Heavy Quark $O(\alpha_s^3)$ Corrections to $F_L(x, Q^2)$

at Large Virtualities

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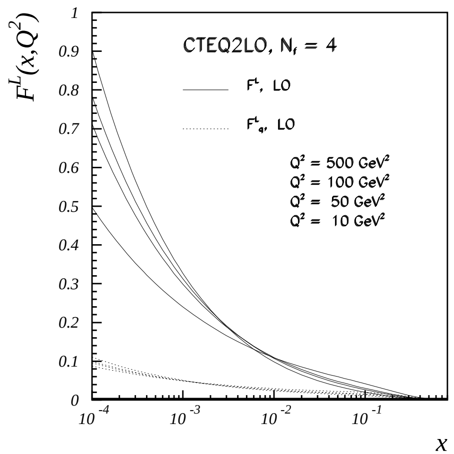
- Motivation
- The Method
- The Heavy Quark Wilson Coefficients for $Q^2 \gg m^2$
- Small x Limit
- Numerical Results

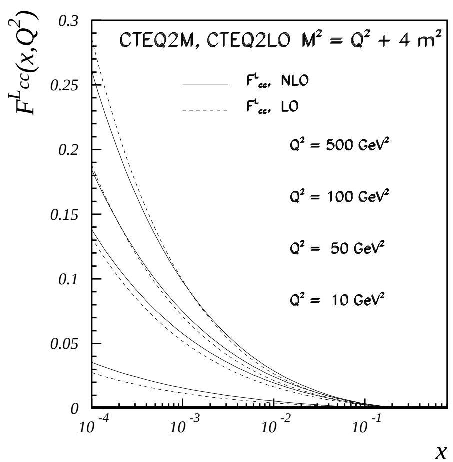
(with A. De Freitas, S. Moch, W.L. van Neerven, S. Klein)

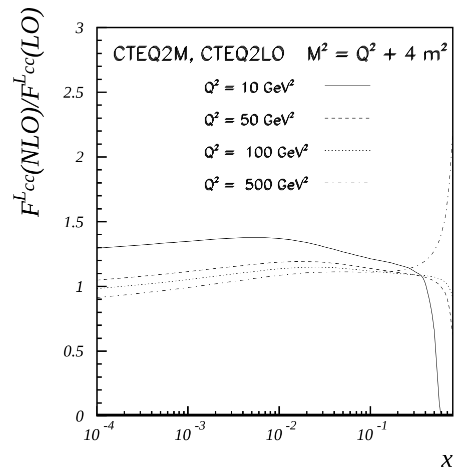
1. Motivation

- Perturbative calculations of scaling violations of QCD structure functions reached 3–loop level
  $\Longrightarrow$ massless contributions.
  (Larin, Vermaseren et al. 1994-2004; Moch, Vermaseren, Vogt 2004/05)

- Heavy Flavor contributions to DIS structure functions are large.







- **New Era :**

Massive Wilson Coefficients for Nucleon Structure Functions
@ $O(\alpha_s^3)$

- First step: RGE & Mass factorization

  Allows to calculate all but the power-suppressed contributions:
  Logarithmic and constant terms.

- Formal structure: harmonic sums.

- Need: Increase accuracy of the perturbative description of DIS structure functions

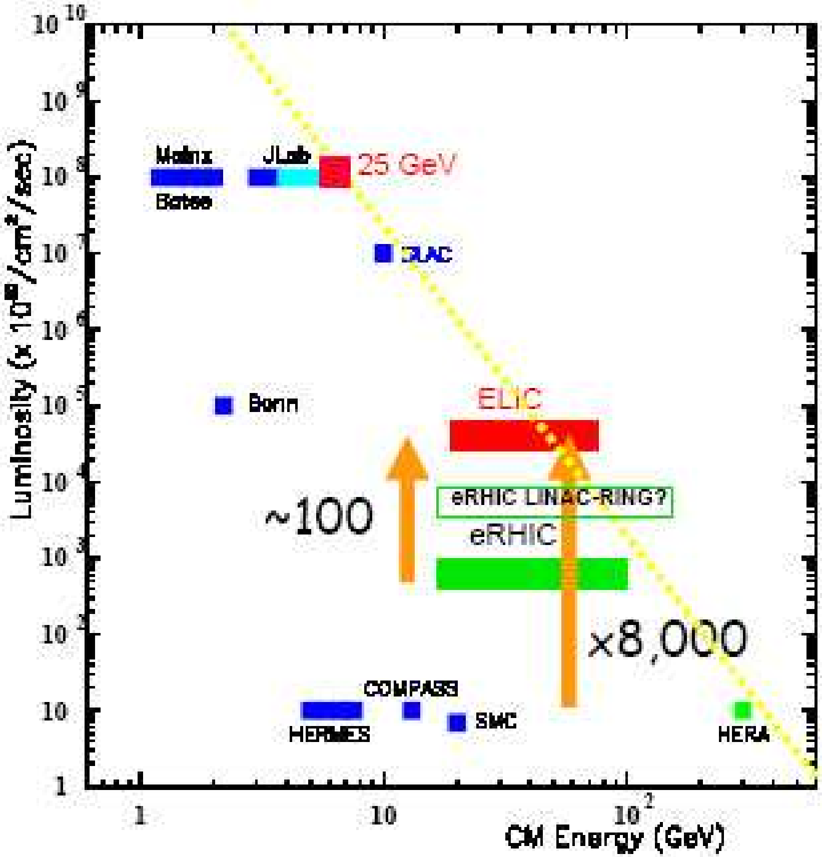
- $\Longleftrightarrow$ QCD Analysis and Determination of $\Lambda_{\rm QCD}$ from DIS data.

- $\Longleftrightarrow$ Future precise determination of the Gluon and Sea Quark Distributions; Scheme-invariant Evolution

- First Example : the longitudinal structure function $F_L(x, Q^2)$

Experimental Perspectives

- HERA : Last running period, cf. DESY PRC 11/05
- ELIC : US Project, JLAB/BNL



R. Ent, 2004.

Status of Heavy Flavor Corrections

Unpolarized DIS :

- LO : (Witten, 1976; Babcock & Sivers, 1978; Shifman, Vainshtein, Zakharov 1978; Leveille & Weiler, 1979)

- NLO : (Laenen, Riemersma, Smith, van Neerven, 1993, 1995)
asymptotic : (Buza, Matiounine, Smith, Migneron, van Neerven, 1996)

Polarized DIS :

- LO : (Watson, 1982; Glück, Reya, Vogelsang, 1991; Vogelsang, 1991)

- NLO : asymptotic: (Buza, Matiounine, Smith, van Neerven, 1996)

Mellin Space Expressions :

(Alekhin, Blümlein, 2003; Vogt).

One-Loop Example

Consider $F_2^{c\overline{c}}(x, Q^2)$:

$$H_{F_2}^{(1)}\left(z, a_s, \frac{m^2}{Q^2}\right) = 8 T_R a_s \left\{ v \left[-\frac{1}{2} + 4z(1-z) + \frac{m^2}{Q^2} z(2z-1) \right] + \left[-\frac{1}{2} + z - z^2 + 2\frac{m^2}{Q^2} z(3z-1) - 4\frac{m^4}{Q^4} z^2 \right] \ln\left(\frac{1-v}{1+v}\right) \right\}$$

$$\lim_{Q^2 \gg m^2} H_{F_2}^{(1)}\left(z, a_s, \frac{m^2}{Q^2}\right) = 4 T_R a_s \left\{ [z^2 + (1-z)^2] \ln\left(\frac{Q^2}{m^2} \frac{1-z}{z}\right) + 8z(1-z) - 1 \right\}$$

$\overline{\rm MS}$ result for $m^2 = 0$:

$$C_{F_2}^{(1)}(z, a_s) = 4 T_R a_s \left\{ [z^2 + (1-z)^2] \ln\left(\frac{Q^2}{m^2} \frac{1-z}{z}\right) + 8z(1-z) - 1 \right\}, \quad \mu^2 = m^2$$

$$\lim_{Q^2 \gg m^2} H_{F_2}^{(1)}\left(z, a_s, \frac{m^2}{Q^2}\right) = C_{F_2}^{(1)}\left(z, a_s, \frac{m^2}{Q^2}\right) + A_{Qg}^{(1)}\left(z, \frac{Q^2}{m^2}\right) .$$

Why do these results agree ? $\Longrightarrow \quad a_{Qg}^{(1)} = 0$

2. The Method

i) massless RGE : mass factorization between Wilson coefficients and parton densities;

ii) parton densities are always massless, i.e. their evolution is free of any quark mass effects.

iii) RGE with a mass : the derivative $m^2\partial/\partial m^2$ acts on the Wilson coefficients only. $\Longrightarrow$ Seek all terms, but power corrections.

iv) For these terms a similar factorization is obtained as in i).
For a more exclusive example in $\mathrm{O}(\alpha^2 \mathrm{L})$ QED, cf : [J.B. & H. Kawamura, 2002]
The non-power mass corrections are process independent and are calculated through partonic operator matrix elements, $\langle i|A_l|j\rangle$. (Likewise, parton densities stem from nucleonic matrix elements. $\langle N|A_l|N\rangle$.)

$$H_{I,l}^{\mathrm{S,NS},g}\left(\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right) = A_{k,l}^{\mathrm{S,NS},g}\left(\frac{m^2}{\mu^2}\right) \otimes C_{I,k}^{\mathrm{S,NS},g}\left(\frac{Q^2}{\mu^2}\right) .$$

(Buza, Matiounine, Smith, Migneron, van Neerven, 1996)

3. The Heavy Quark Wilson Coefficients for $Q^2 \gg m^2$

Mass factorization (m^2) implies :

(after renormalization and separation of UV and collinear singularities)

$$
\begin{aligned}
H_{L,g}^{\mathrm{S}}\left(\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) &= a_s\widehat{C}_{L,g}^{(1)}\left(\frac{Q^2}{\mu^2}\right)+a_s^2\left[A_{Q,g}^{(1)}\left(\frac{\mu^2}{m^2}\right)\otimes C_{L,q}^{(1)}\left(\frac{Q^2}{\mu^2}\right)+\widehat{C}_{L,g}^{(2)}\left(\frac{Q^2}{\mu^2}\right)\right]\\
&+ a_s^3\left[A_{Q,g}^{(2)}\left(\frac{\mu^2}{m^2}\right)\otimes C_{L,q}^{(1)}\left(\frac{Q^2}{\mu^2}\right)+A_{Q,g}^{(1)}\left(\frac{\mu^2}{m^2}\right)\otimes C_{L,q}^{(2)}\left(\frac{Q^2}{\mu^2}\right)+\widehat{C}_{L,g}^{(3)}\left(\frac{Q^2}{\mu^2}\right)\right]\\
H_{L,q}^{\mathrm{PS}}\left(\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) &= a_s^2\widehat{C}_{L,q}^{\mathrm{PS},(2)}\left(\frac{Q^2}{\mu^2}\right)+a_s^3\left[A_{Qq}^{\mathrm{PS},(2)}\left(\frac{\mu^2}{m^2}\right)\otimes C_{L,q}^{(1)}\left(\frac{Q^2}{\mu^2}\right)+\widehat{C}_{L,q}^{\mathrm{PS},(3)}\left(\frac{Q^2}{\mu^2}\right)\right]\\
H_{L,q}^{\mathrm{NS}}\left(\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) &= a_s^2\widehat{C}_{L,q}^{\mathrm{NS},(2)}\left(\frac{Q^2}{\mu^2}\right)+a_s^3\left[A_{qq,Q}^{\mathrm{NS},(2)}\left(\frac{\mu^2}{m^2}\right)\otimes C_{L,q}^{(1)}\left(\frac{Q^2}{\mu^2}\right)+\widehat{C}_{L,q}^{\mathrm{NS},(3)}\left(\frac{Q^2}{\mu^2}\right)\right]~,
\end{aligned}
$$

$$
C_{L,i}^{k}\left(\frac{Q^2}{\mu^2},z\right)=a_S C_{L,i}^{k,(1)}\left(\frac{Q^2}{\mu^2},z\right)+a_s^2 C_{L,i}^{k,(2)}\left(\frac{Q^2}{\mu^2},z\right)+a_s^3 C_{L,i}^{k,(3)}\left(\frac{Q^2}{\mu^2},z\right)
$$

Massive Operator Matrix Elements

$$
\begin{aligned}
A_{Qg}^{(1)} &= -\frac{1}{2}\widehat{P}_{qg}^{(0)} \ln\left(\frac{m^2}{\mu^2}\right) + a_{Qg}^{(1)} \\
A_{Qg}^{(2)} &= \frac{1}{8}\left\{\widehat{P}_{qg}^{(0)} \otimes \left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right]\right\} \ln^2\left(\frac{m^2}{\mu^2}\right) \\
&\quad -\frac{1}{2}\left\{\widehat{P}_{qg}^{(1)} + a_{Qg}^{(1)}\left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right]\right\} \ln\left(\frac{m^2}{\mu^2}\right) \\
&\quad +\overline{a}_{Qg}^{(1)}\left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right] + a_{Qg}^{(2)} \\
A_{Qq}^{\mathrm{PS},(2)} &= -\frac{1}{8}\widehat{P}_{qg}^{(0)} \otimes P_{gq}^{(0)} \ln^2\left(\frac{m^2}{\mu^2}\right) - \frac{1}{2}\left[\widehat{P}_{qq}^{\mathrm{PS},(1)} - a_{Qg}^{(1)} P_{gq}^{(0)}\right] \ln\left(\frac{m^2}{\mu^2}\right) \\
&\quad + a_{Qq}^{\mathrm{PS},(2)} - \overline{a}_{Qg}^{(1)} \otimes P_{gq}^{(0)} \\
A_{qq,Q}^{\mathrm{NS},(2)} &= -\frac{\beta_{0,Q}}{4} P_{qq}^{(0)} \ln^2\left(\frac{m^2}{\mu^2}\right) - \frac{1}{2}\widehat{P}_{qq}^{\mathrm{NS},(1)} \ln\left(\frac{m^2}{\mu^2}\right) + a_{qq,Q}^{\mathrm{NS},(2)} + \frac{1}{4}\beta_{0,Q}\zeta_2 P_{qq}^{0}\ ,
\end{aligned}
$$

with

$$
\widehat{f} = f(N_F + 1) - f(N_F)\ .
$$

Expansion Coefficients

$$
\begin{aligned}
a_{Qg}^{(1)}(N) &= 0 \\
\overline{a}_{Qg}^{(1)}(N) &= -\frac{1}{8}\zeta_2 \widehat{P}_{qg}^{(0)}(N) \\
a_{Qq}^{\mathrm{PS},(2)}(N) &= C_F T_R \Biggl\{ -8 \frac{N^4 + 2N^3 + 5N^2 + 4N + 4}{(N-1)N^2(N+1)^2(N+2)} S_2(N-1) \\
&\quad -4 \frac{(N^2+N+2)^2}{(N-1)N^2(N+1)^2(N+2)}\zeta_2 + \frac{4\ P_4(N)}{(N-1)N^4(N+1)^4(N+2)^3} \Biggr\} \\
a_{qq,Q}^{\mathrm{NS},(2)}(N) &= C_F T_R \Biggl\{ -\left(\frac{224}{27} + \frac{8}{3}\zeta_2\right) S_1(N-1) + \frac{40}{9} S_2(N-1) - \frac{8}{3} S_3(N-1) \\
&\quad + \frac{2(3N+2)(N-1)}{3N(N+1)}\zeta_2 \\
&\quad + \frac{(N-1)(219N^5 + 428N^4 + 517N^3 + 512N^2 + 312N + 72)}{54N^3(N+1)^3} \Biggr\}
\end{aligned}
$$

$$
\begin{aligned}
a_{Qg}^{(2)}(N) \quad = \quad & 4C_F T_R \Bigg\{ \frac{N^2+N+2}{N\,(N+1)\,(N+2)} \Bigg[-\frac{1}{3} S_1^3(N-1) + \frac{4}{3} S_3(N-1) \\
& -S_1(N-1) S_2(N-1) - 2\zeta_2 S_1(N-1) \Bigg] + \frac{2}{N(N+1)} S_1^2(N-1) \\
& + \frac{N^4 + 16\,N^3 + 15\,N^2 - 8\,N - 4}{N^2\,(N+1)^2\,(N+2)} S_2(N-1) \\
& + \frac{3\,N^4 + 2\,N^3 + 3\,N^2 - 4\,N - 4}{2N^2\,(N+1)^2\,(N+2)} \zeta_2 \\
& + \frac{N^4 - N^3 - 16N^2 + 2N + 4}{N^2\,(N+1)^2\,(N+2)} S_1(N-1) + \frac{P_2(N)}{2N^4\,(N+1)^4\,(N+2)} \Bigg\} \\
& + 4C_A T_R \Bigg\{ \frac{N^2+N+2}{N(N+1)(N+2)} \Bigg[4\mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N) + \frac{1}{3} S_1^3(N) + 3 S_2(N) S_1(N) \\
& + \frac{8}{3} S_3(N) + \beta''(N+1) - 4\beta'(N+1) S_1(N) - 4\beta(N+1)\zeta_2 + \zeta_3 \Bigg] \\
& - \frac{N^3 + 8\,N^2 + 11\,N + 2}{N\,(N+1)^2\,(N+2)^2} S_1^2(N) - 2\,\frac{N^4 - 2N^3 + 5N^2 + 2N + 2}{(N-1)\,N^2\,(N+1)^2\,(N+2)} \zeta_2 \\
& - \frac{7N^5 + 21N^4 + 13N^3 + 21N^2 + 18N + 16}{(N-1) N^2\,(N+1)^2\,(N+2)^2} S_2(N) \\
& - \frac{N^6 + 8\,N^5 + 23\,N^4 + 54\,N^3 + 94\,N^2 + 72\,N + 8}{N\,(N+1)^3\,(N+2)^3} S_1(N) \\
& - 4\,\frac{(N^2 - N - 4)}{(N+1)^2\,(N+2)^2} \beta'(N+1) + \frac{P_3(N)}{(N-1) N^4 (N+1)^4 (N+2)^4} \Bigg\}
\end{aligned}
$$

Mellin Transforms

$$
\begin{aligned}
\mathbf{M}[\ln(1+z)](N) &= \frac{1}{N}\left\{\ln(2) - (-1)^N\left[S_{-1}(N) + \ln(2)\right]\right\} \\
&= \frac{1}{N}\left[\ln(2) - \beta(N+1)\right] \\
\mathbf{M}[\ln(z)\ln(1+z)](N) &= -\frac{1}{N^2}\left[\ln(2) - \beta(N+1)\right] - \frac{1}{N}\beta'(N+1) \\
\mathbf{M}[\ln^2(z)\ln(1+z)](N) &= \frac{2}{N^3}\left\{\ln(2) - (-1)^N\left[S_{-1}(N) + \ln(2)\right]\right\} \\
&\quad -(-1)^N\frac{2}{N^2}\left[S_{-2}(N) + \frac{\zeta_2}{2}\right] - (-1)^N\frac{2}{N}\left[S_{-3}(N) + \frac{3}{4}\zeta_3\right] \\
&= \frac{2}{N^3}\left[\ln(2) - \beta(N+1)\right] + \frac{2}{N^2}\beta'(N+1) - \frac{1}{N}\beta''(N+1) \\
\mathbf{M}[\mathrm{Li}_2(-z)](N) &= -\frac{\zeta_2}{2N} + \frac{1}{N^2}\left\{\ln(2) - (-1)^N\left[S_{-1}(N) + \ln(2)\right]\right\} \\
&= -\frac{\zeta_2}{2N} + \frac{1}{N^2}\left[\ln(2) - \beta(N+1)\right] \\
\mathbf{M}[\ln(z)\mathrm{Li}_2(-z)](N) &= \frac{\zeta_2}{2N^2} - \frac{2}{N^3}\left[\ln(2) - \beta(N+1)\right] - \frac{1}{N^2}\beta'(N+1) \\
\mathbf{M}[\mathrm{Li}_2(-z) + \ln(z)\ln(1+z)](N) &= -\frac{1}{2N}\left[\zeta_2 + 2\beta'(N+1)\right] \\
\mathbf{M}[\mathrm{Li}_3(-z)](N) &= -\frac{3}{4N}\zeta_3 + \frac{1}{2N^2}\zeta_2 - \frac{1}{N^3}\left[\ln(2) - \beta(N+1)\right]
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{M}[\Phi_1(z)](N) &= \frac{(-1)^{N+1}}{N}\left\{2S_{1,-2}(N)+\zeta_2\left[S_1(N)-S_{-1}(N)\right]\right\} \\
&\quad +\frac{\left[1+(-1)^{N+1}\right]}{N}\left[\frac{\zeta_3}{4}-\zeta_2\ln(2)\right] \\
&= \frac{1}{N}\left\{2\mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N)-\frac{2}{N}\zeta_2+\frac{2}{N^2}S_1(N)+3\zeta_2\beta(N+1)\right. \\
&\quad \left.+2S_1(N)\beta'(N+1)-\beta''(N+1)+\frac{\zeta_3}{4}-\zeta_2\ln(2)\right\}\ ,
\end{aligned}
$$

$$
\Phi_1(z)=2\mathrm{Li}_2(-z)\ln(1+z)+\ln^2(1+z)\ln(z)+2S_{1,2}(-z)\ .
$$

$$
\begin{aligned}
P_{qq}^{(0)}(N) &= 4C_F\left[-2S_1(N-1)+\frac{(N-1)(3N+2)}{2N(N+1)}\right]\\
P_{qg}^{(0)}(N) &= 8T_RN_F\frac{N^2+N+2}{N(N+1)(N+2)}\\
P_{gg}^{(0)}(N) &= 8C_A\left[-S_1(N-1)-\frac{N^3-3N-4}{(N-1)N(N+1)(N+2)}\right]+2\beta_0\\
P_{gq}^{(0)}(N) &= 4C_F\frac{N^2+N+2}{(N-1)N(N+1)}\\
\widehat{P}_{qq}^{\rm PS,(1)}(N) &= 16C_FT_R\frac{5N^5+32N^4+49N^3+38N^2+28N+8}{(N-1)N^3(N+1)^3(N+2)^2}
\end{aligned}
$$

$$
\begin{aligned}
P_{qq,Q}^{\rm NS,(1)}(N)=\widehat{P}_{qq}^{\rm NS,(1)} &= C_FT_R\left\{\frac{160}{9}S_1(N-1)-\frac{32}{3}S_2(N-1)\right.\\
&\left.-\frac{4}{9}\frac{(N-1)(3N+2)(N^2-11N-6)}{N^2(N+1)^2}\right\}\\
\widehat{P}_{qg}^{(1)}(N) &= 8C_FT_R\left\{2\frac{N^2+N+2}{N(N+1)(N+2)}\left[S_1^2(N)-S_2(N)\right]-\frac{4}{N^2}S_1(N)\right.\\
&\left.+\frac{5N^6+15N^5+36N^4+51N^3+25N^2+8N+4}{N^3(N+1)^3(N+2)}\right\}\\
&+16C_AT_R\left\{-\frac{N^2+N+2}{N(N+1)(N+2)}\left[S_1^2(N)+S_2(N)-\zeta_2-2\beta'(N+1)\right]\right.\\
&\left.+4\frac{2N+3}{(N+1)^2(N+2)^2}S_1(N)+\frac{P_1(N)}{(N-1)N^3(N+1)^3(N+2)^3}\right\}~,
\end{aligned}
$$

Structure of Expression

$$\beta(N+1) = (-1)^N \left[S_{-1}(N) + \ln(2)\right]$$

$$\mathbf{M}\left[\frac{\mathrm{Li}_2(x))}{1+x}\right](N) - \zeta_2\beta(N) = (-1)^N \left[S_{-2,1}(N-1) + \frac{5}{8}\zeta_3\right]$$

- still present in individual terms
- cancels in the complete expression
- $\Longrightarrow$ **no harmonic sum with index {-1}**

Weight	Number of Sums	a-basic sums	Sums $\neg\{-1\}$	a-basic sums	Sums $i > 0$	a-basic sums
1	2	2	1	1	1	1
2	6	3	3	2	2	1
3	18	8	7	4	4	2
4	54	18	17	7	8	3
5	162	48	41	16	16	6
6	486	116	99	30	32	9
7	1458	312	239	68	64	18
8	4374	810	577	140	128	30
9	13122	2184	1393	308	256	56
10	39366	5880	3363	664	512	99

Table 1: Number of harmonic sums and harmonic sums, which do not contain the index $\{-1\}$, and the respective numbers of basic sums by which all sums can be expressed using the algebraic relations in dependence of the weight of the sums.

Final Structure of Wilson Coefficients

$\mu^2 = Q^2$

$$
\begin{aligned}
H_{L,g}^{\rm S}\left(x, a_s, \frac{Q^2}{m^2}\right) &= a_s \hat{c}_{L,g}^{(1)} + a_s^2 \left[\frac{1}{2}\hat{P}_{qg}^{(0)} c_{L,q}^{(1)} \ln\left(\frac{Q^2}{m^2}\right) + \hat{c}_{L,g}^{(2)}\right] \\
&+ a_s^3 \Biggl\{ \Biggl[\frac{1}{8}\hat{P}_{qg}^{(0)}\left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right] \ln^2\left(\frac{Q^2}{m^2}\right) + \frac{1}{2}\hat{P}_{qg}^{(1)} \ln\left(\frac{Q^2}{m^2}\right) \\
&\quad + a_{Qg}^{(2)} + \overline{a}_{Qg}^{(1)}\left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right]\Biggr] c_{L,q}^{(1)} + \frac{1}{2}\hat{P}_{qg}^{(0)} \ln\left(\frac{Q^2}{m^2}\right) c_{L,q}^{(2)} + \hat{c}_{L,g}^{(3)} \Biggr\} \\
H_{L,q}^{\rm PS}\left(x, a_s, \frac{Q^2}{m^2}\right) &= a_s^2 \hat{c}_{L,q}^{\rm PS,(2)} \\
&+ a_s^3 \left\{ \left[-\frac{1}{8}\hat{P}_{qg}^{(0)} P_{gq}^{(0)} \ln^2\left(\frac{Q^2}{m^2}\right) + \frac{1}{2}\hat{P}_{qq}^{\rm PS,(1)} \ln\left(\frac{Q^2}{m^2}\right) + a_{Qq}^{\rm PS,(2)} - \overline{a}_{Qg}^{(1)} P_{gq}^{(0)}\right] c_{L,q}^{(1)} + \hat{c}_{L,q}^{\rm PS,(3)} \right\} \\
H_{L,q}^{\rm NS}\left(x, a_s, \frac{Q^2}{m^2}\right) &= a_s^2 \left[-\beta_{0,Q} c_{L,q}^{(1)} \ln\left(\frac{Q^2}{m^2}\right) + \hat{c}_{L,q}^{\rm NS,(2)}\right] \\
&+ a_s^3 \Biggl\{ \left[-\frac{1}{4}\beta_{0,Q} P_{qq}^{(0)} \ln^2\left(\frac{Q^2}{m^2}\right) + \frac{1}{2}\hat{P}_{qq}^{\rm NS,(1)} \ln\left(\frac{Q^2}{m^2}\right) + a_{qq,Q}^{\rm NS,(2)} + \frac{1}{4}\beta_{0,Q}\zeta_2 P_{qq}^{(0)}\right] \\
&\quad \times c_{L,q}^{(1)} + \hat{c}_{L,q}^{\rm NS,(3)} \Biggr\}~.
\end{aligned}
$$

4. Small x Limit

Anticipated result : $\mu^2 = Q^2$

$$H_L^S(z) = a_s^2 \frac{d_1^{(1)}}{z} + \sum_{k=2}^{\infty} a_s^{k+1} \left[d_k^{(1)} \frac{\ln^{k-1}(z)}{z} + d_k^{(2)} \frac{\ln^{k-2}(z)}{z} + \dots \right]$$

$$d_{1,i}^{(1)} = -32 C_i T_R \frac{1}{9}$$

(Catani, Ciafaloni, Hautmann, 1990; Buza, Matiounine, Smith, Migneron, van Neerven, 1996)

$$\begin{aligned}
d_{2,i}^{(1)} &= \frac{128}{3} C_A C_i T_R \left[-\frac{34}{9} + \zeta_2 \right] \\
d_{2,A}^{(2)} &= -32 C_A C_F T_R \left[\frac{1}{3} \ln^2 \left(\frac{Q^2}{m^2} \right) - \frac{10}{9} \ln \left(\frac{Q^2}{m^2} \right) + \frac{28}{27} \right] \\
& -\frac{256}{27} C_F T_R^2 (2N_F + 1) \ln \left(\frac{Q^2}{m^2} \right) \\
& +\frac{32}{3} C_A^2 T_R \left[-\frac{2756}{27} + \frac{65}{3} \zeta_2 + 20 \zeta_3 \right] + \frac{64}{3} C_A C_F T_R \left[\frac{56}{9} - \zeta_2 - 4 \zeta_3 \right] \\
& +C_F T_R^2 (2N_F + 1) \frac{64}{9} \left[\frac{121}{9} - 4 \zeta_2 \right] + C_A T_R^2 (2N_F + 1) \frac{32}{9} \left[\frac{101}{9} - 8 \zeta_2 \right] . \\
d_{2,F}^{(2)} &= -32 C_F^2 T_R \left[\frac{1}{3} \ln^2 \left(\frac{Q^2}{m^2} \right) - \frac{10}{9} \ln \left(\frac{Q^2}{m^2} \right) + \frac{28}{27} \right] \\
& +32 C_A C_F T_R \left[-\frac{899}{27} + 7 \zeta_2 + \frac{20}{3} \zeta_3 \right] + \frac{64}{3} C_F^2 T_R \left[\frac{56}{9} - \zeta_2 - 4 \zeta_3 \right] \\
& +C_F T_R^2 (2N_F + 1) \frac{256}{9} \left[\frac{53}{9} - \zeta_2 \right] .
\end{aligned}$$

blue :this paper

purple : Moch, Vermaseren, Vogt, $m = 0, \overline{\mathrm{MS}}$, 2005.

True HQ corrections show sub-leading small-z behaviour @ $O(a_s^3)$.

5. Numerical Results

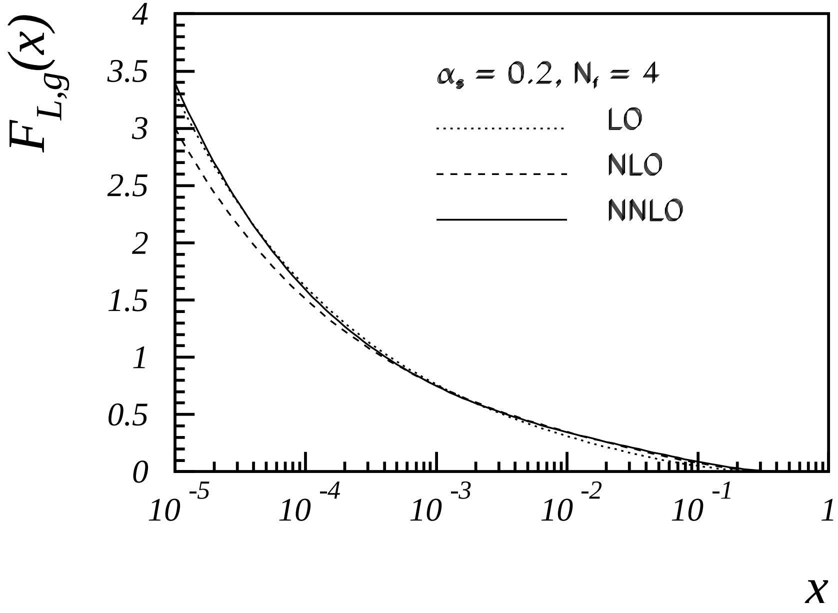
Numerical evaluation for
Gluon, quark-singlet & non-singlet HQ contributions

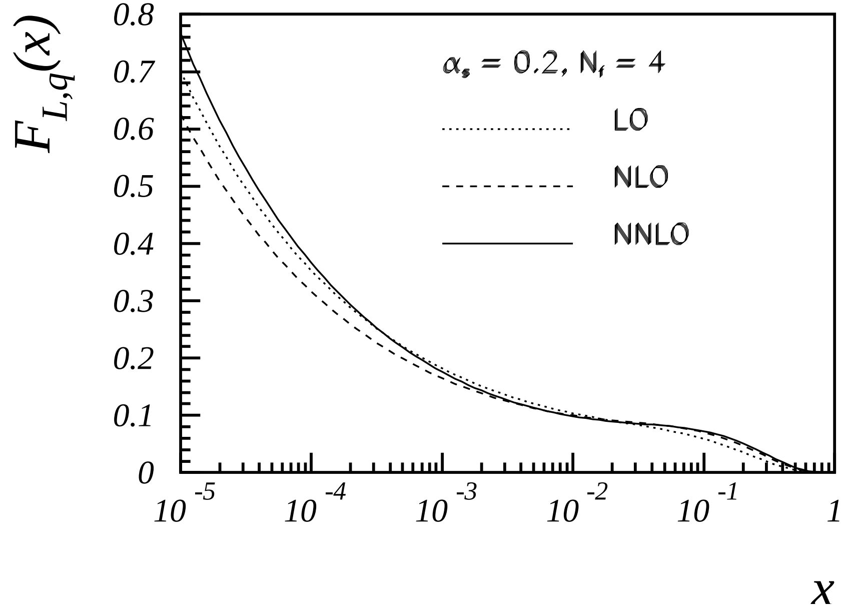
Ansatz :

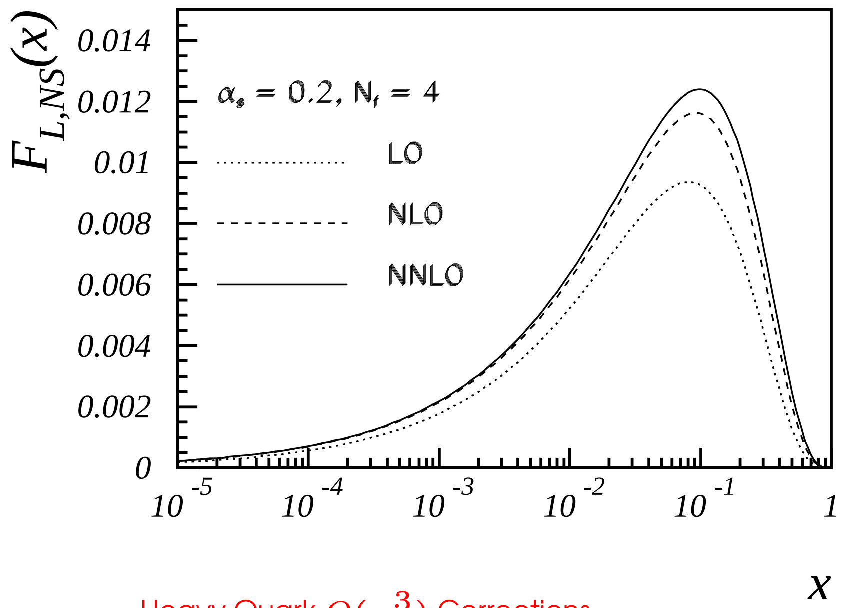
$$\begin{aligned} xg(x) &= 1.6x^{-0.3}(1-x)^{4.5}\left(1-0.6x^{0.3}\right) \\ xq(x) &= 0.6x^{-0.3}(1-x)^{3.5}\left(1+5x^{0.8}\right) \\ xq_{\rm NS}(x) &= x^{0.5}(1-x)^3, \qquad @ \quad Q^2 = 30\,\text{GeV}^2 \end{aligned}$$

5. Numerical Results

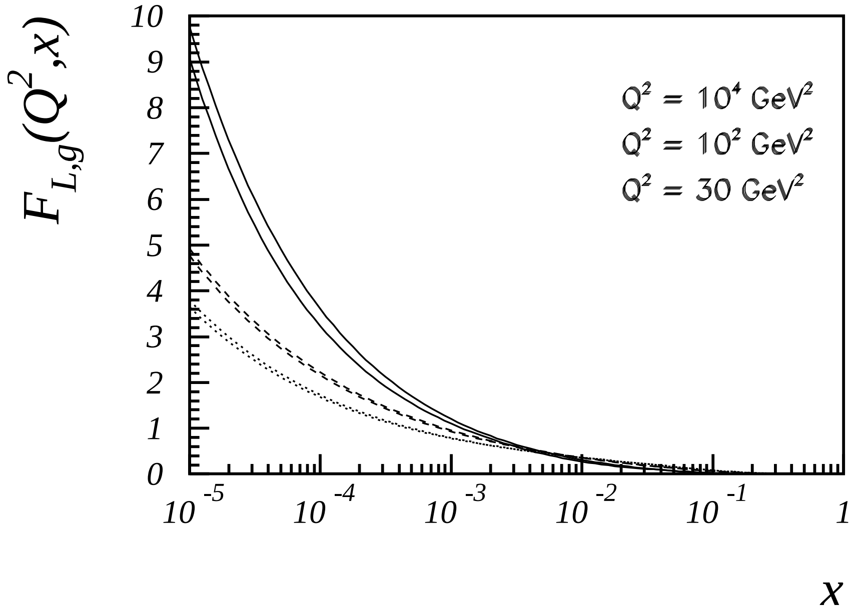
Massless case : (Moch, Vermaseren, Vogt, 2004)

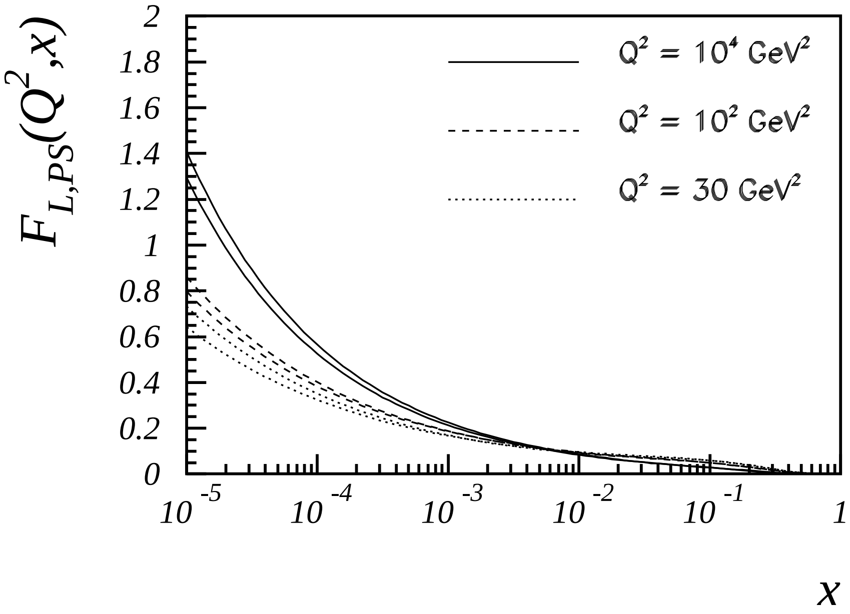






Heavy Flavor Contributions :





Correction to NS distribution very small.

6. Conclusions

- Massive Operator Matrix-Elements are a tool to calculate the heavy flavor contributions to QCD structure functions in the region $Q^2 \gg m^2$ relating the massless to the massive result.

- All contributions but power corrections $(m^2/Q^2)^k$ are obtained by this method.

- The method was applied to $F_L^{Q\overline{Q}}(x, Q^2)$ to $O(\alpha_s^3)$. Here the heavy flavor contributions are determined by the 2–loop OME's unlike $F_2(x, Q^2)$, which depends on the 3-loop OME's.

- The small x asymptotics of the corresponding Wilson coefficients has been determined. The $O(\alpha_s^2)$ true heavy quark corrections belong to the next-to-leading class.

- The numerical results presented apply to the high Q^2 terms and will be supplemented by the lower Q^2 corrections.