Electroweak corrections to H → WW/ZZ → 4 fermions

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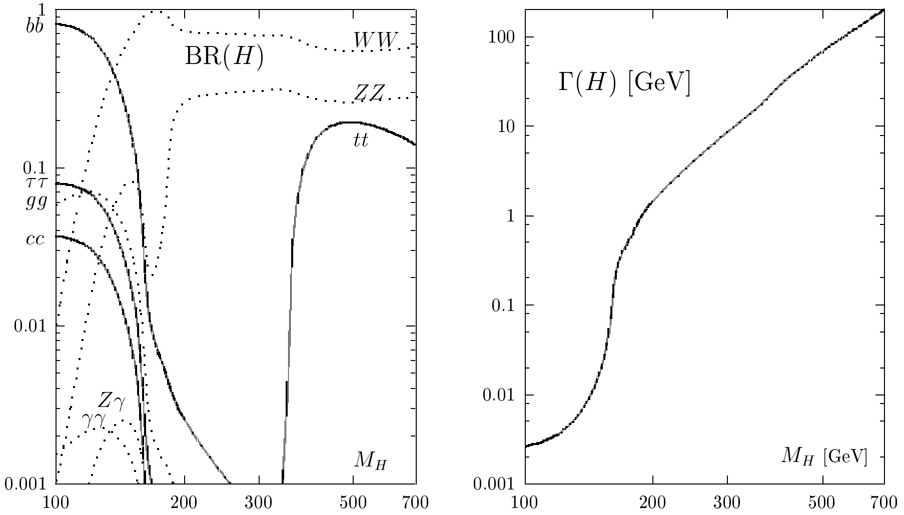
- Relevance of $\mathrm{H} \to \mathrm{WW}^{(*)}/\mathrm{ZZ}^{(*)}$
- Virtual corrections: tensor reduction, unstable particles
- Handling of soft and collinear divergences:
  dipole subtraction and phase-space slicing
- Numerical results for $\mathrm{H} \to \mathrm{WW}/\mathrm{ZZ} \to$ 4 leptons

$\mathrm{H} \to \mathrm{WW}^{(*)}/\mathrm{ZZ}^{(*)}$



Tesla TDR

LHC: $\mathrm{H} \to \mathrm{WW}^{(*)}/\mathrm{ZZ}^{(*)}$:

largest discovery potential

for $M_\mathrm{H} \gtrsim 125\,\mathrm{GeV}$

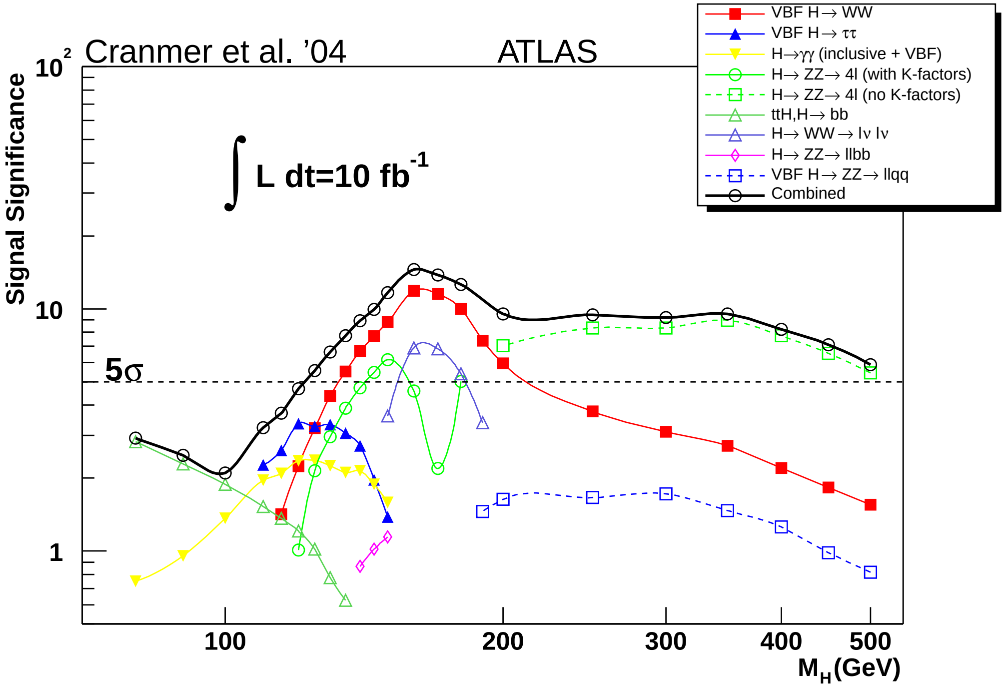
$\mathrm{H} \to \mathrm{ZZ} \to 4l$:

most accurate measurement

of Higgs mass for $M_\mathrm{H} \gtrsim 130\,\mathrm{GeV}$

ILC: measurement of branching ratios etc.

at percent level



$\mathbf{H} \to \mathbf{WW^{(*)}/ZZ^{(*)}}$

Current theoretical predictions:

above $\mathrm{WW/ZZ}$ threshold:

$\mathcal{O}(\alpha)$ corrections known for stable $\mathrm{W/Z}$ (Kniehl '91; Bardin et al. '91)

below threshold (and in transition region): only tree-level predictions

(e.g. by HDecay: Djouadi, Kalinowski, Spira '97)

→ corrections to $\mathrm{H} \to \mathrm{WW/ZZ} \to 4f$ necessary

Relevance of distributions:

- reconstruction (γ radiation)
- correlation of decay angles → verification of spin 0 (Choi et al. '02)

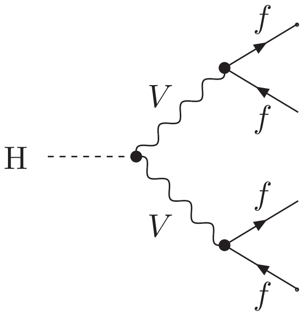
→ Monte Carlo generator for $\mathrm{H} \to \mathrm{WW/ZZ} \to 4f$ required

This talk: $\mathcal{O}(\alpha)$ EW corrections to $\mathrm{H} \to \mathrm{WW/ZZ} \to 4f$

See also talk of Carloni-Calami: $\mathcal{O}(\alpha)$ QED corrections to $\mathrm{H} \to \mathrm{ZZ} \to 4l$

$\mathbf{H \to WW/ZZ \to 4\mathit{f}}$

Tree level:

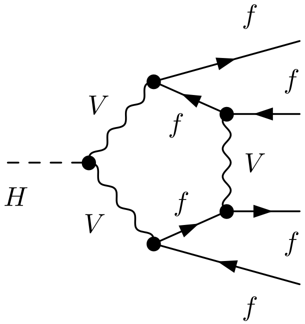


external fermion masses are neglected

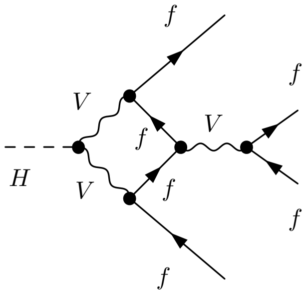
$\mathcal{O}(\alpha)$: virtual + real corrections

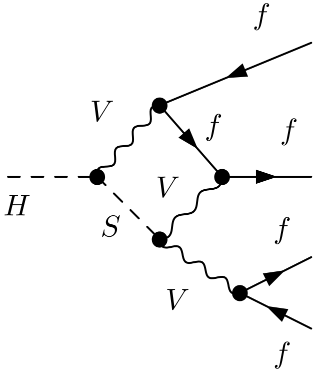
classification of virtual corrections in

pentagon

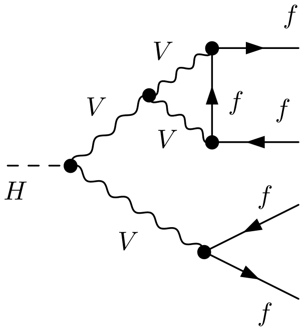


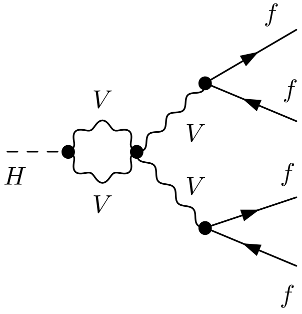
box



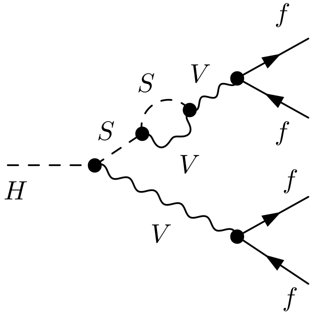


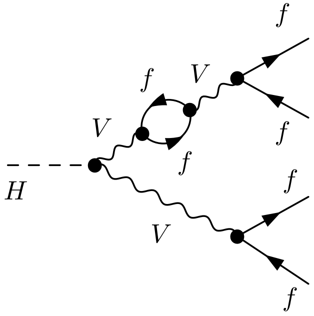
vertex





self energy





diagrams, #=$\mathcal{O}(200-400)$

Virtual corrections: technical challenges

Reduction of tensor integrals:

appearance of small Gram determinants in denominator in standard Passarino-Veltman reduction

two different approaches to circumvent this problem → talk of S. Dittmaier

Same methods used as in $\mathrm{e}^+\mathrm{e}^- \longrightarrow 4f$ Denner, Dittmaier, Roth, Wieders '05 :

(see Denner, Dittmaier '05)

- 5-point integrals are reduced to 4-point integrals without inverse Gram determinant
- 3-/4-point integrals:

  special treatment of phase-space points with small Gram determinant

  2 different methods:

  - ◇ semi-numerical method + analytical special cases

    $\longrightarrow$ avoids inverse Gram determinant
  - ◇ expansion in small Gram and other kinematical determinants

Virtual corrections: conceptual issues

Unstable particles:

Dyson summation for description of resonances

→ potential violation of gauge invariance

tree level: various schemes (e.g. naive fixed width, complex-mass scheme, fermion-loop scheme)

one loop: pole expansion (Aeppli et al. '93, '94; Stuart '91; Beenakker et al. '98; Denner et al. '00; Beneke et al. '04)

$M_\mathrm{H} \ll 2\mathrm{M}_\mathrm{W/Z}$: single-pole approximation

$M_\mathrm{H} \gg 2\mathrm{M}_\mathrm{W/Z}$: double-pole approximation

$M_\mathrm{H} \sim 2\mathrm{M}_\mathrm{W/Z}$: not reliable in threshold region

unified description: complex-mass scheme at one loop Denner, Dittmaier, Roth, Wieders '05

$M^2 \to \mu^2 = M^2 - \mathrm{i}M\Gamma$ everywhere in Feynman rules, also in loop integrals

renormalization conditions also modified:

e.g. $\hat{\Sigma}_T(\mu^2) = 0$ (on-shell scheme)

→ talk of A. Denner

Phase-space integration

multi-channel Monte Carlo integration with adaptive optimization

Berends, Kleiss, Pittau '94 Kleiss, Pittau '94

- mappings for propagators (e.g. resonances) → integrand is flattened
- "coherent" combination of different mappings (channels)
- adaptive optimization finds most important channels

tree-level and virtual corrections: 1(2) channels (# of Born diagrams),
adaptive optimization reduces statistical error for mixed-current processes

full bremsstrahlung process $\mathrm{H} \to \mathrm{WW}/\mathrm{ZZ} \to 4f + \gamma$: 4–10 channels
below threshold: additional mappings for non-resonant propagators

regularization of soft and collinear singularities: small mass parameters
(tensor reduction would also allow for dimensional regularization)

matching of singularities between virtual and real corrections:

→ dipole subtraction, phase-space slicing

Subtraction method

Basic idea: subtract and re-add the quantity $|M_{\mathrm{sub}}|^2$

$$\int \mathrm{d}\phi_{4f\gamma}\,\left(|\mathcal{M}_{\mathrm{real}}|^2 - |\mathcal{M}_{\mathrm{sub}}|^2\right) + \int \mathrm{d}\phi_{4f}|\mathcal{M}_{\mathrm{virt}}|^2 + \int \mathrm{d}\phi_{4f\gamma}|\mathcal{M}_{\mathrm{sub}}|^2$$

$|\mathcal{M}_{\mathrm{sub}}|^2 \sim |\mathcal{M}_{\mathrm{real}}|^2$ for $k \to 0$ or $p_i k \to 0$ $k = \gamma$ momentum

$\Rightarrow \int \mathrm{d}\phi_{4f\gamma}\,\left(|\mathcal{M}_{\mathrm{real}}|^2 - |\mathcal{M}_{\mathrm{sub}}|^2\right)$ is finite (no regulators needed, $m_f = 0, m_\gamma = 0$)

define mapping $\phi_{4f\gamma} \to \tilde{\phi}_{4f}$ such that

$$p_i \underset{k\to 0}{\widetilde{\qquad}} \tilde{p}_i, \qquad p_i + k \underset{kp_i\to 0}{\widetilde{\qquad}} \tilde{p}_i, \qquad p_j \underset{kp_i\to 0}{\widetilde{\qquad}} \tilde{p}_j$$

$$\int \mathrm{d}\phi_{4f\gamma} = \int \mathrm{d}\tilde{\phi}_{4f} \otimes \mathrm{d}\phi_\gamma$$

$$\int \mathrm{d}\phi_{4f\gamma}|\mathcal{M}_{\mathrm{sub}}(\phi_{4f\gamma})|^2 = \int \mathrm{d}\tilde{\phi}_{4f} \otimes \mathrm{d}\phi_\gamma\, \underbrace{g(p_i,p_j,k)}_{\text{universal}}\, |\mathcal{M}_0(\tilde{\phi}_{4f})|^2$$

$$G = \int \mathrm{d}\phi_\gamma g(p_i,p_j,k)$$

$\Rightarrow \int \mathrm{d}\tilde{\phi}_{4f}\,\left(|\mathcal{M}_{\mathrm{virt}}|^2 + G|\mathcal{M}_0|^2\right)$ is finite due to KLN theorem

G contains singularities analytically

Explicit algorithm/method: dipole subtraction Catani,Seymour '96; Dittmaier '99; Roth '00

Subtraction method

Generalization to non-collinear safe observables

KLN theorem: no mass singularities for inclusive quantities

inclusive: fermion+photon = one quasi particle for $p_i k \rightarrow 0$

energy fraction $z_i = \frac{p_i^0}{p_i^0 + k^0}$ is fully integrated over

inclusiveness achieved e.g. by photon recombination ($p_i + k = \tilde{p}_i$ for $p_i k \rightarrow 0$)

cuts or histogram bins $\rightarrow$ integration over z_i is constrained,

mass singularities do not cancel between real and virtual corrections

$\rightarrow \alpha \log m_f$ terms

$\Rightarrow z_i$ cannot be integrated analytically,

has to be part of numerical phase-space integration

generalization straightforward for phase-space slicing,

but more involved for dipole subtraction

Subtraction method

Dipole subtraction has to be generalized (A.B., Dittmaier, Roth ’05)

step function $\Theta(\phi)$ describes cuts or histogram bins:

$$\int \mathrm{d}\phi_{4f\gamma}\left(|\mathcal{M}_{\mathrm{real}}|^2\Theta(\phi_{4f\gamma}) - |\mathcal{M}_{\mathrm{sub}}|^2\Theta(\tilde{\phi}_{4f})\right)$$

remember: subtraction function defined via mapping $\phi_{4f\gamma} \to \tilde{\phi}_{4f}$

photon recombination $\Rightarrow$ collinear-safe observable, $\Theta(\phi_{4f\gamma}) \xrightarrow[p_i k \to 0]{} \Theta(\tilde{\phi}_{4f})$

non-collinear safe observables:

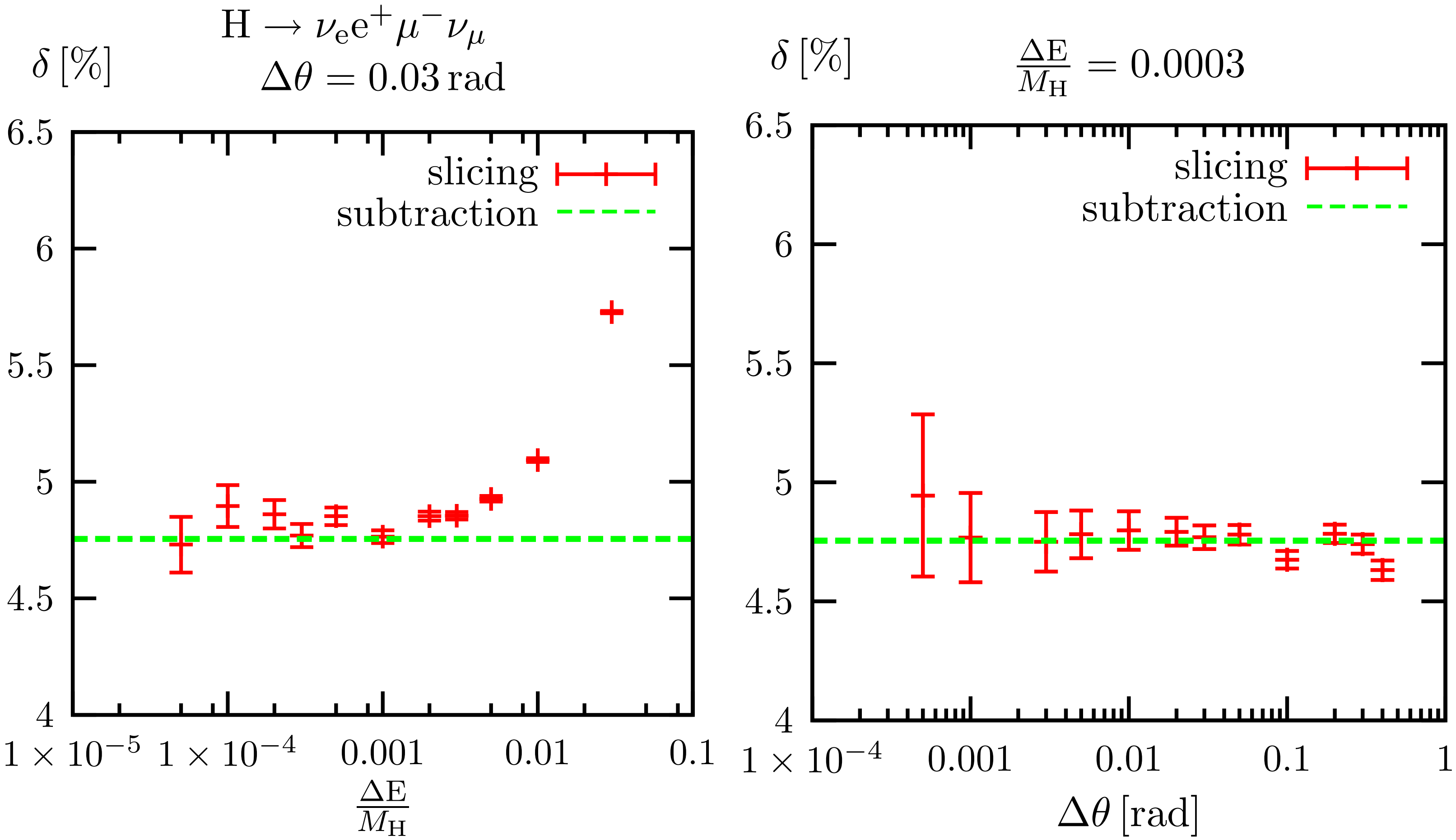
keep information on energy fraction z_i in each part of the subtraction function:

- $\Theta(\tilde{\phi}_{4f}) \to \Theta\Big(p_i = z_{ij}\tilde{p}_i, k = (1 - z_{ij})\tilde{p}_i, \{\tilde{p}_{k\neq i}\}\Big)$ ($z_{ij} \xrightarrow[p_i k \to 0]{} z_i$)
- new subtraction functions $\int \mathrm{d}z_{ij} G(z_{ij}) = \int \mathrm{d}\phi_\gamma g(p_i, p_j, k)$
- numerical integration over z_{ij}

Phase-space slicing

$$\int \mathrm{d}\phi_{4f\gamma}\,|\mathcal{M}_{\mathrm{real}}|^2 = \int \mathrm{d}\phi^{\mathrm{finite}}_{4f\gamma}\,|\mathcal{M}_{\mathrm{real}}|^2 + \int_{\substack{E_\gamma<\Delta E \\ \text{or } \theta(\gamma,f_i)<\Delta\theta}} \mathrm{d}\phi^{\mathrm{sing}}_{4f\gamma}\,|\mathcal{M}_{\mathrm{real}}|^2$$

analytical integration over $\mathrm{d}\phi_\gamma$

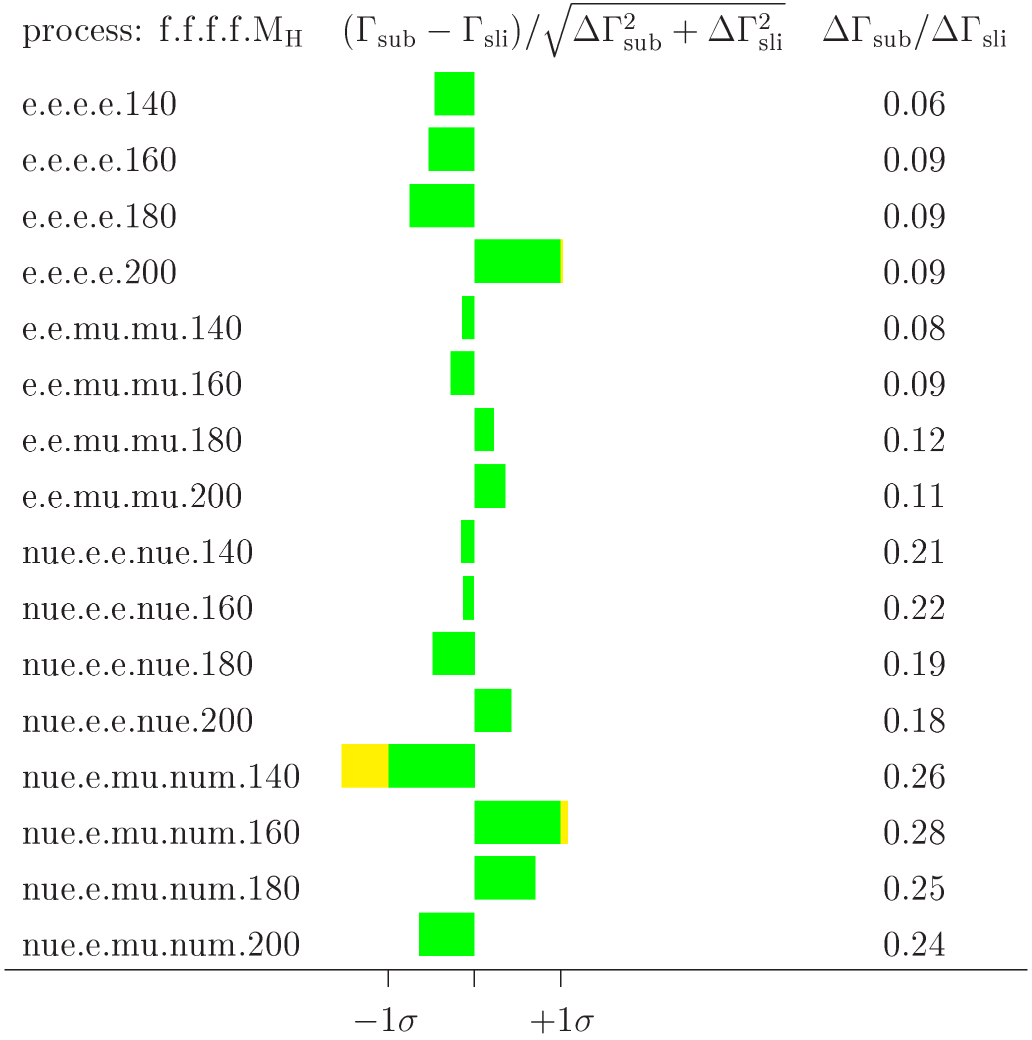


good agreement between subtraction and slicing method

$5\cdot10^7$ events

Subtraction – Slicing

decay width for various processes and Higgs masses



process: f.f.f.f.M_H	$(\Gamma_{\rm sub}-\Gamma_{\rm sli})/\sqrt{\Delta\Gamma^2_{\rm sub}+\Delta\Gamma^2_{\rm sli}}$	$\Delta\Gamma_{\rm sub}/\Delta\Gamma_{\rm sli}$
e.e.e.e.140		0.06
e.e.e.e.160		0.09
e.e.e.e.180		0.09
e.e.e.e.200		0.09
e.e.mu.mu.140		0.08
e.e.mu.mu.160		0.09
e.e.mu.mu.180		0.12
e.e.mu.mu.200		0.11
nue.e.e.nue.140		0.21
nue.e.e.nue.160		0.22
nue.e.e.nue.180		0.19
nue.e.e.nue.200		0.18
nue.e.mu.num.140		0.26
nue.e.mu.num.160		0.28
nue.e.mu.num.180		0.25
nue.e.mu.num.200		0.24

good agreement between two methods

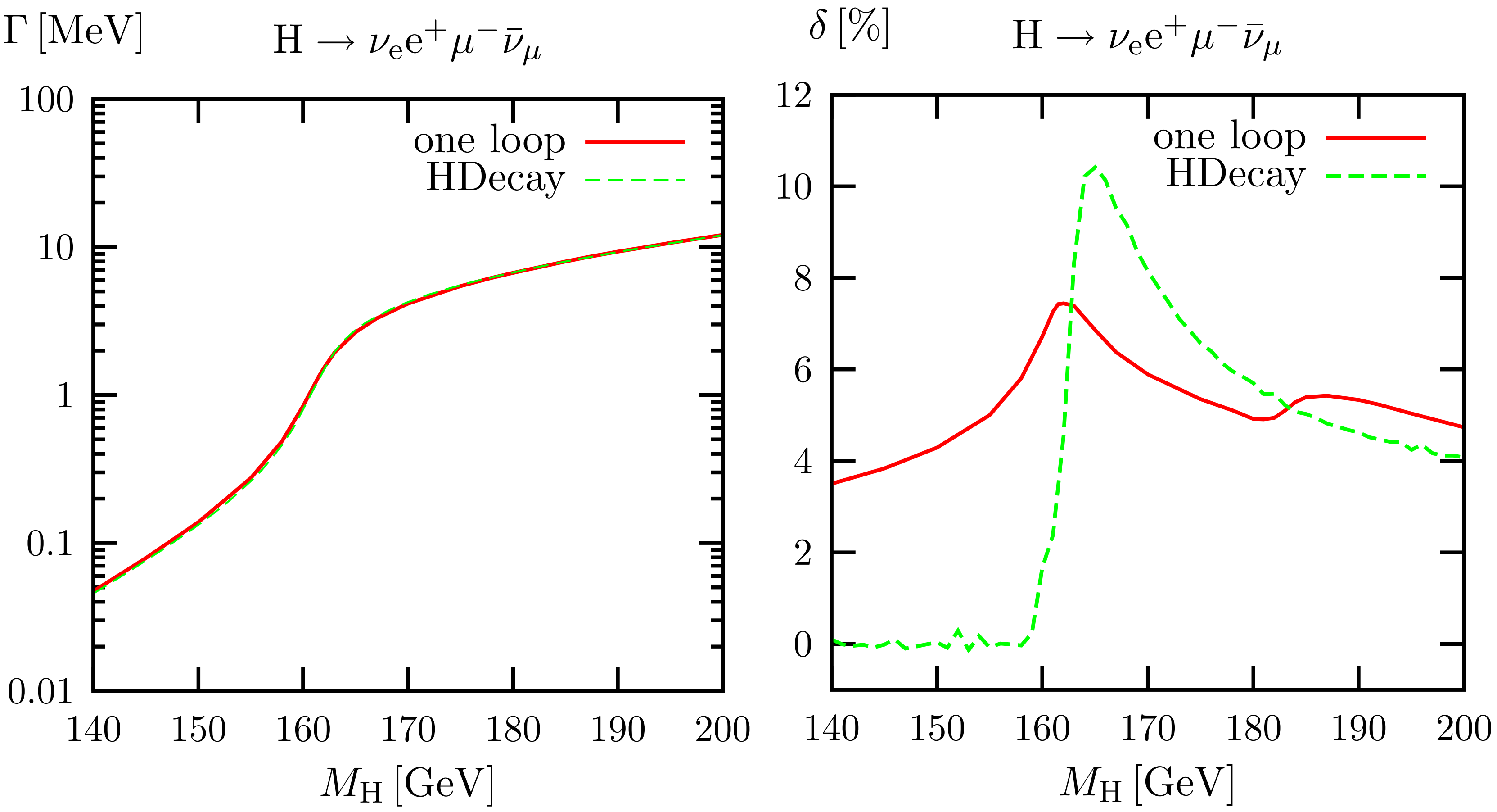
statistical error of subtraction method $\Delta\sigma_{\rm sub} < 0.1\%$ for 10^7 events

Consistency checks

- UV: independence of μ in dimensional regularization
- Soft IR: independence of photon mass (regulator for soft singularities)
- Collinear IR: independence of external fermion masses (needed to describe collinear singularities) in inclusive case
- Gauge independence: calculation in ’t Hooft–Feynman and background-field gauge (Denner, Dittmaier, Weiglein ’94)
- Different methods for combining soft and collinear singularities: dipole subtraction and phase-space slicing
- Last but not least: two completely independent calculations

Impact of $\mathcal{O}(\alpha)$ corrections

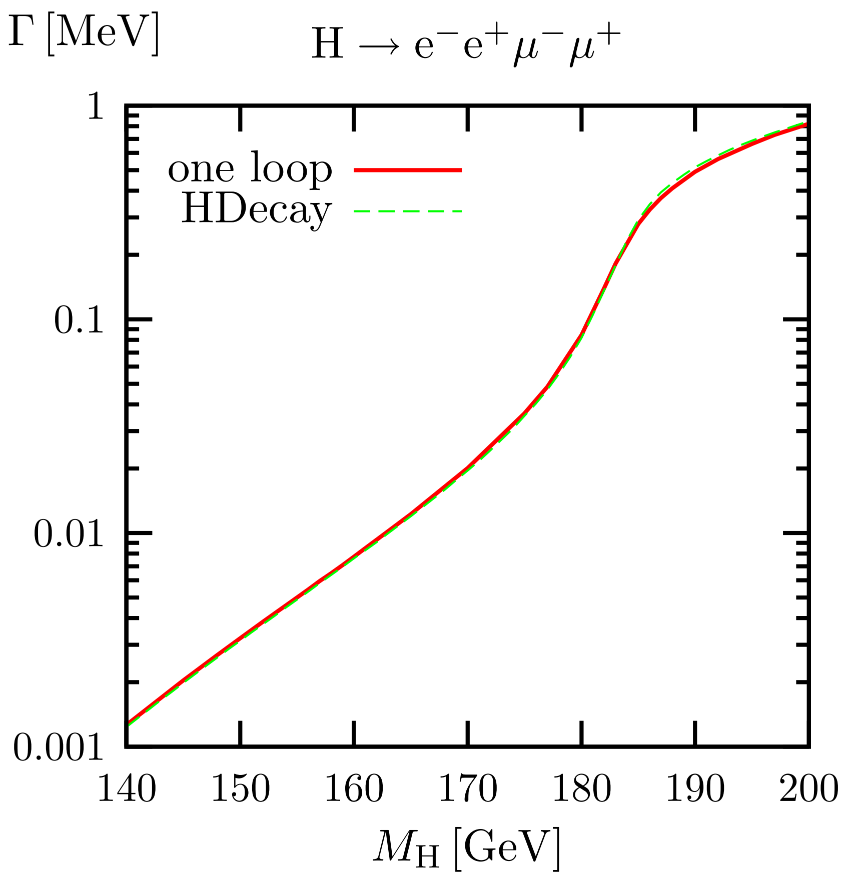
Partial decay width for $\mathrm{H} \to \mathrm{WW} \to \nu_\mathrm{e}\mathrm{e}^+\mu^-\bar{\nu}_\mu$ G_μ-scheme

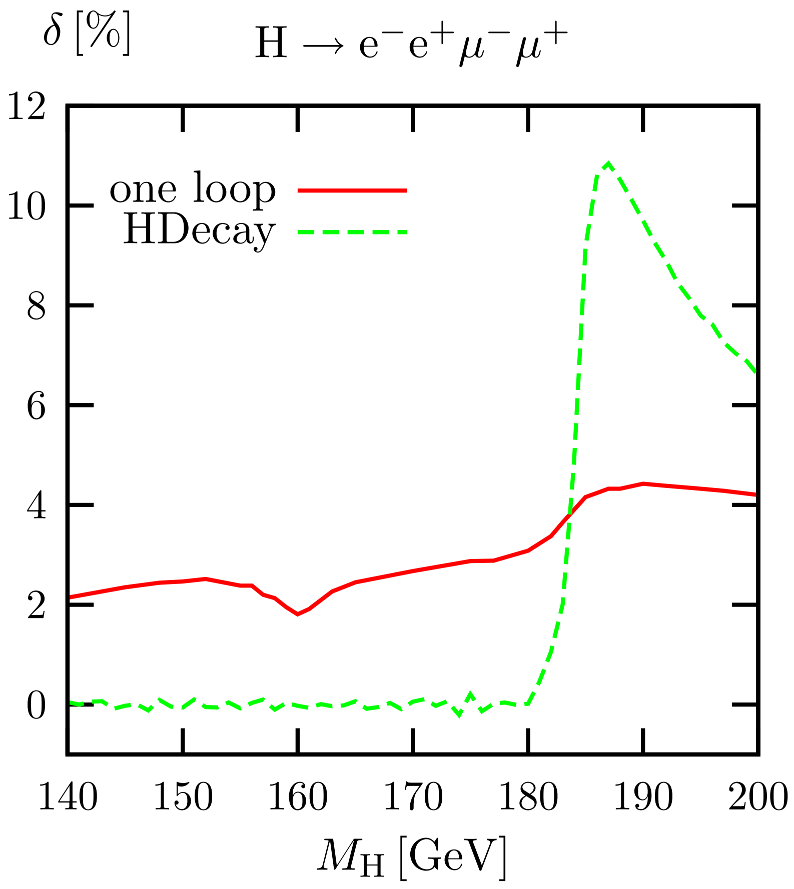


$$\delta = \frac{\Gamma}{\Gamma_{\mathrm{H}\to 4f,\mathrm{Born}}} - 1$$

Impact of $\mathcal{O}(\alpha)$-corrections

Partial decay width for $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{e}^-\mathrm{e}^+\mu^-\mu^+$ G_μ-scheme

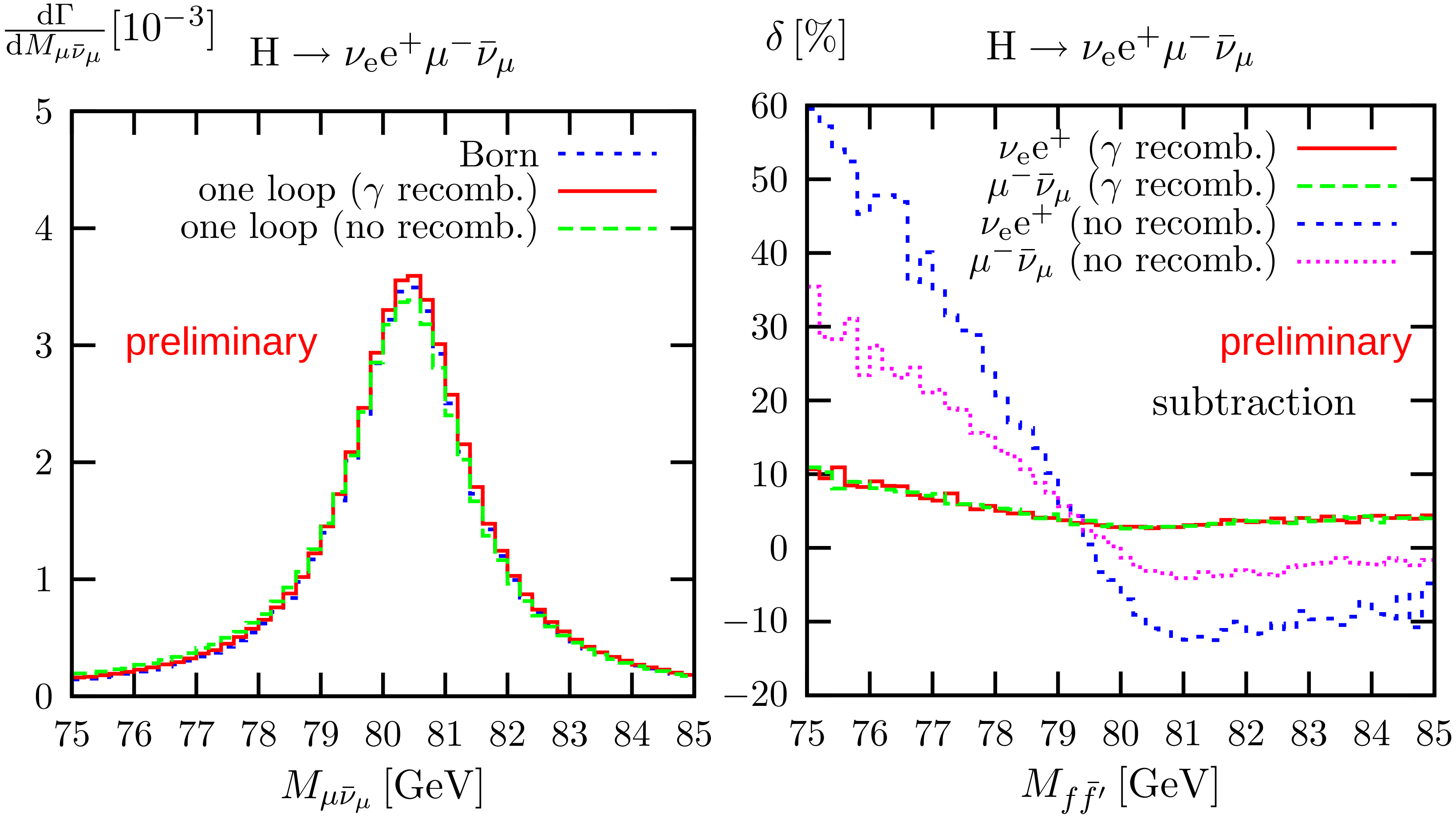




$$\delta = \frac{\Gamma}{\Gamma_{\mathrm{H}\to 4f,\mathrm{Born}}} - 1$$

Impact of $\mathcal{O}(\alpha)$-corrections

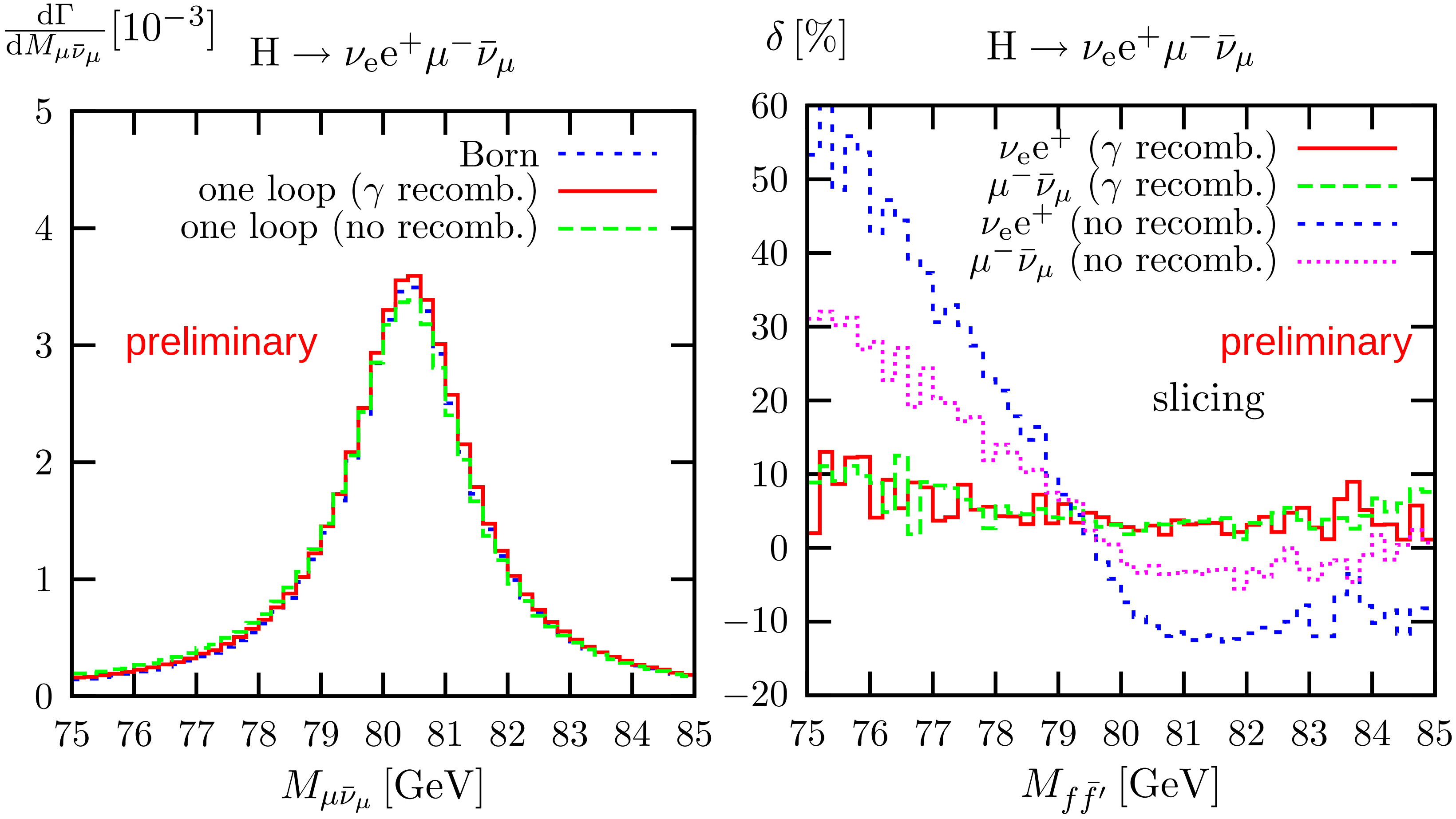
W-invariant-mass distribution for $\mathrm{H} \to \mathrm{WW} \to \nu_\mathrm{e}\mathrm{e}^+\mu^-\bar{\nu}_\mu$ G_μ-scheme



γ recombination if $M_{\mathrm{e}^+\gamma/\mu^-\gamma} < 5\,\mathrm{GeV}$

Impact of $\mathcal{O}(\alpha)$-corrections

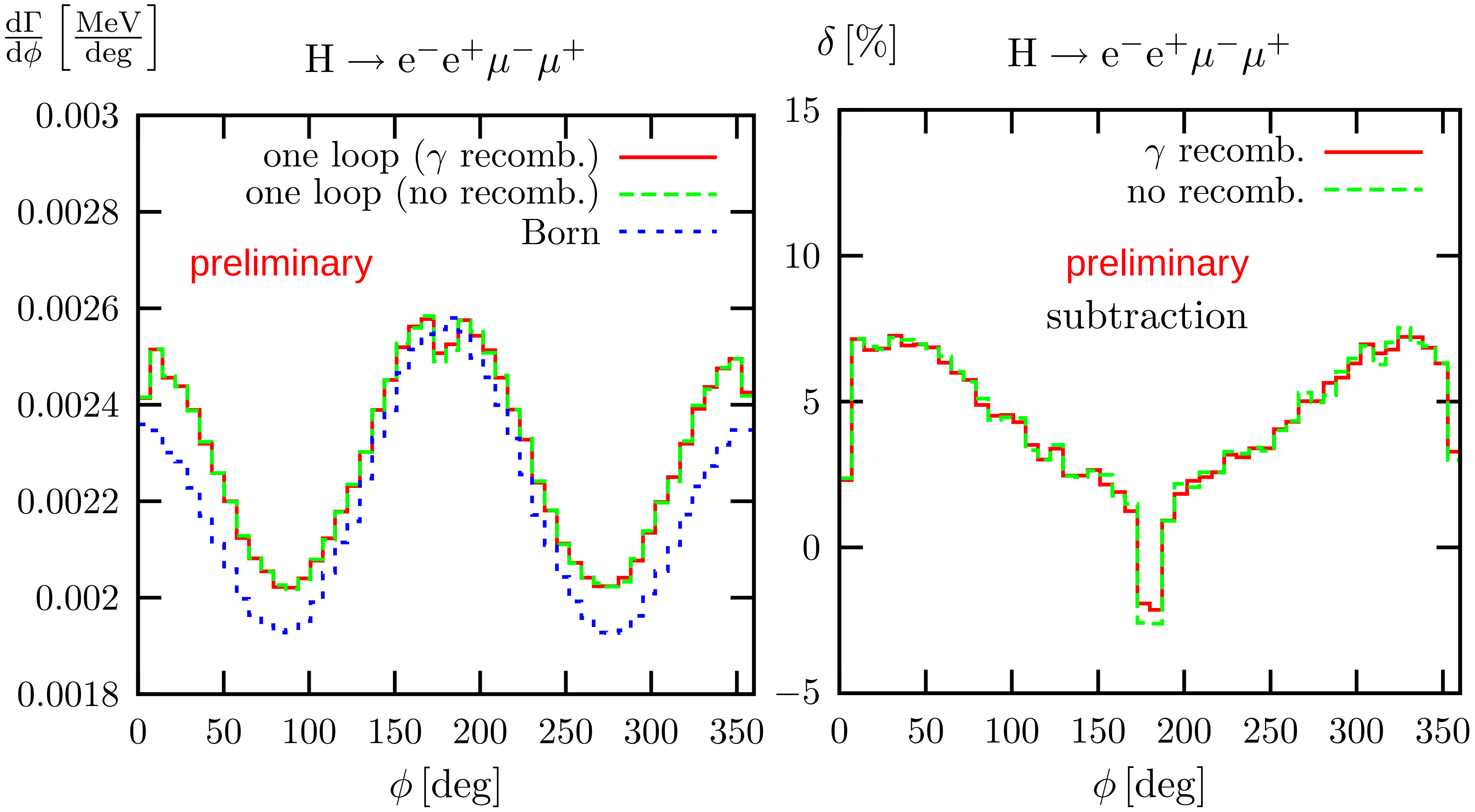
W-invariant-mass distribution for $\mathrm{H} \to \mathrm{WW} \to \nu_\mathrm{e}\mathrm{e}^+\mu^-\bar{\nu}_\mu$ G_μ-scheme



γ recombination if $M_{\mathrm{e}^+\gamma/\mu^-\gamma} < 5\,\mathrm{GeV}$

Impact of $\mathcal{O}(\alpha)$-corrections

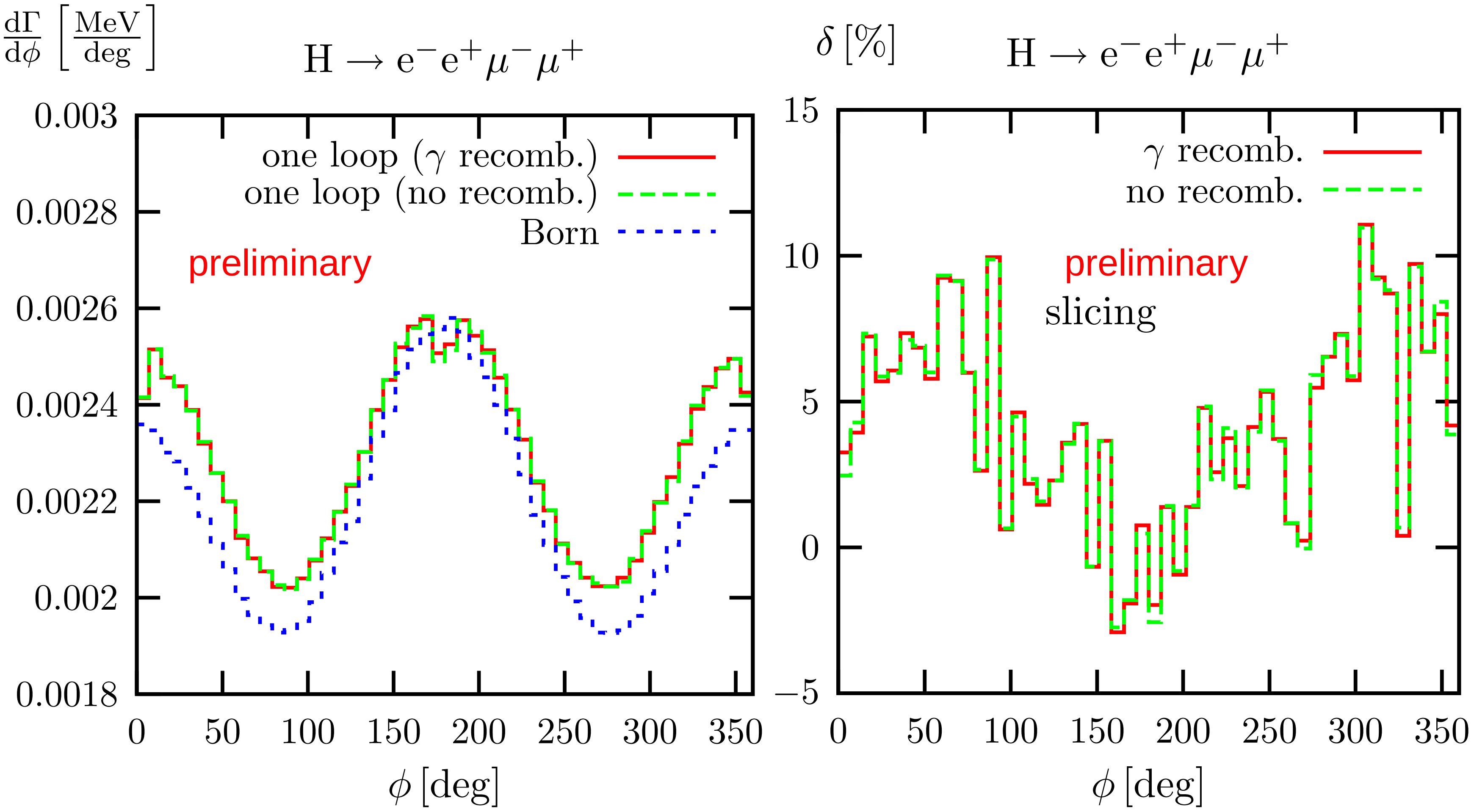
Angle between decay planes for $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{e}^-\mathrm{e}^+\mu^-\mu^+$ G_μ-scheme



$$\cos\phi = \frac{((p_1+p_2)\times p_1)(-(p_3+p_4)\times p_3)}{|(p_1+p_2)\times p_1||-(p_3+p_4)\times p_3|}, \quad p_{\mathrm{H}} = p_1 + p_2 + p_3 + p_4 (+p_\gamma)$$

Impact of $\mathcal{O}(\alpha)$-corrections

Angle between decay planes for $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{e}^-\mathrm{e}^+\mu^-\mu^+$ G_μ-scheme



$$\cos\phi = \frac{((p_1+p_2)\times p_1)(-(p_3+p_4)\times p_3)}{|(p_1+p_2)\times p_1||-(p_3+p_4)\times p_3|}, \quad p_\mathrm{H} = p_1+p_2+p_3+p_4(+p_\gamma)$$

Summary

- Precise description of $\mathrm{H} \to \mathrm{WW}^{(*)}/\mathrm{ZZ}^{(*)}$ requires calculation of radiative corrections to $\mathrm{H} \to \mathrm{WW}/\mathrm{ZZ} \to 4f$, in particular at and below threshold
- Implementation of new techniques for tensor reduction (Denner, Dittmaier ’05) and complex-mass scheme at one loop
- Monte Carlo generator with multi-channel integration with adaptive optim.
- Matching of soft and collinear singularities is done with the dipole subtraction or phase-space slicing method
- First results presented, RCs $\sim \mathcal{O}(2-8\%)$ in $\Gamma_{\mathrm{H}\to 4f}$, much larger in distributions (depending on photon treatment)
- Things to be done:
  - ◇ QCD corrections to $\mathrm{H} \to 2q2l, 4q$
  - ◇ final-state radiation beyond leading order
  - ◇ comparison with narrow-width approximation