LHC phenomenology on the loop level

A new method for one-loop amplitudes.

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Content:

- Motivation: LHC@NLO
- Reduction formalism for one-loop diagrams
- Numerical evaluation of Feynman parameter integrals
- Summary

Introduction

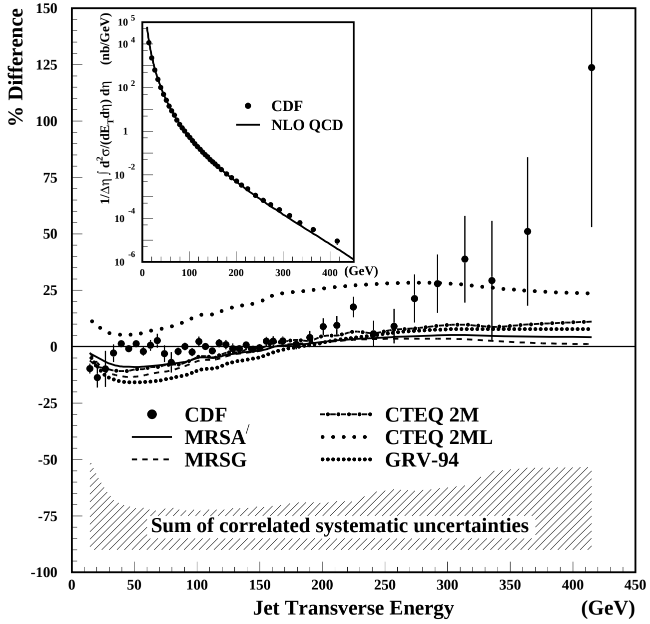
The decade of hadron colliders at the TeV scale

Tevatron: $P\bar{P}$ collider at Fermilab:

Run I: $\sqrt{s} = 1.8$ TeV (1992-1996)

Run II: $\sqrt{s} = 1.96$ TeV (2001-2007 $\pm$ x)

- discovery of the top quark (1994/95)
- electroweak, b and jet physics
- new physics searches: leptoquarks, SUSY, extra dimensions...



CDF, Abe et al., Phys. Rev. Lett. 77, 438 (1996).

The decade of hadron colliders at the TeV scale

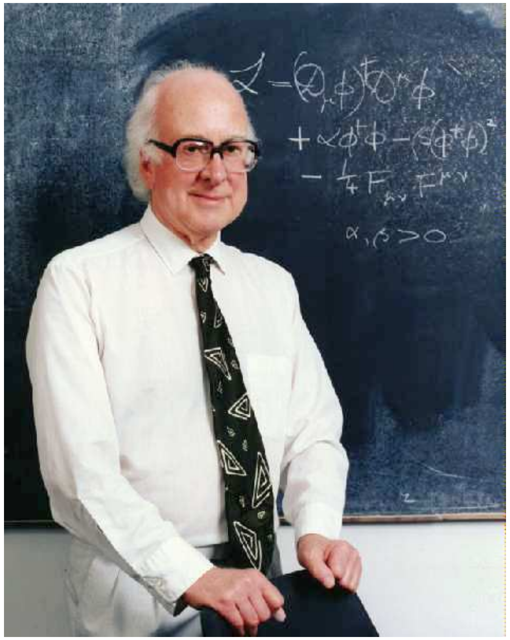
LHC: PP collider at CERN, $\sqrt{s} = 14$ TeV, start 2007 + x

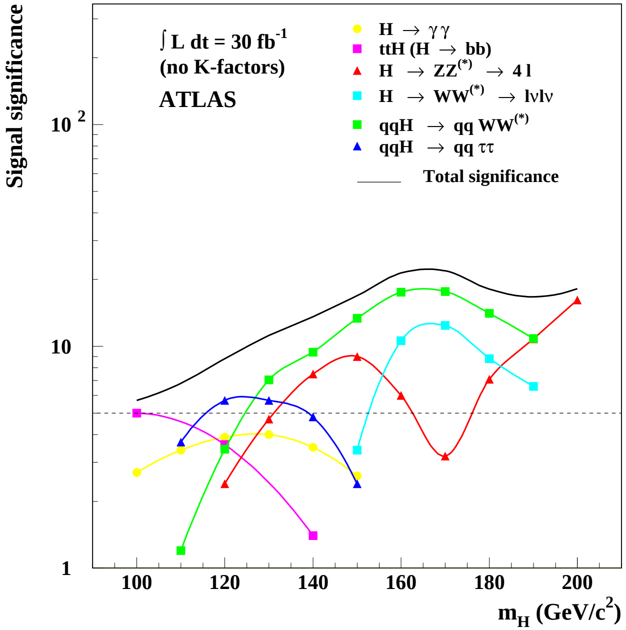
- Higgs mechanism, electroweak symmetry breaking
- low energy SUSY models
- strong interaction scenarios like Technicolour theories
- models with extra space time dimensions
- ...

LHC may give hints on physics we did not even think about !!!

We have to keep tuned for the unexpected !!!

But first get the Higgs...





"There appears to be some hope that [. . .] the combination of spontaneous symmetry breakdown with the gauge principle may provide the basis for an understanding of the broken symmetries of high–energy physics."

[P.W. Higgs, Phys.Rev. 145:1156 (1966)]

P.W. Higgs, Phys.Lett. 12 (1964), Phys.Rev.Lett. 13 (1964), Phys.Rev. 145 (1966).
F. Englert, R. Brout, Phys.Rev.Lett. 13 (1964).
G.S. Guralnik, C.R. Hagen, T.W. Kibble, Phys.Rev.Lett. 13 (1964).

Theoretical control over backgrounds needed:

The Higgs boson, new physics etc. might come as:

- well detectable resonances: $S/B >> 1$
  $\Rightarrow$ Good control of B necessary for a quantitative understanding of new and standard physics (e.g. measurement of couplings).
- faint irregularities: $S/B << 1$
  $\Rightarrow$ Good control of B needed to see them at all.

– Accurate measurement of the background not possible in all kinematical regions.

– Extrapolation to the signal region needs theoretical knowledge of distribution shapes, e.g. $H \to W^*W^* \to l\nu l'\nu'$.

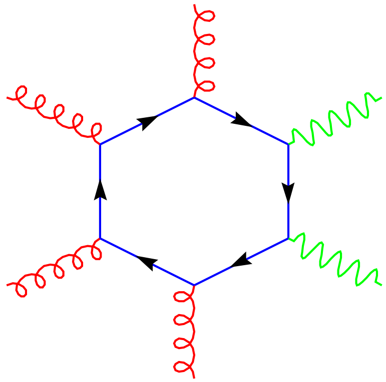
Signal/background processes for Higgs boson search:

To make reliable estimates of backgrounds for Higgs signals

- $PP \to H$ by Gluon Fusion (NNLO), $H \to \gamma\gamma$, $H \to WW^{(*)}$
- $PP \to H + 1, 2\, j$ (NLO $m_{Top} \to \infty$)
- $PP \to Hjj$ by Weak Boson Fusion (NLO), $H \to \tau^+\tau^-$, $H \to WW^{(*)}$, $H \to \gamma\gamma$

the following NLO computations are needed:

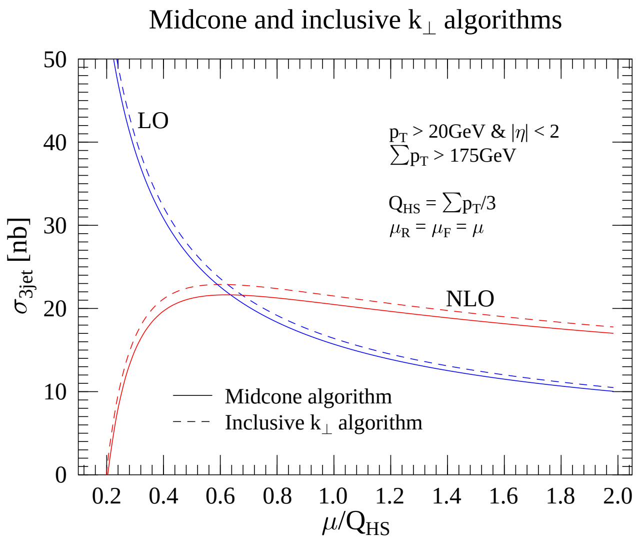
- $PP \to \gamma\gamma + 0, 1, 2$ jets
- $PP \to WW^*, ZZ^* + 0, 1, 2$ jets
- $PP \to t\bar{t} + 0, 1, 2$ jets
- $PP \to \tau^+\tau^- + 0, 1, 2$ jets
- $PP \to 1, 2, 3\, V + 3, 2, 1$ jets $\quad (V = \gamma, W, Z)$

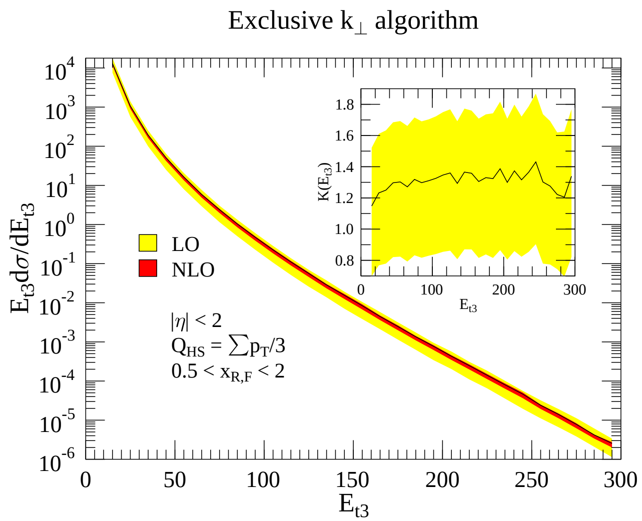


Efficient methods for partonic 5-/6-point–processes mandatory!

Scale uncertainties:

Example: 3 jet cross section at NLO





Z. Nagy, Phys.Rev. D68 (2003).

> Higher order QCD calculations are mandatory to soften scale dependence of phenomenological predictions !!!

Observables have to be defined infrared safe !!!

i.e. insensitive to emission of an extra soft/collinear quark or gluon.

Status QCD@NLO for LHC:

$2 \to 2$: everything you want

$2 \to 3$: $PP \to 3\,j,\, Vjj,\, \gamma\gamma j,\, Vb\bar{b},\, t\bar{t}H,\, b\bar{b}H,\, jjH,\, (t\bar{t}j)$

Bern ,Campbell, De Florian, Del Duca, Dixon, R.K. Ellis, Giele, Glover, Kilgore, Kosower, Kunszt, Maltoni, Miller, Nagy, Trocsanyi, Beenakker, Dittmaier, Plümper, Spira, Zerwas, Dawson, Orr, Reina, Wackeroth, Brandenburg, Uwer, Weinzierl, Zanderighi. . .

$2 \to 4$: everything remains to be done !
Efficient methods needed

(for $e^+e^- \to 4f$ see talks of S. Dittmaier, A. Denner)

Why another method?

- Evaluation of one-loop amplitudes is bottleneck for efficient calculations of multi-particle cross sections at NLO

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- $\Rightarrow$ Suitable method should be numerically robust and easy to automate
- Activ field: Bern, Dixon, Kosower; Del Aguila, Pittau; Van Hameren, Vollinga, Weinzierl; Soper, Nagy; R.K. Ellis, Giele, Glover, Zanderighi; Denner, Dittmaier,. . .

A new method for one-loop multi-leg amplitudes

T.B., J.Ph. Guillet, G. Heinrich, E. Pilon, C. Schubert: hep-ph/0504267

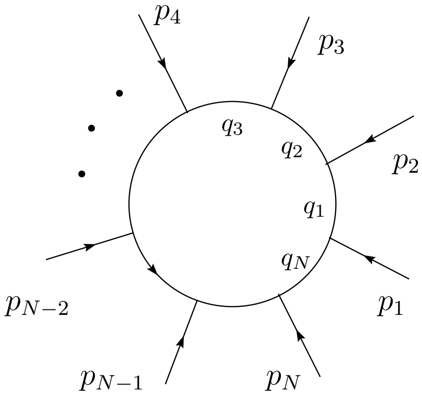
Method is

- valid for an arbitrary number N of external legs
- valid for massless and massive particles (uses dimensional regularisation for IR poles)
- designed to be numerically robust and fast due to a combination of semi-numerical and algebraic approaches

Tensor integrals

$$I_N^{n,\mu_1\ldots\mu_r}(a_1,\ldots,a_r) = \int \frac{d^n k}{i\,\pi^{n/2}}\,\frac{q_{a_1}^{\mu_1}\ \ldots q_{a_r}^{\mu_r}}{(q_1^2-m_1^2+i\delta)\ldots(q_N^2-m_N^2+i\delta)}$$

$q_i = k + r_i\,,\ \ r_j - r_{j-1} = p_j\,,\ \sum_{j=1}^N p_j = 0$



advantages of this representation:

- combinations $q_i = k + r_i$ appear naturally (e.g. fermion propagators)
- allows for a manifestly translation invariant formulation Lorentz structure carried by difference vectors $\Delta_{i\,j}^{\mu} = r_i^{\mu} - r_j^{\mu}$ and $g^{\mu\nu}$

Lorentz structure and form factors

$$I_N^{n,\,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;\,S) =$$

$$\sum_{l_1\cdots l_r\in S}\left[\Delta^{\cdot}_{l_1\cdot}\cdots\Delta^{\cdot}_{l_r\cdot}\right]^{\{\mu_1\cdots\mu_r\}}_{\{a_1\cdots a_r\}}\;A^{N,r}_{l_1\ldots,l_r}(S)$$

$$+\sum_{l_1\cdots l_{r-2}\in S}\left[g^{\cdot\cdot}\Delta^{\cdot}_{l_1\cdot}\cdots\Delta^{\cdot}_{l_{r-2}\cdot}\right]^{\{\mu_1\cdots\mu_r\}}_{\{a_1\cdots a_r\}}\;B^{N,r}_{l_1\ldots,l_{r-2}}(S)$$

$$+\sum_{l_1\cdots l_{r-4}\in S}\left[g^{\cdot\cdot}g^{\cdot\cdot}\Delta^{\cdot}_{l_1\cdot}\cdots\Delta^{\cdot}_{l_{r-4}\cdot}\right]^{\{\mu_1\cdots\mu_r\}}_{\{a_1\cdots a_r\}}\;C^{N,r}_{l_1\ldots,l_{r-4}}(S)$$

example

$$I_N^{n,\mu_1\mu_2}(a_1,a_2;S)=\sum_{l_1,l_2\in S}\Delta^{\mu_1}_{l_1\,a_1}\,\Delta^{\mu_2}_{l_2\,a_2}\,A^{N,2}_{l_1\,l_2}(S)+g^{\mu_1\,\mu_2}\,B^{N,2}(S)$$

kinematical information in matrix: $\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$

$$I_N^n(S) \;=\; (-1)^N \Gamma(N - \frac{n}{2}) \int \prod_{i=1}^{N} dz_i \, \delta(1 - \sum_{l=1}^{N} z_l) \, \left(R^2\right)^{\frac{n}{2}-N}$$

$$R^2 = -\frac{1}{2} \sum_{i,j=1}^{N} z_i \, \mathcal{S}_{ij} z_j \; - i\delta$$

Notation:

matrix $\mathcal{S}$ is defined by ordered set S of propagator labels

matrix corresponding to integral with propagator $q_j^2 - m_j^2$ missing is denoted by $\mathcal{S}^{\{j\}}$, corresponding to set $S \setminus \{j\}$

Reduction of scalar integrals:

$$I_N^n(S) = \sum_{i\in S} b_i(S) \int d^n\bar{k} \, \frac{(q_i^2 - m_i^2)}{\prod_{j\in S}(q_j^2 - m_j^2 + i\,\delta)}$$

$$+ \int d^n\bar{k} \, \frac{1 - \sum_{i\in S} b_i(S)\,(q_i^2 - m_i^2)}{\prod_{j\in S}(q_j^2 - m_j^2 + i\,\delta)}$$

$$\stackrel{!}{=} I_{div}(S) + I_{fin}(S)$$

If $\sum_{i\in S} b_i(S)\,\mathcal{S}_{ij} = 1$ then

$$I_{fin}(S) = -B(S)\,(N - n - 1)\,I_N^{n+2}(S)$$

$$B(S) = \sum_{i\in S} b_i(S)\;, \; B\,\det\mathcal{S} = (-1)^{N+1}\det G, \; G_{ij} = 2\,r_i\cdot r_j$$

Reduction of tensor integrals:

$$
\begin{aligned}
& I_N^{n,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S) = \\
& -\sum_{j\in S} \mathcal{C}^{\mu_1}_{j a_1} \int d\bar{k}\, \frac{(q_j^2 - m_j^2)\, q_{a_2}^{\mu_2} \ldots q_{a_r}^{\mu_{a_r}}}{\prod_{i\in S}(q_i^2 - m_i^2 + i\delta)} \\
& + \int d\bar{k}\, \frac{\left[q_{a_1}^{\mu_1} + \sum_{j\in S} \mathcal{C}^{\mu_1}_{j a_1}\,(q_j^2 - m_j^2)\right]\; q_{a_2}^{\mu_2} \ldots q_{a_r}^{\mu_r}}{\prod_{i\in S}(q_i^2 - m_i^2 + i\delta)} \\
\stackrel{!}{=}\; & I_{div} + I_{fin}
\end{aligned}
$$

If $\sum_{i\in S} \mathcal{C}^{\mu}_{i\,a}(S)\,\mathcal{S}_{ij} = \Delta^{\mu}_{j\,a} = r^{\mu}_{j} - r^{\mu}_{a}$ then

$$I_{fin}(S) = \int d\vec{Z} \int d^n\bar{l}\, \frac{\tilde{q}^{\mu_1}_{a_1} \ldots \tilde{q}^{\mu_{r-1}}_{a_{r-1}}}{(l^2 - R^2)^N}$$

$$\left[l_\nu \left(\mathcal{T}^{\mu_r \nu}_{a_r d} + 2\, \mathcal{V}^{\mu_r}_{a_r} \sum_{i \in S} z_i \Delta^{\nu}_{d\,i} \right) + \mathcal{V}^{\mu_r}_{a_r} \,(l^2 + R^2) \right]$$

where

$$l = k + \sum_{i \in S} z_i\, r_i, \quad \tilde{q}^{\mu}_{a} = l^{\mu} + \sum z_i \Delta^{\mu}_{ai}$$

$$\mathcal{V}^{\mu}_{a} = \sum_{j \in S} \mathcal{C}^{\mu}_{ja} = \sum_{k \in S} b_k\, \Delta^{\mu}_{k\,a}$$

$$\mathcal{T}^{\mu\nu}_{a_1\, a_2} = g^{\mu\nu} + 2 \sum_{j \in S} \mathcal{C}^{\mu}_{j\, a_1}\, \Delta^{\nu}_{j\, a_2}$$

solving the defining equation for arbitrary N

if $\mathcal{S}$ is invertible, i.e. for $N < 7$ (non-exceptional kinematics):

$$\sum_{i \in S} \mathcal{C}^{\mu}_{i\,a}(S)\,\mathcal{S}_{ij} \;=\; \Delta^{\mu}_{j\,a}$$

$$\Leftrightarrow\; \mathcal{C}^{\mu}_{i\,a}(S) \;=\; \sum_{j \in S} \left(\mathcal{S}^{-1}\right)_{ij} \Delta^{\mu}_{ja}$$

Otherwise use pseudo-inverse H_{ij} to Gram matrix

$$\mathcal{C}^{\mu}_{i\,b} \;=\; -\sum_{j \in S \setminus \{a\}} H_{ij}\,\Delta^{\mu}_{j\,a} + W^{\mu}_{i} \quad , \quad i \in S \setminus \{a\}$$

$$\mathcal{C}^{\mu}_{a\,b} \;=\; -\sum_{j \in S \setminus \{a\}} \mathcal{C}^{\mu}_{j\,b}$$

The case $N \geq 5$

Due to d=4 kinematics:

$$\mathcal{V}_b^{\mu} \equiv 0 \;\text{ and }\; \mathcal{T}^{\mu\nu}_{a_1\,a_2} \equiv 0 \;\text{ for } N > 5$$

$\Rightarrow$ NO higher dimensional integrals for $N > 5$!

$$I_N^{n,\,\mu_1\ldots\mu_r}(a_1,\ldots,a_r\,;S) \sim \sum_j \mathcal{C}^{\mu_r}_{j\,a_r}\, I_{N-1}^{n,\,\mu_1\ldots\mu_{r-1}}(a_1,\ldots,a_{r-1}\,;S\backslash\{j\})$$

$\Rightarrow$ N and tensor rank are reduced at the same time !

case $N = 5$ more tricky, proof relies on $\mathcal{T}^{\mu\,\nu}_{a\,b} = 2\,\mathcal{V}_a^{\mu}\,\mathcal{V}_b^{\nu}/B$

Result:

- no higher dimensional integrals I_N^{n+2m} for $N > 4$
- no inverse Gram determinants

N=5 rank 2 example

$$I_5^{n,\,\mu_1\mu_2}(a_1,a_2;S) = g^{\mu_1\,\mu_2}\,B^{5,2}(S) + \sum_{l_1,l_2\in S} \Delta^{\mu_1}_{l_1\,a_1}\,\Delta^{\mu_2}_{l_2\,a_2}\,A^{5,2}_{l_1\,l_2}(S)$$

$$B^{5,2}(S) = -\frac{1}{2}\sum_{j\in S} b_j\, I_4^{n+2}(S\setminus\{j\})$$

$$A^{5,2}_{l_1 l_2}(S) =$$

$$\sum_{j\in S}\left(\mathcal{S}^{-1}{}_{j\,l_1}\,b_{l_2} + \mathcal{S}^{-1}{}_{j\,l_2}\,b_{l_1} - 2\,\mathcal{S}^{-1}{}_{l_1\,l_2}\,b_j + b_j\,\mathcal{S}^{\{j\}-1}{}_{l_1\,l_2}\right) I_4^{n+2}(S\setminus\{j\})$$

$$+\frac{1}{2}\sum_{j\in S}\sum_{k\in S\setminus\{j\}}\left[\mathcal{S}^{-1}{}_{j\,l_2}\,\mathcal{S}^{\{j\}-1}{}_{k\,l_1} + \mathcal{S}^{-1}{}_{j\,l_1}\,\mathcal{S}^{\{j\}-1}{}_{k\,l_2}\right] I_3^{n}(S\setminus\{j,k\})$$

- Algebraic separation of IR poles, contained in 3-point integrals

N=6 rank 1 example

$$I_6^{n,\,\mu}(a;S) = -\sum_{j\in S} \mathcal{C}_{j\,a}^{\mu}\, I_5^n(S\setminus\{j\})$$

$$= -\sum_{j,l\in S} \Delta_{la}^{\mu}\, \mathcal{S}_{lj}^{-1} \sum_{k\in S} b_k^{\{j\}}$$

$$\left[B^{\{j,k\}}\, I_4^{n+2}(S\setminus\{j,k\}) + \sum_{m\in S\setminus\{j,k\}} b_m^{\{j,k\}}\, I_3^n(S\setminus\{j,k,m\}) \right]$$

- For rank $R > 2$ reduction to pure $N = 3, 4$ scalar integrals not longer possible $\rightarrow$ Feynman parameters in the numerator
- Explicite representations for all formfactors for $r \leq N \leq 5$ worked out

Basis integrals

$$I_3^n(j_1,\dots,j_r) = -\Gamma\left(3-\frac{n}{2}\right)\int_0^1\prod_{i=1}^{3}dz_i\,\delta(1-\sum_{l=1}^{3}z_l)\,\frac{z_{j_1}\dots z_{j_r}}{(-\frac{1}{2}\,z\cdot\mathcal{S}\cdot z-i\delta)^{3-n/2}}$$

$$I_3^{n+2}(j_1) = -\Gamma\left(2-\frac{n}{2}\right)\int_0^1\prod_{i=1}^{3}dz_i\,\delta(1-\sum_{l=1}^{3}z_l)\,\frac{z_{j_1}}{(-\frac{1}{2}\,z\cdot\mathcal{S}\cdot z-i\delta)^{2-n/2}}$$

$$I_4^{n+2}(j_1,\dots,j_r) = \Gamma\left(3-\frac{n}{2}\right)\int_0^1\prod_{i=1}^{4}dz_i\,\delta(1-\sum_{l=1}^{4}z_l)\,\frac{z_{j_1}\dots z_{j_r}}{(-\frac{1}{2}\,z\cdot\mathcal{S}\cdot z-i\delta)^{3-n/2}}$$

$$I_4^{n+4}(j_1) = \Gamma\left(2-\frac{n}{2}\right)\int_0^1\prod_{i=1}^{4}dz_i\,\delta(1-\sum_{l=1}^{4}z_l)\,\frac{z_{j_1}}{(-\frac{1}{2}\,z\cdot\mathcal{S}\cdot z-i\delta)^{2-n/2}}$$

and scalar integrals $I_3^n, I_3^{n+2}, I_4^{n+2}, I_4^{n+4}$.   Note: $r_{\max}=3$

two alternatives for evaluation:

1. algebraic reduction to scalar integrals (differentiation by parts in parameter space)

2. direct numerical evaluation (contour deformation→ see below)

Algebraic reduction: example N=4

$\to$ inverse Gram determinants $\sim 1/B \sim \det \mathcal{S} / \det G$

$$I_4^{n+2}(l;S) \quad = \quad \frac{1}{B}\left\{ b_l\, I_4^{n+2}(S) + \frac{1}{2}\sum_{j\in S} \mathcal{S}^{-1}{}_{j\,l}\, I_3^n(S\setminus\{j\}) - \frac{1}{2}\sum_{j\in S\setminus\{l\}} b_j\, I_3^n(l;S\setminus\{j\}) \right\}$$

- does not pose a problem in most regions of phase space
- allows fast evaluation
- $\Rightarrow$ use numerical method only in regions where $B = \sum_j b_j$ becomes small

Numerical evaluation of Feynman parameter integrals

see also: D. Soper Phys.Rev.D62 (2000); Y. Kurihara, T. Kaneko, hep-ph/0503003

Note: A numerically stable evaluation of the integrals

$$I_N^D(j_1,\ldots,j_r) \quad = \quad (-1)^N \Gamma(N-\frac{D}{2}) \int_0^\infty d^N x\, \delta(1-\sum_{l=1}^N x_l) \frac{x_{j_1}\ldots x_{j_r}}{(x\cdot \mathcal{S}\cdot x/2+i\varepsilon)^{N-D/2}}$$

would completly solve the 1-loop problem!!!

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Focus on basis integrals: $D=n-2\epsilon$, $N=3$ and $D=6$, $N=4$

- IR divergencies: Only for $D=4 \Rightarrow$ analytical evaluation
- Kinematical singularities: $x\cdot\mathcal{S}\cdot x$ changes sign.

Contour deformation for parameter integrals:

To get rid of $\delta(1 - \sum x_l)$ make sector decomposition:

$$1 = \sum_{l=1}^{N} \theta(x_l > x_1, \ldots, x_{l-1}, x_{l+1}, \ldots, x_N)$$

Integral decays into N terms:

$$I_N^D(j_1, \ldots, j_r) = (-1)^N \Gamma(N - D/2) \sum_{l=1}^{N} J_l(N, D, j_1, \ldots, j_r)$$

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In sector l make variable transform: $x_j = t_j x_l$ for $(j < l)$, $x_j = t_{j-1} x_l$ for $(j > l)$,
Integrate out x_l with δ distribution. Defining $\vec{T} = (t_1, \ldots, t_{l-1}, 1, t_l, \ldots, t_{N-1})$ gives:

$$J_l(N, D, j_1, \ldots, j_r) = \int_0^1 d^{N-1}t \left(\sum_{j=1}^{N} T_j\right)^{N-D-r} \frac{T_{j_1} \ldots T_{j_r}}{\left(T \cdot \mathcal{S} \cdot T/2 - i\delta\right)^{N-D/2}}$$

Denominator $Q(t) = T \cdot \mathcal{S} \cdot T$ singular if

$$Q(t) = \frac{1}{2} \sum_{j,k=1}^{N-1} A_{jk} t_j t_k + \sum_{j=1}^{N-1} B_j t_j + C = 0$$

A, B, C defined by $\mathcal{S}$.

View $Q(t)$ as value of complex 2-form $Q(x)$ along integration "contour", which is the whole $N-1$ dim. hyper cube.

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A, B, C defined by $\mathcal{S}$.

View $Q(t)$ as value of complex 2-form $Q(x)$ along integration "contour", which is the whole $N-1$ dim. hyper cube.

To avoid crossing a pole make Ansatz: $\vec{x} = \vec{t} - i\vec{\tau}(\vec{t})$:

$$Q(\vec{x}) = Q(\vec{t}) - \frac{1}{2} \sum_{j,k=1}^{N-1} A_{jk} \tau_j \tau_k - i \sum_{k=1}^{N-1} \tau_k \sum_{j=1}^{N-1} (A_{jk} t_j + B_k)$$

choose $\vec{\tau}$ such that always: $\mathrm{Im}(Q(x)) < 0$!

The following choice is doing the job:

$$\vec{x}(\vec{t}) = \vec{t} - i\,\vec{\tau}(\vec{t})$$

$$\tau_k = \lambda\, t_k^{\alpha}\, (1 - t_k)^{\beta} \sum_{j=1}^{N-1} (A_{jk} t_j + B_k)$$

contour deformation parametrized by: α, β>0, $\lambda \geq 0$.

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contour deformation parametrized by: α, β>0, $\lambda \geq 0$.
Invariance under homeomorphisms of the contour, $\mathcal{C}_\lambda$:

$$\int_{\mathcal{C}_0} d^{N-1}x f(x) = \int_{\mathcal{C}_\lambda} d^{N-1}x f(x)$$

In the given parametrisation:

$$\int_0^1 d^{N-1}t\; f(\vec{t}) = \int_0^1 d^{N-1}t\; \det\left(\frac{\partial x_i}{\partial t_j}\right)\; f(\vec{t} - i\vec{\tau}(\vec{t}))$$
$$\frac{\partial x_l}{\partial t_j} = \delta_{lj} - i\,\lambda\, t_l^{\alpha-1}(1-t_l)^{\beta-1}\Big[\delta_{lj}[\alpha(1-t_l) - \beta t_l]\Big(\sum_{k=1}^{N-1} A_{lk} t_k + B_l\Big) + t_l(1-t_l)A_{lj}\Big]$$

Comments:

- $\lambda \nabla \cdot Q$ controls the size of the deformation, α, β control the smoothness of the deformation at the integration boundaries.

Comments:

- $\lambda \nabla \cdot Q$ controls the size of the deformation, α, β control the smoothness of the deformation at the integration boundaries.
- $Q = 0$ and $\nabla \cdot Q = 0$: exceptional kinematics $\Leftrightarrow$ normal or anomalous thresholds Split integration at: $t_j = -\sum_{l=1}^{N-1} A_{jl}^{-1} B_l$.

Comparison between numerical/algebraical method:

- All necessary code can be produced by "Code Generators" from algebraic programs
- Analytic representation of $I_4^{n+2}(j_1, \ldots, j_r) \sim 1/\det(G)^r$

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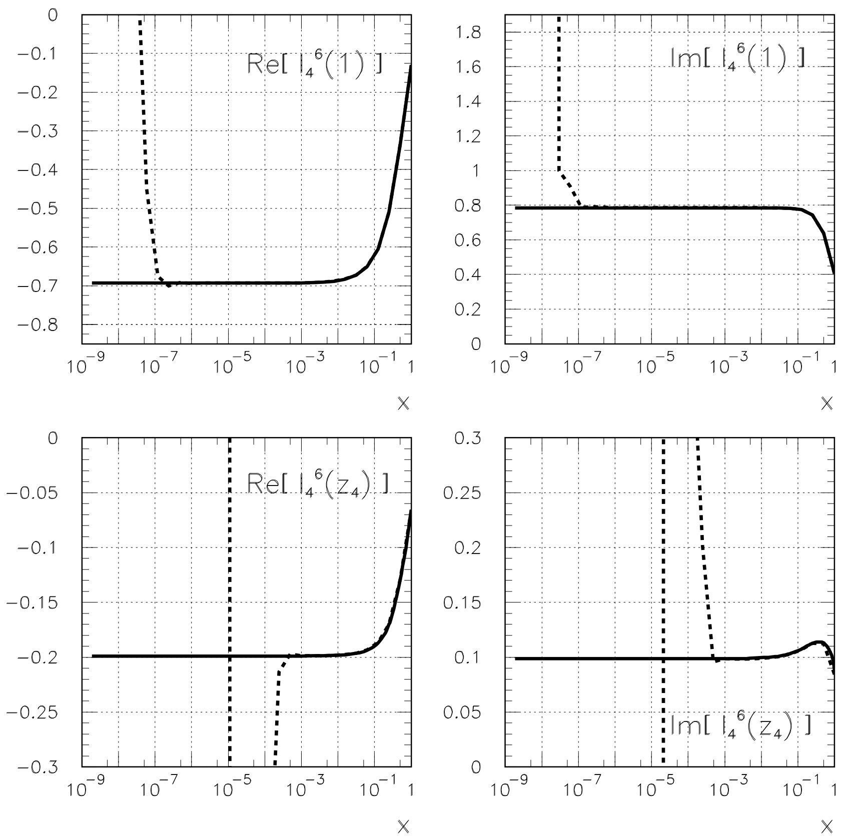
Consider the following $2 \to 2$ kinematics:

$$
\begin{aligned}
p_1 &= (E(x),0,0,xM) \\
p_2 &= (E(x),0,0,-xM) \\
p_3 &= E(x)\,(1,0,\sin(\theta),\cos(\theta)) \\
p_4 &= E(x)\,(1,0,-\sin(\theta),-\cos(\theta)) \\
E(x) &= M\,\sqrt{1+x^2}
\end{aligned}
\tag{2}
$$

Phase space boundaries: $x \to 0$, $\theta \to 0$

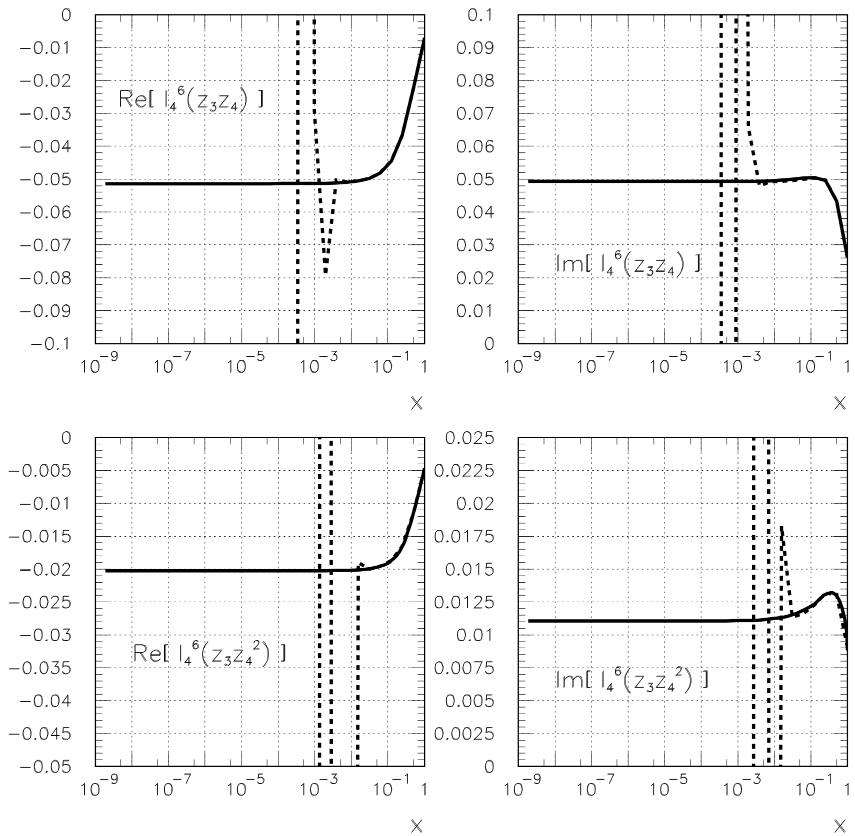
$\Rightarrow \det(G) = 32M^6\,(1+x^2)^2\,x^2\,\sin^2\theta = 0.$

Real and imaginary parts of the basis integrals, $I_4^6(1)$, $I_4^6(z_4)$:



- Full line: numerical implementation
- Dashed line: algebraical implementation

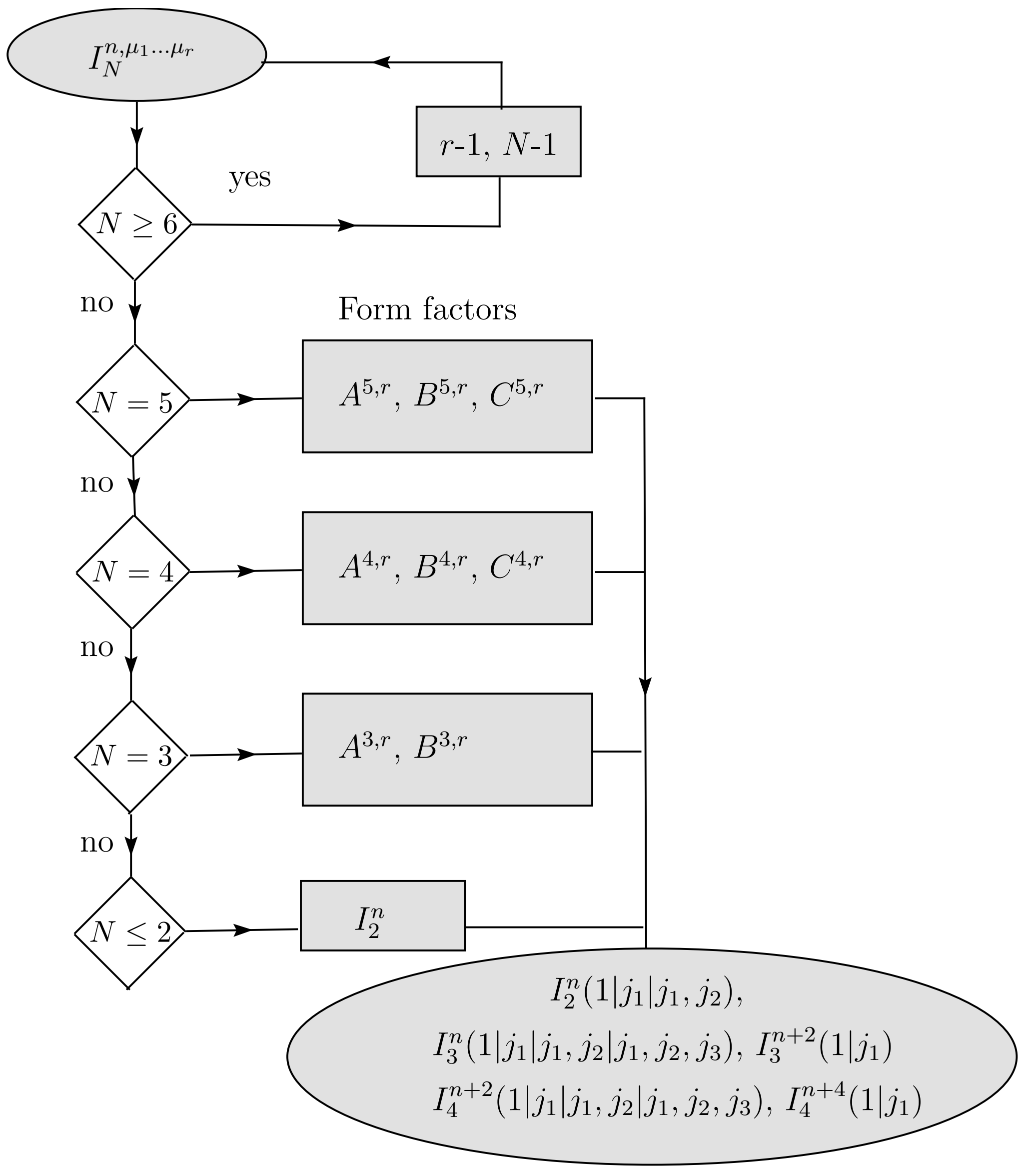
Real and imaginary parts of the basis integrals, $I_4^6(z_3 z_4)$, $I_4^6(z_3 z_4^2)$:



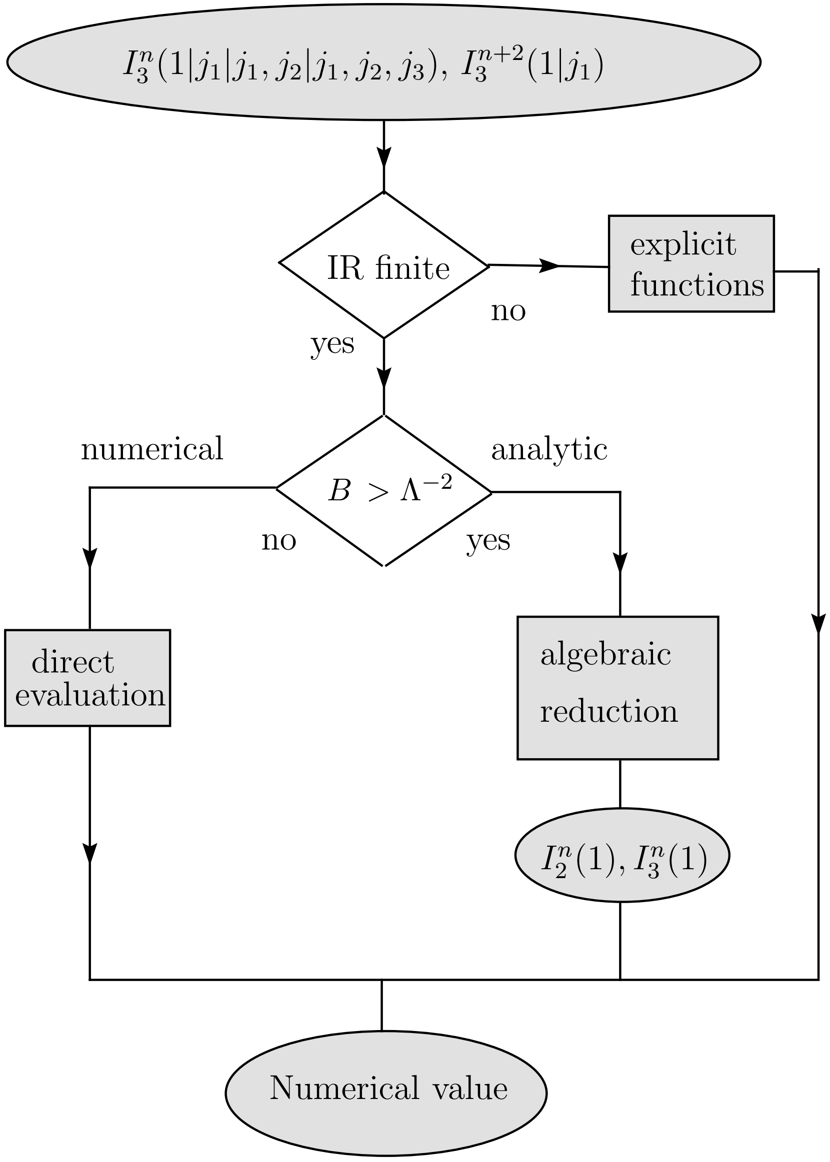
Conclusion for the practioner:

Use fast and accurate algebraic formulas for the "bulk" of the phase space, switch to slow but reliable numerical evaluation at the phase space boundary!

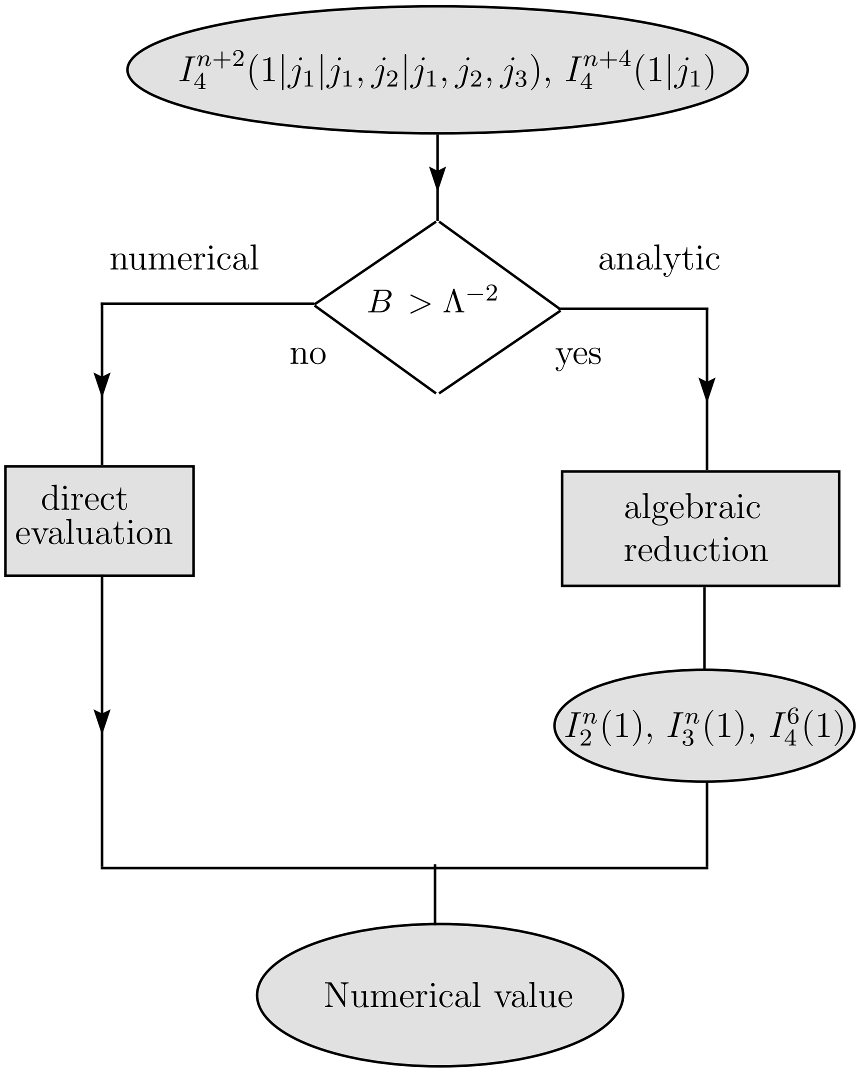
Schematic overview:



Treatment of basis integrals: N=3



Treatment of basis integrals: N=4



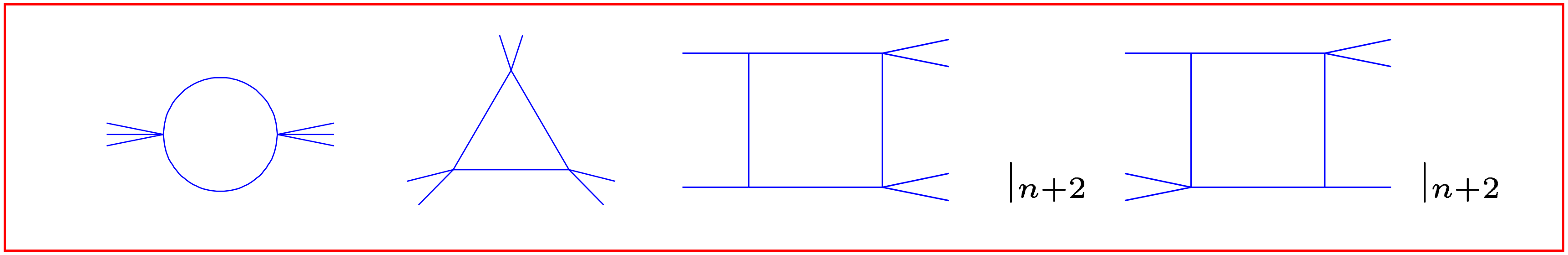
The $gg \to W^*W^*$ amplitude

- missing background for $gg \to H \to W^*W^*$
  [T.B., M. Ciccolini, N. Kauer, M. Kramer, JHEP0503:065, 2005.]
- On-shell amplitude known since a long time

  [N. Glover, J.J. van der Bij (1989) $m_q = 0$; C. Kao, D. A. Dicus (1991) $m_q \neq 0$]

Helicity amplitudes Γ^{++}, Γ^{+-} computed in a modular way:

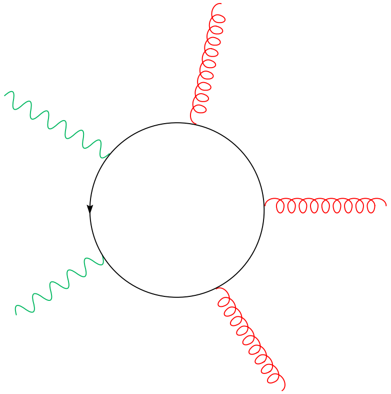
- Decomposition of amplitude into gauge invariant structures
- Sorted by analytical structures
- Stable numerical representation for $m_q = 0$, $m_q \neq 0$ underway



The $\gamma\gamma \to ggg$ amplitude

[T.B., J.-Ph. Guillet, F. Mahmoudi, JHEP 0402:057, 2004]

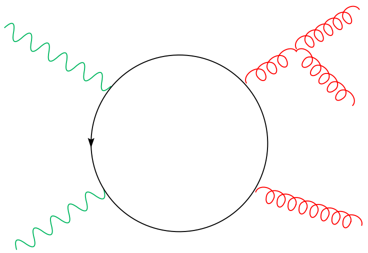
- Relevant for $\gamma\gamma + \text{jet}$ background for Higgs+jet production
  [D. de Florian, Z. Kunszt, Phys.Lett.B460 (1999)]
- Amplitude indirectly known from $gg \to ggg$
  [Z. Bern, L. Dixon, D. Kosower, Phys.Rev.Lett.70 (1993)]



Independent helicity structures:
$\Gamma^{+++++}, \Gamma^{++++-}, \Gamma^{+++--}, \Gamma^{+-+++}, \Gamma^{+-++-}, \Gamma^{--+++}$

All helicity amplitudes calculated in a modular way

- Box, pentagon topologies
- One colour structure: $\sim f^{abc}$
- Sorted by analytically independent structure



Summary

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- New algebraic/numerical approach for 1-loop multi-leg processes
  - formalism valid for arbitrary N
  - massive and massless particles
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  - Gram determinants can be avoided
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- Multidimensional contour deformation method for 1-loop parameter integrals, [full potential to be exploited]
- applications for LHC under construction

Jet rates at the LHC

Number of jets:	3	4	5	6	7	8
σ/nb	91.4	6.54	0.46	0.032	0.002	0.0002

$p_T(jet) > 60$ GeV, $\theta_{ij} > 30^0$, $|\eta_j| < 3$

[Draggiotis,Kleiss,Papadopoulos, EPJ C24 (2002)]

Multi–particles/jet production plays a very important role !!!

Problems with leading order predictions:

- Scale dependence: N–jet cross sections behave $\sim \alpha_s(\mu)^N$
  $\Rightarrow$ To have predictions for jet rates NLO corrections have to be included
- Peripheral phase space regions: degenerate partonic configurations at LO are sensitive to extra parton emission $\Rightarrow$ estimates for backgrounds — especially after severe cuts — may be considerably underestimated
- Jet structure: the more partons are in the amplitudes the more precise the jet structure is described