Radiative Correction of Eighth- and Tenth-Orders to Lepton g-2

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APPLICATION OF QUANTUM FIELD THEORY TO PHENOMENOLOGY

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1. Introduction

- **Deviation of electron g value from 2 was first discovered in atomic spectrum.**

P. Kusch and H. M. Foley, PR 72, 1256 (1947)

- **Schwinger showed that it can be explained as QED effect.**

J. Schwinger, PR 73, 416L (1948)

- **Together with the Lamb shift, it provided convincing experimental evidence that (until then discredited) QED is the correct theory of electromagnetic interaction, provided that it is renormalized.**

- **As precision of measurement of g-2 improves by 7 orders of magnitude from 5×10^{-2} to 4×10^{-9}, theory of radiative correction has been pushed to order α^4 to match measurement.**

- **Their comparison provides the most stringent test of the validity of QED.**

2. Electron $g-2$: Measurement.

- In 1987 the value of electron g-2 was improved over previous best value by three orders of magnitude in a Penning trap experiment by Dehmelt et al. at U. of Washington.

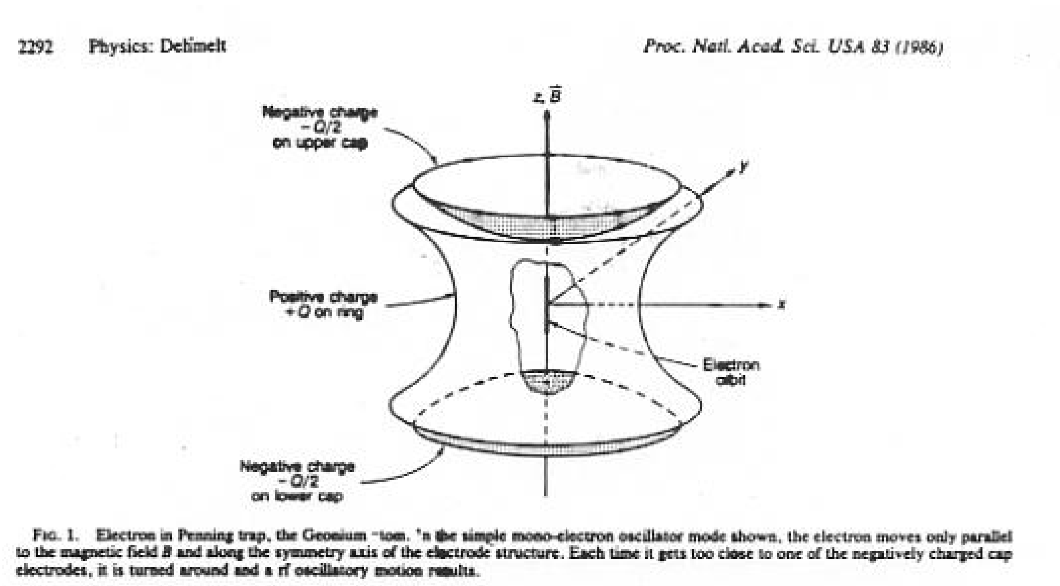

Figure 1: **Penning trap with hyperboloid electrodes.**

- **Best results reported from Seattle were:**

$$\mathbf{a_{e^-} = 1\ 159\ 652\ 188.4\ (4.3) \times 10^{-12}}$$

$$\mathbf{a_{e^+} = 1\ 159\ 652\ 187.9\ (4.3) \times\ 10^{-12}}$$

Van Dyck *et al.*, PRL 59, 26 (1987)

- **Uncertainty of this measurement was dominated by cavity shift due to interaction of electron with hyperboloid cavity which has complicated resonance structure.**

• **Several ways to reduce this error examined:**

a) Use cavity with smaller Q.

Preliminary result:

$$a_{e^-} = \mathbf{1\ 159\ 652\ 185.5\ (4.0) \times 10^{-12}}$$

Van Dyck *et al.*, 1991, unpublished.

b) Study the cavity shift by many (∼ 1000)-electron cluster which magnifies the cavity shift.

Mittleman *et al.* PRL 75, 2839 (1995)

c) Use cylindrical cavity, whose property is known analytically.

Brown, Gabrielse, PRL 55, 44 (1985)

• **Gabrielse's new measurement of a_e is based on c).**

• **It is in an advanced stage.**

(See Figures, next pages.)

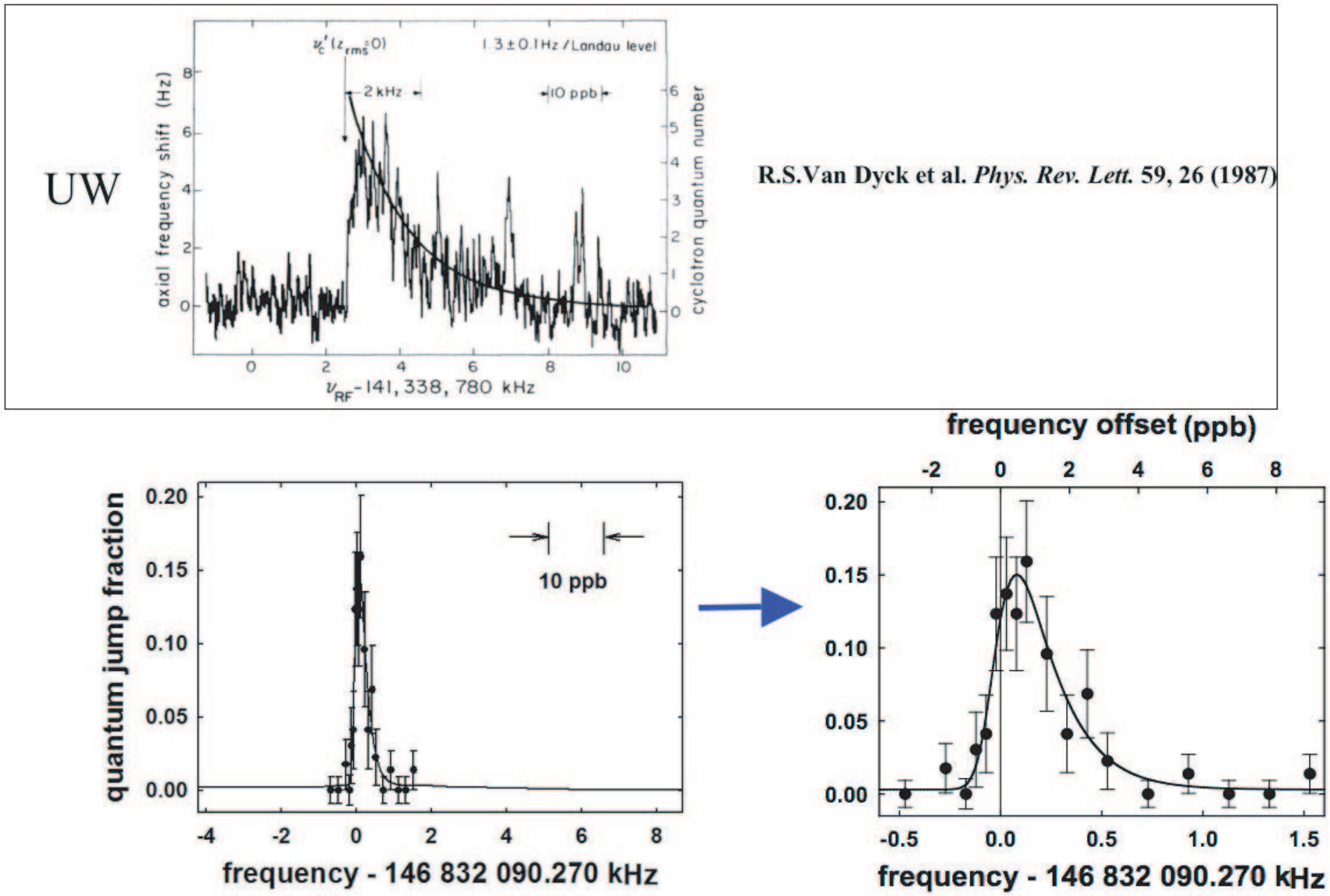

Figure 2: **Cyclotron Resonance Line (Gabrielse).**

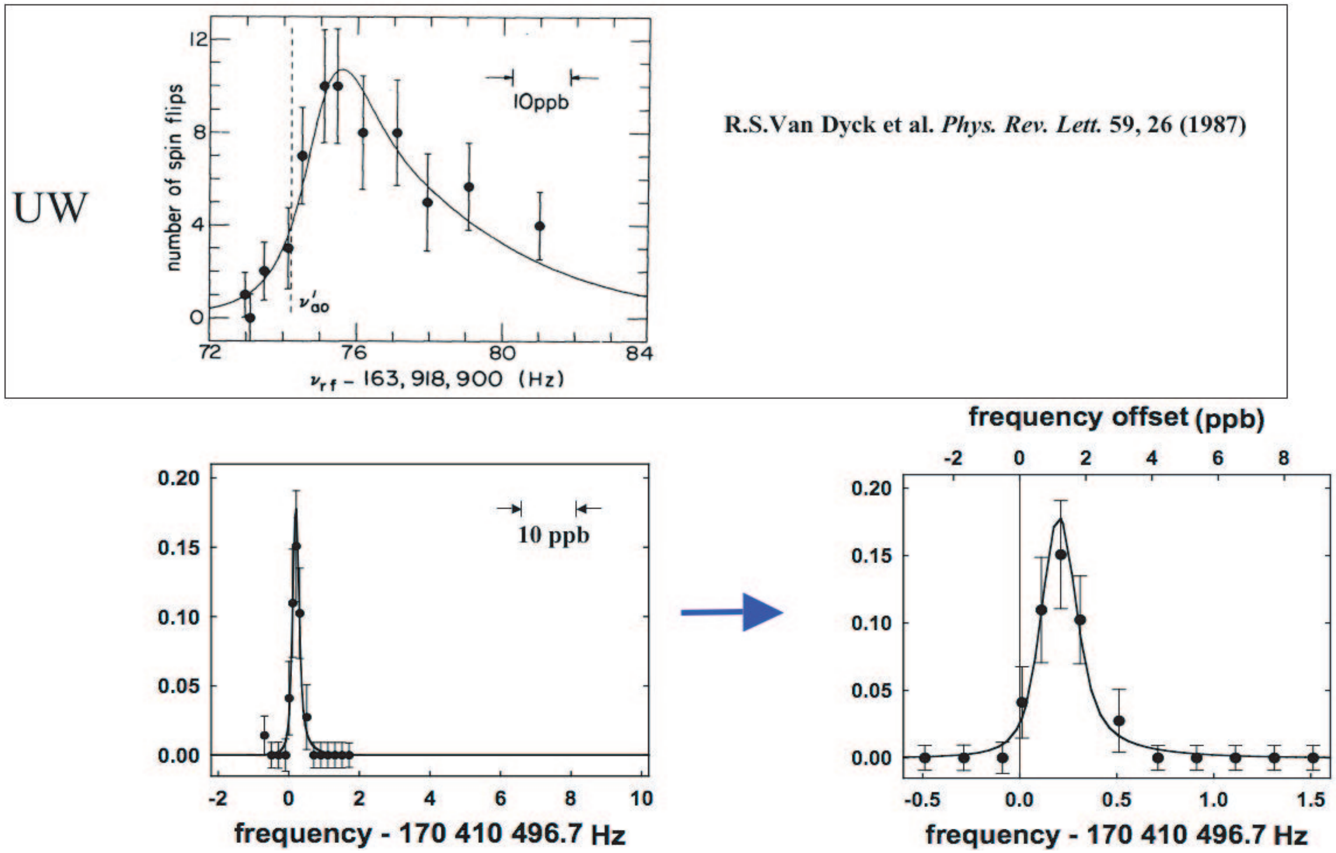

Figure 3: **Anomaly Resonance Line (Gabrielse).**

- **Recently a preliminary value was reported:**

$$\mathbf{a_{e^-} = 1\ 159\ 652\ 180.86\ (0.57) \times 10^{-12} \qquad (0.49\ ppb)}$$

B. Odom, PhD thesis, Harvard University, 2005

- **7.5 times more precise than the Seattle result.**

- **Gabrielse thinks that this was premature: Error analysis is not yet finished.**

- **Final published value may be more conservative.**

3. Electron $g-2$: Current Status of Theory.

- **QED contribution**

$$\mathbf{a_e(QED) = A_1 + A_2(m_e/m_\mu) + A_2(m_e/m_\tau) + A_3(m_e/m_\mu, m_e/m_\tau)}$$

$$\mathbf{A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + \ldots,\ i = 1, 2, 3}$$

$\mathbf{A_1^{(2)} = \ \ 0.5}$ **1 diagram (analytic)**

$\mathbf{A_1^{(4)} = -0.328\,478\,965\ \ldots}$ **7 diagrams (analytic)**

$\mathbf{A_1^{(6)} = \ \ 1.181\,241\,456\ \ldots}$ **72 diagrams (numerical, analytic)**

Kinoshita, PRL 75, 4728 (1995)
Laporta, Remiddi, PLB 379, 283 (1996)

$\mathbf{A_1^{(8)} = -1.728\,3\,(35)}$ **891 diagrams (numerical)**

Kinoshita, Nio, arXiv:hep-ph/0507249 v1 21 Jul 2005.

$\mathbf{A_1^{(10)} = 0\,(3.8)}$ **12672 diagrams (guess by Mohr, Taylor)**

- $A_1^{(8)}$ **is our new result.**
  Its error has been reduced by 10 compared with old one.

- **Thus far** $A_1^{(8)}$ **has been evaluated by one method only.**
- **However, there are extensive cross-checking among diagrams of 8th-order and also with 6th-, 4th-, 2nd-order diagrams.**

• $\mathbf{A_2}$ **terms are small** :

$$\mathbf{A_2^{(4)}(m_e/m_\mu)(\alpha/\pi)^2 = \ 2.804 \times 10^{-12}}$$

$$\mathbf{A_2^{(4)}(m_e/m_\tau)(\alpha/\pi)^2 = \ 0.010 \times 10^{-12}}$$

$$\mathbf{A_2^{(6)}(m_e/m_\mu)(\alpha/\pi)^3 = -0.924 \times 10^{-13}}$$

$$\mathbf{A_2^{(6)}(m_e/m_\tau)(\alpha/\pi)^3 = -0.825 \times 10^{-15}}$$

• **The** $\mathbf{A_3}$ **term is even smaller** $(\sim \mathbf{2.4 \times 10^{-21}})$.

• **Non-QED contribution (Standard Model).**

a) $\quad \mathbf{a_e}(\text{hadron}) = \mathbf{1.645}\ (\mathbf{42})\ \times \mathbf{10^{-12}}$

Jegerlehner, priv. com. 1996

Krause, priv. com. 1996

b) $\quad \mathbf{a_e}(\text{weak}) = \mathbf{0.030}\ \times \mathbf{10^{-12}}$

estimated by scaling down from a_μ**(weak).**

Czarnecki *et al.*, **PRL 76, 3267 (1996)**

- **To compare theory with measured value of a_e one needs α obtained by some non-QED measurements.**

- **Best α available at present is one obtained by atom interferometry combined with cesium D_1 line measurement:**

$$\alpha^{-1}(\mathbf{h}/M_{C_s}) = \mathbf{137.036\ 000\ 3\ (10)} \qquad \mathbf{(7.4\ ppb)}$$

A. Wicht et al. in Proc. of 6th Symp. on Freq. Standards and Metrology (World Sci., 2002), pp.193 -212

- **This leads to**

$$\mathbf{a_e(h/M_{Cs}) = 1\ 159\ 652\ 174.19\ (0.11)(0.26)(8.48) \times\ 10^{-12}}$$
$$\mathbf{(8th)(10th)(}\alpha(h/M_{Cs})\mathbf{)}$$

$$\mathbf{a_e(exp) - a_e(h/M_{Cs}) = 11.8\ (9.5) \times\ 10^{-12}},$$

assuming $\mathbf{A_1^{(10)} = 0.0(3.8)}$.

- **Error 8.48 of $\alpha(h/M_{Cs})$ is still large but comparable with error of Seattle experiment.**

• **Very recent developments:**

(1) Relative uncertainty of Wicht et al. has been reduced to 4 ppb.

(2) Measurement of recoil velocity of $^{87}R_b$ atom based on Bloch oscillations in a vertically accelerated optical lattice by French group gives

$$\alpha^{-1}(\mathbf{h}/M_{R_b}) = \mathbf{137.035\ 998\ 78\ (91)} \qquad \mathbf{(6.7\ ppb)}$$

P. Claré et al. submitted to PRL.

• **Statistical uncertainty: 4.4 ppb, Systematic uncertainty: 5.0 ppb.**

• **These uncertainties will be reduced further.**

4. Muon $g-2$: Measurement and Interpretation.

- **First precision measurement (7ppm) at CERN.**

- **After years of hard work muon g-2 measurement at BNL has come close to the design goal (0.35 ppm).**

- **The current world-average is**

$$\mathbf{a}_\mu(\mathbf{exp}) = \mathbf{11\ 659\ 208\ (6) \times 10^{-10}} \qquad (\mathbf{0.5\ ppm})$$

Bennett *et al.*, **PRL 92, 161802 (2004)**

- **Few years ago apparent disagreement with Standard Model caused a lot of excitement as indicator of possible new physics.**

- **By now it is clear that prediction of SM must be known more precisely in order to explore physics beyond SM.**

- **SM prediction consists of QED, electroweak, and hadronic parts.**

- **Largest uncertainty comes from hadronic v-p term.**
- **It is calculated from three types of measurements:**

  (1) $e^+e^- \to$ **hadrons,**

  (2) $\tau^\pm \to \nu + \pi^\pm + \pi^0$ **(with isospin invariance) ,**

  (3) $e^+e^- \to \gamma +$ **hadrons (radiative return) .**

- **Process (3), the latest arrival, seems to agree with (1).**
- **Process (1) has been analyzed by many groups over years. Some recent results are**

$$\mathbf{a}_\mu(\mathbf{had.vp}) = \mathbf{6934}\ (\mathbf{53})_{\mathbf{exp}}\ (\mathbf{35})_{\mathbf{rad}} \times \mathbf{10^{-11}}$$

Höcker, hep-ph/0410081 (2004)

$$\mathbf{a}_\mu(\mathbf{had.vp}) = \mathbf{6924}\ (\mathbf{59})_{\mathbf{exp}}\ (\mathbf{24})_{\mathbf{rad}} \times \mathbf{10^{-11}}$$

Hagiwara et al., PRD 69, 093003 (2004)

$$\mathbf{a}_\mu(\mathbf{had.vp}) = \mathbf{6944}\ (\mathbf{48})_{\mathbf{exp}}\ (\mathbf{10})_{\mathbf{rad}} \times \mathbf{10^{-11}}$$

Trocóniz,Ynduráin, hep-ph/0402285 (2004)

- **Recent estimate of hadronic l-l contribution:**

$$\mathbf{a}_\mu(\mathbf{had.ll}) = \mathbf{136}\ (\mathbf{25}) \times \mathbf{10}^{-11}$$

Melnikov, Vainshtein, arXiv:hep-ph/0312226

- **NLO hadronic contribution:**

$$\mathbf{a}_\mu(\mathbf{had.NLO}) = -\mathbf{101}\ (\mathbf{6}) \times \mathbf{10}^{-11}$$

Hagiwara et al., PRD 69, 093003 (2004)

- **Electroweak contribution to 2-loop order:**

$$\mathbf{a}_\mu(\mathbf{weak}) = \mathbf{152}\ (\mathbf{1}) \times \mathbf{10}^{-11}$$

Knecht et al., JHEP 11, 003 (2002)

$$\mathbf{a}_\mu(\mathbf{weak}) = \mathbf{154}\ (\mathbf{1})(\mathbf{2}) \times \mathbf{10}^{-11}$$

Czarnecki et al., PRD 67, 073006 (2003)

5. Muon $g-2$: Current Status of QED Correction.

- a_μ(QED) can be written in the general form:

$$\mathbf{a_\mu(QED) = A_1 + A_2(m_\mu/m_e) + A_2(m_\mu/m_\tau) + A_3(m_\mu/m_e, m_\mu/m_\tau)}$$

$$\mathbf{A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + \ldots,\ i = 1, 2, 3.}$$

- $\mathbf{A_1}$ is common to $\mathbf{a_e}$ and $\mathbf{a_\mu}$. $\mathbf{A_2^{(4)}}$, $\mathbf{A_2^{(6)}}$, $\mathbf{A_3^{(6)}}$ evaluated by numerical int., analytic int., asymptotic expansion in $\mathbf{m_\mu/m_e}$, or power series expansion in $\mathbf{m_\mu/m_\tau}$. Errors are due to α, $\mathbf{m_\mu}$ and $\mathbf{m_\tau}$ only.

$$\mathbf{A_2^{(4)}(m_\mu/m_e) = 1.094\ 258\ 311\ 1(84)}$$

$$\mathbf{A_2^{(4)}(m_\mu/m_\tau) = 7.8064\ (25) \times 10^{-5}}$$

$$\mathbf{A_2^{(6)}(m_\mu/m_e) = 22.868\ 380\ 02(20)}$$

$$\mathbf{A_2^{(6)}(m_\mu/m_\tau) = 36.051(21) \times 10^{-5}}$$

$$\mathbf{A_3^{(6)}(m_\mu/m_e, m_\mu/m_\tau) = 0.527\ 66\ (17) \times 10^{-3}}$$

Kinoshita NCB 51, 140(1967)
Laporta NCB 106, 675(1993)
Laporta, Remiddi, PLB 301, 440(1993)
Czarnecki, Skrzypek, PLB 449, 354(1999)
Updated by Passera, hep – ph/0411168

- $A_2^{(8)}(m_\mu/m_e)$ **and** $A_2^{(8)}(m_\mu/m_e, m_\mu/m_\tau)$ **have been evaluated thus far by numerical method only.**
- **Evaluated by Monte-Carlo integration code VEGAS.**

Lepage, J. Comput. Phys. 27, 192 (1978).

- **Latest results are**

$$\mathbf{A_2^{(8)}(m_\mu/m_e) = 132.6823(72)}$$

$$\mathbf{A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.0376(1)}$$

Kinoshita, Nio, PRD 70, 113001(2004)

- **Improved by more than an order of magnitude over previous results.**
- **As is discussed later we also obtained an improved value of** $A_2^{(10)}$**:**

$$\mathbf{A_2^{(10)}(m_\mu/m_e) = 652(20)},$$

Kinoshita, Nio, preliminary.

- **Total QED contribution to a_μ, including α^5 term, is**

$$\mathbf{a}_\mu(\mathbf{QED}) = \mathbf{116\ 584\ 717.72}\ (\mathbf{0.02})(\mathbf{0.14})(\mathbf{0.85}) \times \mathbf{10}^{-11}$$

$$(\mathbf{8th})(\mathbf{10th})(\alpha(h/M_{Cs}))$$

using

$$\alpha^{-1}(\mathbf{h}/M_{C_s}) = \mathbf{137.036\ 000\ 3\ (10)} \qquad \mathbf{(7.4\ ppb)}$$

- **Including hadronic v-p and l-l terms and electroweak term, the SM value of a_μ is**

$$\mathbf{a}_\mu(\mathbf{SM}) = \mathbf{116\ 591\ 870.7}\ (\mathbf{76.2}) \times \mathbf{10}^{-11}$$

$$\mathbf{a}_\mu(\mathbf{exp}) - \mathbf{a}_\mu(\mathbf{SM}) = \mathbf{209}\ (\mathbf{97}) \times \mathbf{10}^{-11}$$

where uncertainty in "theory" is mostly due to hadronic v-p terms.

Summary: Relative contribution of various terms.

α **term**	**994623 ppm**	**known exactly**
α^2 **term**	**5064 ppm**	**known exactly**
α^3 **term**	**246 ppm**	**known exactly**
$a_\mu(had)$	$\sim$ **60 ppm**	$\sim$ **0.6 ppm**
α^4 **term**	**3.9 ppm**	**0.0002 ppm**
$a_\mu(e-w)$	**1.3 ppm**	$\sim$ **0.02 ppm**
α^5 **term**	**0.044 ppm**	**0.0014 ppm**
expt. uncertainty		**0.5 ppm**

6. Tenth-order term: Why needed ?

- **Very important byproduct of study of a_e is that it is the best source of α.**
- **If we use Odom's report we find**

$$\alpha^{-1}(a_e) = 137.035\ 999\ 708\ (12)\ (31)\ (68) \quad \textbf{(0.55 ppb)}$$
$$(\alpha^4)\ (\alpha^5)\ \textbf{(expt)}$$

- **This is almost an order of magnitude better than any other measurements of α.**
- **Uncertainty of this measurement is only factor 2 larger than that of theory, which is mostly from the α^5 term, since α^4 is known with small error.**
- **Thus, when measurement improves further, reduction of uncertainty of α^5 term will become necessary in order to obtain a better $\alpha(a_e)$.**

- **For muon, old estimate of $A_2^{(10)}(m_\mu/m_e)$ was 930 (170), which contributes only 0.054 ppm to a_μ, well within current experimental uncertainty.**

- **Thus improving $A_2^{(10)}(m_\mu/m_e)$ is not urgent.**

- **But it will become important source of error in next generation of a_μ measurement.**

- **This is why we tried to obtain a better estimate of $A_2^{(10)}(m_\mu/m_e)$.**

- **The number of diagrams contributing to $A_2^{(10)}(m_\mu/m_e)$ is**

  9080 !

- **Fortunately, we found out that it is not too difficult to improve $A_2^{(10)}(m_\mu/m_e)$ substantially.**

  Kinoshita, Nio, in preparation.

- **For the electron $A_1^{(10)}$ (12672 Feynman diagrams) is much harder to evaluate, but we developed an algorithm that makes it feasible.**

  Aoyama, Hayakawa, Kinoshita, Nio, in progress

- **This will be discussed by the next speaker.**

- **Anyway the first step is to classify all tenth-order diagrams into gauge-invariant sets.**

- **There are 32 g-i sets within 6 supersets.**

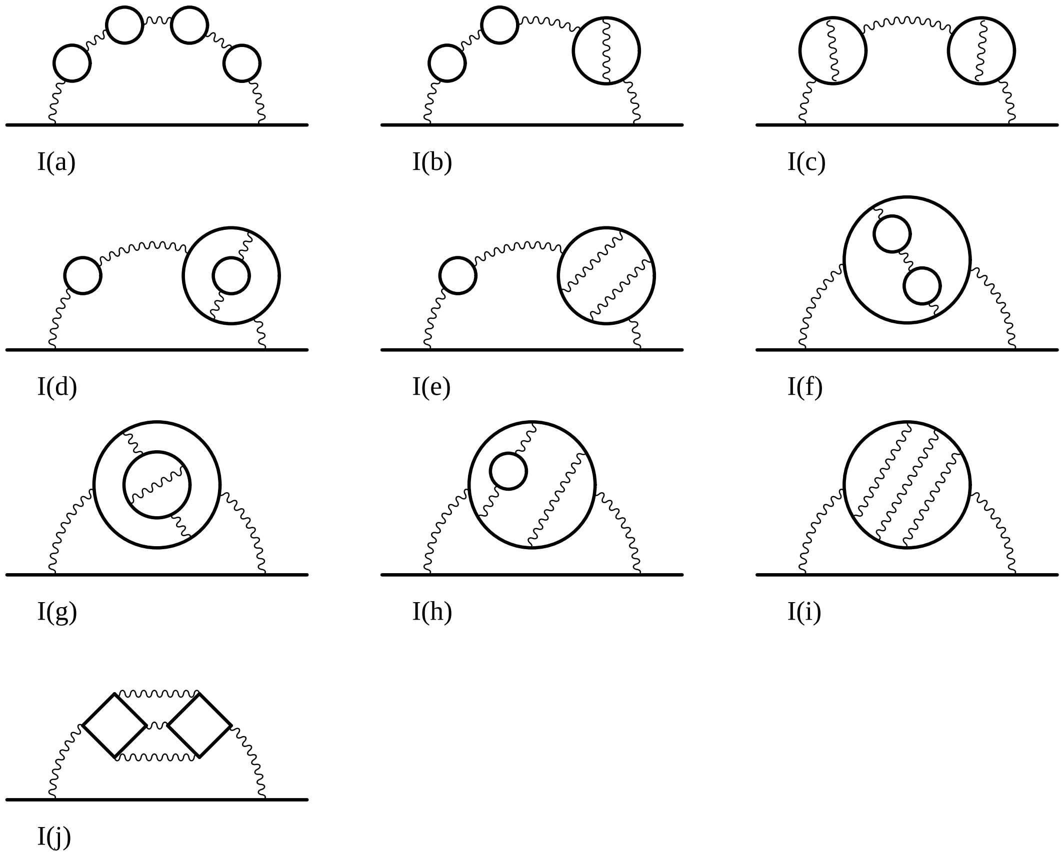

Figure 4: **Some diagrams of Set I.**

Set I is built from a second-order vertex. 208 diagrams contribute to $A_1^{(10)}$. 498 diagrams contribute to $A_2^{(10)}(m_\mu/m_e)$.

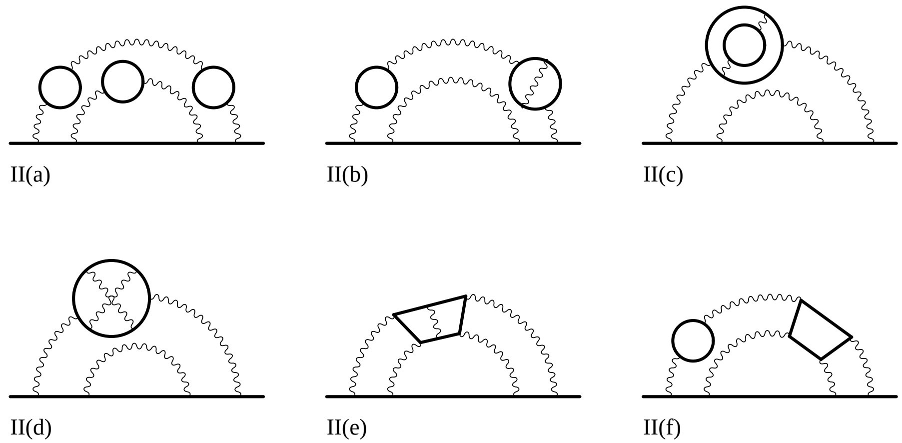

Figure 5: **Diagrams of Set II.**

Set II is built from fourth-order proper vertices. 600 diagrams contribute to $A_1^{(10)}$. 1176 diagrams ontribute to $A_2^{(10)}(m_\mu/m_e)$.

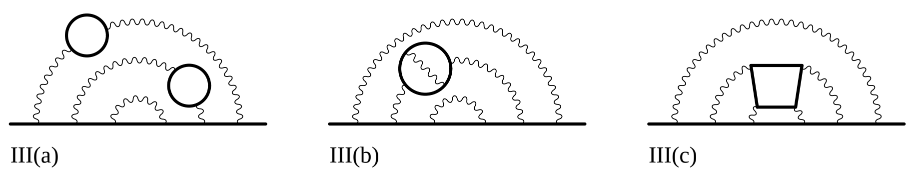

Figure 6: **Diagrams of Set III.**

Set III is built from sixth-order proper vertices. 1140 diagrams contribute to $A_1^{(10)}$. 1740 diagrams contribute to $A_2^{(10)}(m_\mu/m_e)$.

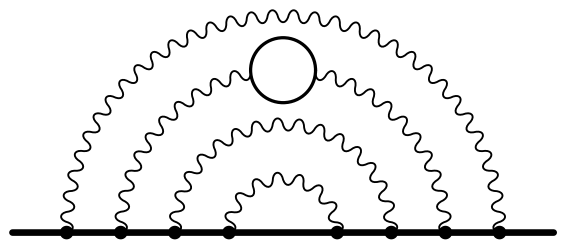

Figure 7: **Diagrams of Set IV.**

Set IV is built from eighth-order proper vertices. 2072 diagrams contribute to both $A_1^{(10)}$ and $A_2^{(10)}(m_\mu/m_e)$.

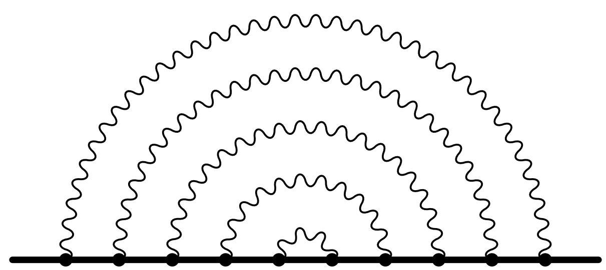

Figure 8: **Diagrams of Set V.**

Set V consists of 10th-order proper vertices with no closed lepton loop. 6354 Feynman diagrams. They contribute only to $A_1^{(10)}$.

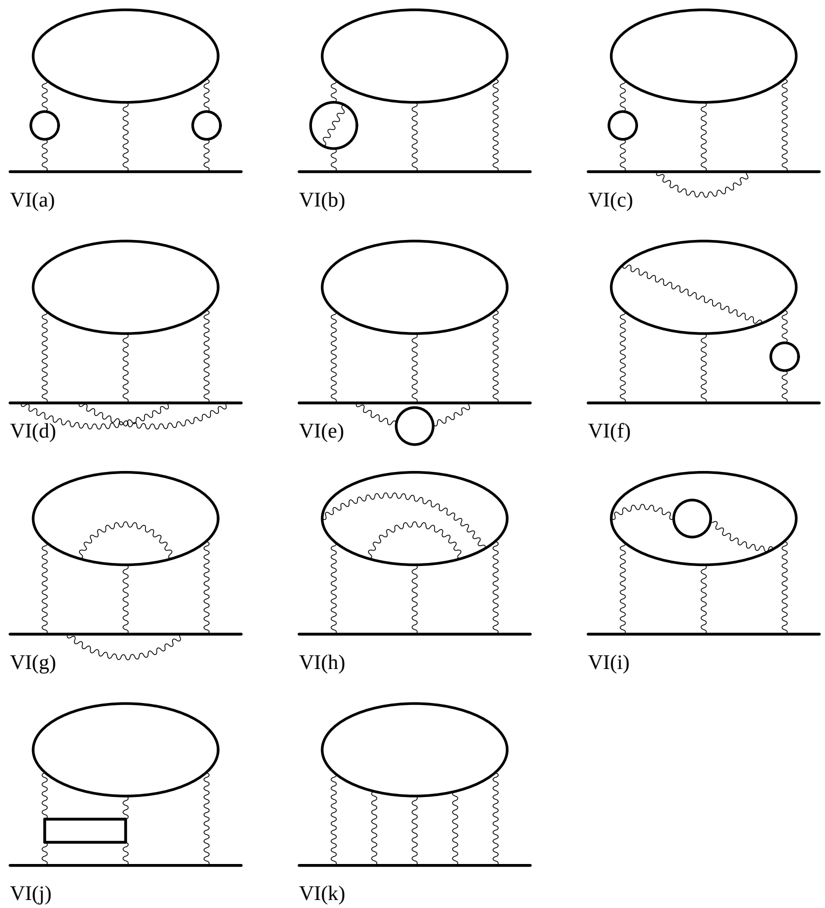

Figure 9: **Diagrams of Set VI.**

Set VI consists of diagrams containing various *l-l-scattering* **subdgrams. 2298 contribute to $A_1^{(10)}$. 3594 contribute to $A_2^{(10)}(m_\mu/m_e)$.**

7. $A_2^{(10)}(m_\mu/m_e)$ of muon g-2

- **Fortunately, for $A_2^{(10)}(m_\mu/m_e)$, it is not difficult to see which g-i sets give large contribution.**
- **They are the sets containing light-by-light scattering subdiagram and/or vacuum-polarization subdiagram, both of which are sources of $\ln(m_\mu/m_e)$.**

Kinoshita et al., PRD 41, 593 (1990)

Karshenboim, Yad. Phys. 56, 252 (1993)

- **Largest contribution comes from Set VI(a) [252 diagrams].**
- **Next largest comes from Set VI(b) [162 diagrams].**
- **We have evaluated them precisely:**

$$\mathbf{A_2[VI(a)] = 629.1407\ (118)} \quad \textbf{(it was 570 (140))}^*$$

$$\mathbf{A_2[VI(b)] = 181.1285\ (\ 51)} \quad \textbf{(it was 176 (35))}^*$$

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* **Still preliminary. Please don't quote until posted on the web.**

- **Largest remaining contribution is likely to come from VI(k) [120 diagrams] whose leading log term was studied previously :**

$$\mathbf{A_2}[\mathbf{VI}(\mathbf{k})] \simeq \mathbf{C_n}\pi^4 \log(\mathbf{m}_\mu/\mathbf{m_e}) + \ldots.$$

Elkhovskii, Yad. Phys. 49, 1059 (1989)

$$\mathbf{C_n} = \mathbf{0.438}\ldots.$$

Milstein,Elkhovskii, Phys. Lett. 233B, 11 (1989)

- **This led to the estimate**

$$\mathbf{A_2}[\mathbf{VI}(\mathbf{k})] = \mathbf{185}\ (\mathbf{85}).$$

Karshenboim, Yad. Phys. 56, 252 (1993)

- **The huge π^4 factor is due to the fact that $\log(m_e)$ comes from the integration domain where all exchanged photons have $|k| << m_e$.**

- **In this domain, electron moves non-relativistically in the Coulomb potential of the muon, each Coulomb photon contributing a factor $i\pi$ when its momentum is integrated out.**

- **This estimate will be too crude since $m_\mu/m_e \simeq 206$ is far from asymptotic. It may have to be reduced substantially, as was the case with sixth-order light-by-light-scattering diagram.**

- **To answer this question it is best to evaluate them explicitly.**

- **It turned out that this is not difficult.**

- **Our first step is to reduce the number of integrals to 12 using**

$$\mathbf{\Lambda}^\nu(\mathbf{p}, \mathbf{q}) \simeq -\mathbf{q}_\mu \left[\frac{\partial \mathbf{\Lambda}_\mu(\mathbf{p},\mathbf{q})}{\partial \mathbf{q}_\nu}\right]_{\mathbf{q}=\mathbf{0}} - \frac{\partial \mathbf{\Sigma}(\mathbf{p})}{\partial \mathbf{p}_\nu},$$

and reduce it further to 9 using time-reversal symmetry.

- **Starting from RHS of this identity, FORM generates more than 90,000 terms occupying about 30,000 lines of FORTRAN code.**

- **Not too bad: It is huge but only 30 times larger than eighth-order integrals.**

- **Numerical integration (over 13-dim. Feynman parameter space) is carried out by VEGAS.**

- **Our result is**

$$\mathbf{A_2[VI(k)] = 86.692\ (91).}^*$$

- **Clearly previous value was overestimate by ~ 100.**

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* Still preliminary. Please don't quote until posted on the web.

- **Another possibly large term is VI(j) [162 vertex diagrams].**
- **Karshenboim's guess:**

$$\mathbf{A_2[VI(j)] = 0 \pm 40},$$

- **We decided to evaluate contribution of these diagrams explicitly.**
- **With the help of W-T transformation and time-reversal invariance they can be represented by 4 independent integrals.**
- **FORM generated about 42,000 terms occupying about 18,000 lines of FORTRAN code.**
- **Our result is**

$$\mathbf{A_2[VI(j)] = -25.5024\ (24)^*.}$$

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* **Still preliminary. Please don't quote until posted on the web.**

Other sets computed (not yet fully double-checked) are:

$$\begin{aligned}
\mathbf{A_2[I(a)]} &= \mathbf{22.553\ 17\ (25)^*} \\
\mathbf{A_2[I(b)]} &= \mathbf{30.667\ 54\ (33)^*} \\
\mathbf{A_2[I(c)]} &= \mathbf{5.141\ 38\ (15)^*} \\
\mathbf{A_2[I(d)]} &= \mathbf{8.892\ 07\ (102)} \\
\mathbf{A_2[I(e)]} &= \mathbf{-1.219\ 20\ (71)} \\
\mathbf{A_2[I(f)]} &= \mathbf{3.685\ 10\ (13)} \\
\mathbf{A_2[II(a)]} &= \mathbf{-70.4717\ (38)^*} \\
\mathbf{A_2[II(b)]} &= \mathbf{-34.7717\ (26)^*} \\
\mathbf{A_2[II(f)]} &= \mathbf{-77.5224\ (414)} \\
\mathbf{A_2[VI(c)]} &= \mathbf{-36.5763\ (1141)} \\
\mathbf{A_2[VI(e)]} &= \mathbf{-4.3215\ (1341)} \\
\mathbf{A_2[VI(f)]} &= \mathbf{-38.1502\ (1545)} \\
\mathbf{A_2[VI(i)]} &= \mathbf{-27.3373\ (1147)}
\end{aligned}$$

Parts of data with * contain analytic results.

Laporta, PLB 328, 522 (1994)

- **Thus far we have evaluated 2958 Feynman diagrams of $A_2^{(10)}(m_\mu/m_e)$.**

- **Remaining 6122 diagrams are not likely to give large contribution.**

- **Our provisional estimate for the 10th-order term is**

$$\mathbf{A_2^{(10)}(m_\mu/m_e) = 652(20)}^*,$$

which is smaller by about 280 than the old crude estimate

$$\mathbf{A_2^{(10)}(m_\mu/m_e) = 930(170)}.$$

- **To improve it further we have to evaluate all remaining Feynman diagrams.**

- **It is a matter of time to finish it.**

- **It is put on hold temporarily because our attention is now focused on the electron g-2.**

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* **Still preliminary. Please don't quote until posted on the web.**

8. $A_1^{(10)}$ term of electron g-2

- Electron g-2 is much harder to evaluate.
- Besides its huge size, none of 12672 diagrams is dominant so that every term must be evaluated accurately.
- Very large and difficult diagrams are mostly in Set V, set of 6354 Feynman diagrams without closed lepton loop.
- This number can be reduced to 706 by W-T transform.
- Time reversal invariance cuts it down further to 389.

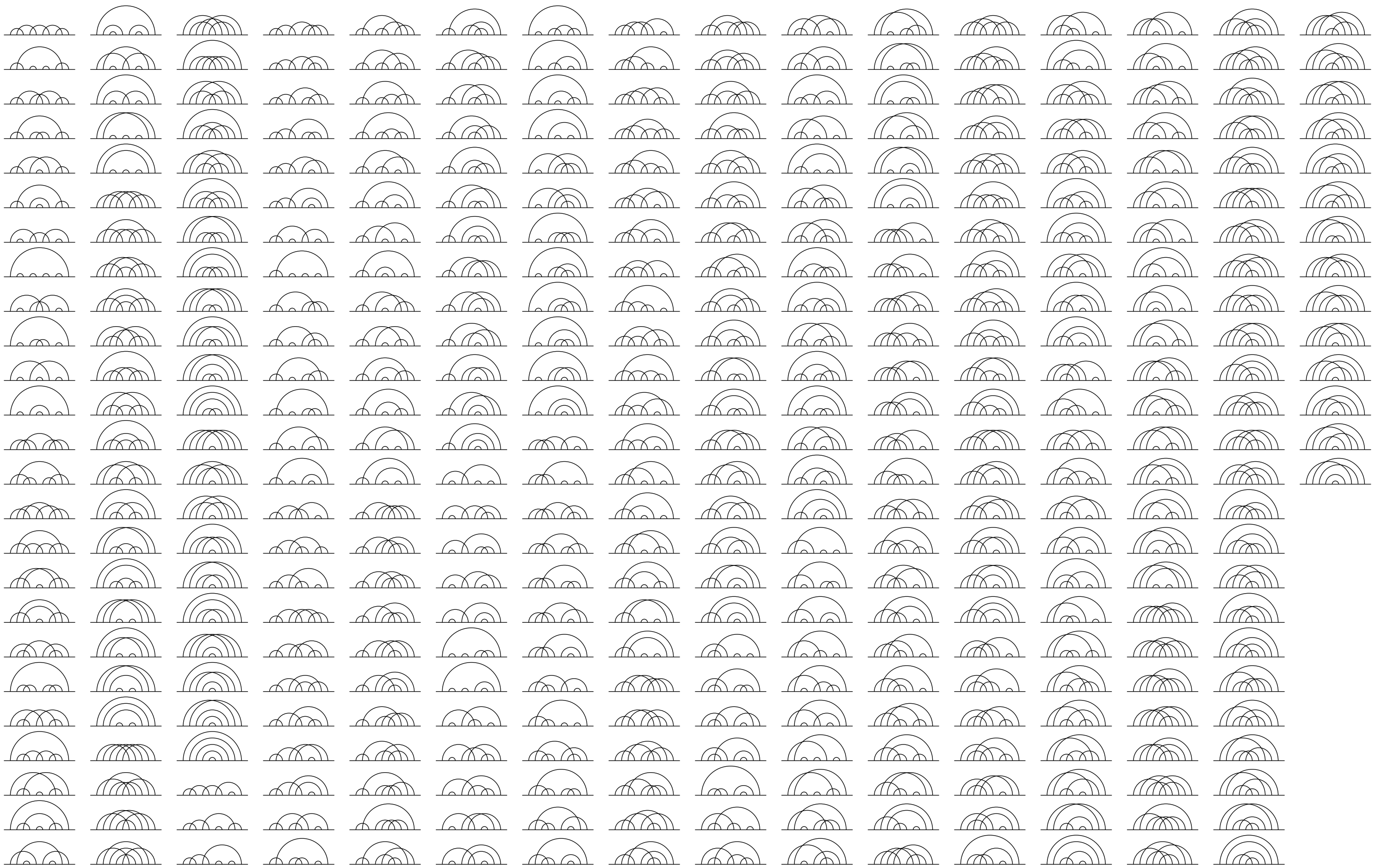

Figure 10: **Overview of all diagrams contributing to Set V.**

- **Both the number of diagrams and size of their integrals are an order of magnitude larger than the α^4 case.**

- **Thus job will be two to three orders of magnitude more difficult.**

- **While the method for α^4 applies to α^5 equally well, its execution for α^4 was rather pedestrian.**

- **It would take more than 100 years if α^5 were handled at the same pace as the α^4 case.**

- **Obviously, to finish α^5 case within reasonal time, we must automate the computation as much as possible.**

- **This is the subject of next talk by Aoyama.**

- **As a prelude to his talk let me outline how previous approach was formulated.**

(I) Identify diagrams contributing to g-2 and their UV- and IR-divergent subdiagrams.

(II) Convert momentum integral obtained by Feynman-Dyson rule to integral, whose integrand is function of Feynman parameters z_i and functions A_i , B_{ij}, $i, j = 1, ..., N$.

(III) Express A_i and B_{ij} as explicit functions of z_i.

(IV) Build counter terms of UV- and IR-divergences.

- **Step II is the most difficult one and was carried out analytically by FORM for α^3 and α^4.**
- **Part of Step IV was automated, too.**
- **In the α^3 and α^4 cases other steps were not difficult and carried out manually.**

- **For the α^5 case all these steps are so huge that it is close to impossible without full automation.**

- **The goal of our project is to make all steps, including renormalization, controled entirely by input information which is <u>one-line</u> computer representation of Feynman diagram.**

- **We have been able to achieve such an automation by a code which faithfully simulates analysis of forest of diagrams by Zimmermann.**

- **Thus far we generated FORTRAN codes for all 2232 diagrams which contain only vertex subdiagrams.**

- **Numerical evaluation of these integrals gives rough idea how large and difficult they actually are.**

- **Typical integrand has about quarter million terms occupying about 80,000 line of FORTRAN code.**

- **This is about 3 times larger than Set VI(k) diagrams, which is still manageable.**

- **Integration by VEGAS had no problem thus far.**

- **Complete automation, including diagrams containing self-energy subdiagrams, will be achieved shortly.**

- **Although this project is far from finished, we might say that we have come a long way to realize physicists' pipe dream since Feynman diagram was invented more than half century ago.**