPT, CIPT vs. Borel sum for BJ model
$w(0,0)$

$w_2$

Maltman weight $w_2$
* Test expectations with combined FOPT, CIPT $w_2$-$w_3$ fit
  
  - combined fit yields $\alpha_s$, $C_6$, $C_8$, hence OPE integrals fixed for any degree $\leq 3$ $w(y)$

  - test agreement of CIPT, FOPT OPE with corresponding spectral integrals for $w_{(0,0)}$, $y(1-y)^2$

  * find good (not good) CIPT (FOPT) consistency (contrary to model expectations) (FIGURE)

  * suggests alternate non-minimal modelling possible using such observations as constraints
FOPT vs CIPT $w_2$-$w_3$ joint fit V+A fit qualities