Precision Predictions for Deep-Inelastic Scattering

Sven-Olaf Moch

Sven-Olaf.Moch@desy.de

DESY, Zeuthen

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in collaboration with **J.A.M. Vermaseren** (NIKHEF) and **A. Vogt** (IPPP Durham)

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Plan

- One loop (Introduction)

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- Two loops (Methods)

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- Two loops (Methods)
- Three loops (Results)

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- Two loops (Methods)
- Three loops (Results)
- Four loops (Applications)

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- One loop (Introduction)
- Two loops (Methods)
- Three loops (Results)
- Four loops (Applications)
- Five loops (Summary and Outlook)

Introduction

Basic concepts of perturbative QCD

- QCD theory predictions at high energies rely on few basic concepts
  - infrared safety
  - factorization
  - evolution

Introduction

Basic concepts of perturbative QCD

- QCD theory predictions at high energies rely on few basic concepts
  - infrared safety
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Infrared safety

- Small class of cross sections at high energies directly calculable in perturbation theory
- Infrared safe quantities
  - free of long range dependencies at leading power in large momentum scale Q Kinoshita '62; Lee, Nauenberg '64
- General structure of cross section
  - large momentum scale Q, renormalization scale μ

$$Q^2\,\hat{\sigma}\left(Q^2,\mu^2,\alpha_s(\mu^2)\right) = \sum_n \alpha_s^n\, c_n(Q^2/\mu^2)$$

Factorization

- Large class of hard-scattering reactions (e.g. initial state hadrons)
  - sensitive to dynamics form different scales (e.g. soft and collinear)
- Structure of factorized cross section
  - large momentum scale Q, factorization scale μ, soft scale m

$$Q^2\, \sigma_{\mathrm{phys}}\, (Q, m) = \hat{\sigma}_{\mathrm{pt}}\, (Q/\mu, \alpha_s(\mu))\; \otimes\; \phi\, (\mu, m)$$

  - convolution $\otimes$ in suitable kinematical variables
  - generalization of operator product expansion

Evolution

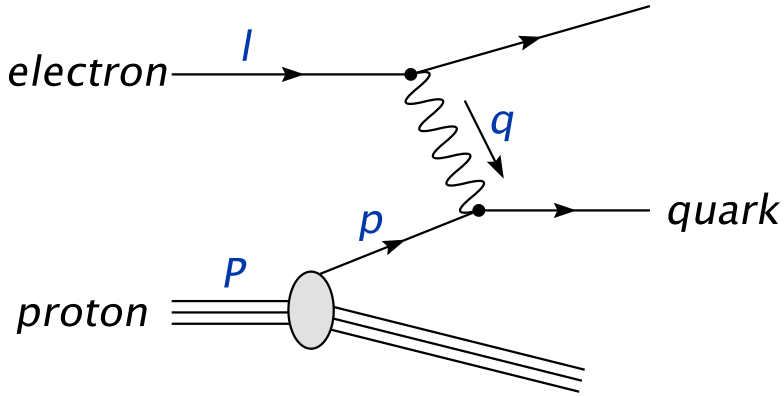
- Dependence of cross sections for observable on momentum transfer
- Physical cross section in factorization ansatz cannot depend on μ

$$\mu \frac{d\sigma_{\mathrm{phys}}}{d\mu} = 0$$

(factorization scale μ arbitrary)

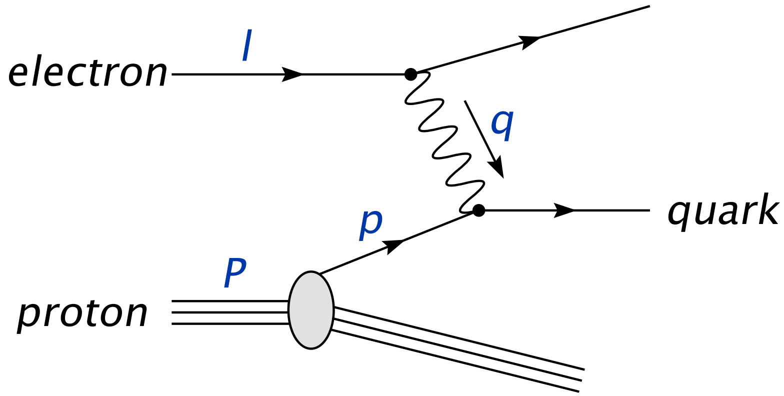
- Classic example: QCD corrections to deep-inelastic scattering
  - scaling violations Gross, Wilczek '73; Politzer '73
  - evolution of parton densities Altarelli, Parisi '77

Deep-inelastic scattering



- **Kinematic variables**
  - momentum transfer $Q^2 = -q^2$
  - Bjorken variable $x = Q^2/(2p \cdot q)$

Deep-inelastic scattering



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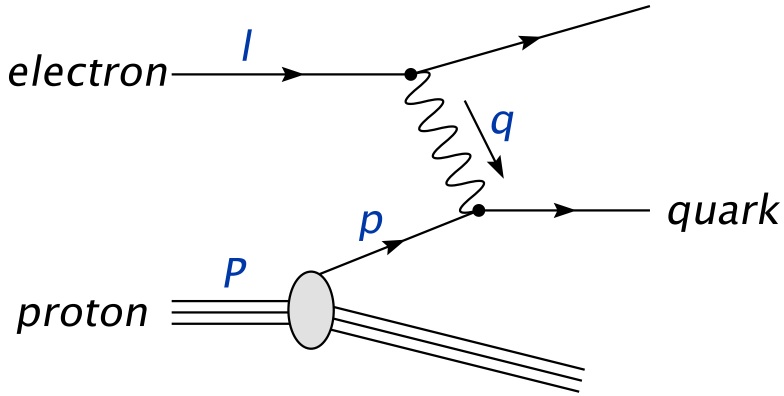
- Structure functions (up to order $\mathcal{O}(1/Q^2)$)

$$F_a\left(x, Q^2\right) = \sum_i \left[c_{a,i}\left(\alpha_s(\mu^2), \mu^2/Q^2\right) \otimes PDF\left(\mu^2\right)\right](x)$$

  - coefficient functions $c_{a,i}$

Deep-inelastic scattering



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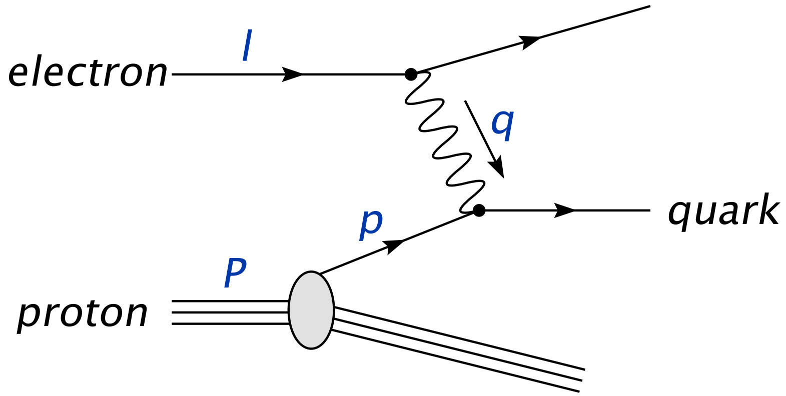
- Evolution equations
  - non-singlet ($2n_f - 1$ scalar) and singlet (2×2 matrix) equations

$$\frac{d}{d \ln \mu^2} PDF(x, \mu^2) \quad = \quad \left[P(\alpha_s(\mu^2)) \otimes PDF(\mu^2)\right](x)$$

  - splitting functions P_{ij}

Deep-inelastic scattering



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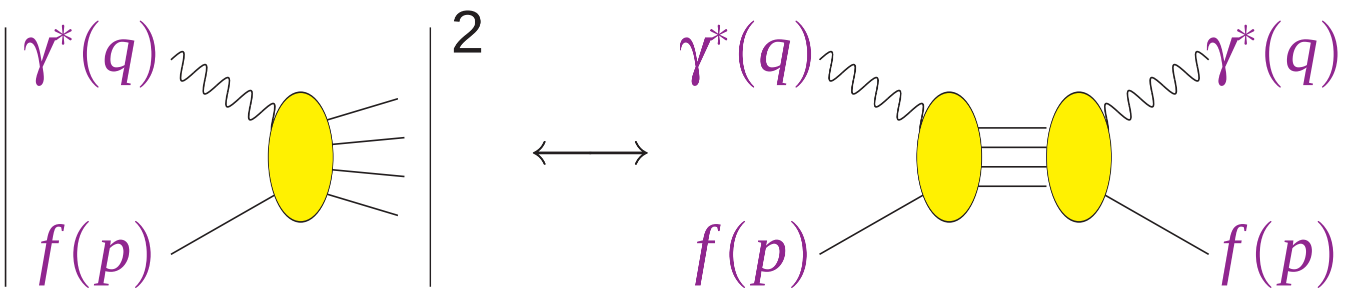
Methods

Optical theorem

- Total cross section $\longleftrightarrow$ imaginary part of Compton amplitude
  - three-loop calculation in DIS with help of **loop technology**



- Coefficient of $(2p \cdot q)^N$ gives N-th Mellin moment

$$A^N = \int_0^1 dx \; x^{N-1} A(x)$$

- UV and mass singularities in dimensional regularization $D = 4 - 2\epsilon$
  - splitting functions: $\frac{1}{\epsilon}$-poles
  - coefficient functions: ϵ^0-part

Three-loop calculcation of DIS

- Splitting functions $P_{\rm gg}$, $P_{\rm gq}$
  - DIS with scalar ϕ coupling to $G^a_{\mu\nu} G_a^{\mu\nu}$ (cf. Higgs)
  - gluon polarization sum $\longleftrightarrow$ diagrams with external ghost h

	tree	1-loop	2-loop	3-loop
qγ	1	3	25	359
gγ		2	17	345
hγ			2	56
qW	1	3	32	589
qϕ		1	23	696
gϕ	1	8	218	6378
hϕ		1	33	1184
sum	3	18	350	9607

- Highly optimised symbolic manipulation of formulae
  - computer algebra system FORM Vermaseren '89-'05
  - capabilities substantially extended for this project

Results

NNLO non-singlet splitting functions

S.M., Vermaseren, Vogt ‘04

$$
\begin{aligned}
P_{\rm ns}^{(2)+}(x) \;=\;& 16C_AC_Fn_f\Big(\frac{1}{6}p_{\rm qq}(x)\Big[\frac{10}{3}\zeta_2-\frac{209}{36}-9\zeta_3-\frac{167}{18}H_0+2H_0\zeta_2-7H_{0,0}-2H_{0,0,0}\\
&+3H_{1,0,0}-H_3\Big]+\frac{1}{3}p_{\rm qq}(-x)\Big[\frac{3}{2}\zeta_3-\frac{5}{3}\zeta_2-H_{-2,0}-2H_{-1}\zeta_2-\frac{10}{3}H_{-1,0}-H_{-1,0,0}\\
&+2H_{-1,2}+\frac{1}{2}H_0\zeta_2+\frac{5}{3}H_{0,0}+H_{0,0,0}-H_3\Big]+(1-x)\Big[\frac{1}{6}\zeta_2-\frac{257}{54}-\frac{43}{18}H_0-\frac{1}{6}H_{0,0}-H_1\Big]\\
&-(1+x)\Big[\frac{2}{3}H_{-1,0}+\frac{1}{2}H_2\Big]+\frac{1}{3}\zeta_2+H_0+\frac{1}{6}H_{0,0}+\delta(1-x)\Big[\frac{5}{4}-\frac{167}{54}\zeta_2+\frac{1}{20}\zeta_2^{\,2}+\frac{25}{18}\zeta_3\Big]\Big)\\
&+16C_AC_F^{\,2}\Big(p_{\rm qq}(x)\Big[\frac{5}{6}\zeta_3-\frac{69}{20}\zeta_2^{\,2}-H_{-3,0}-3H_{-2}\zeta_2-14H_{-2,-1,0}+3H_{-2,0}+5H_{-2,0,0}\\
&-4H_{-2,2}-\frac{151}{48}H_0+\frac{41}{12}H_0\zeta_2-\frac{17}{2}H_0\zeta_3-\frac{13}{4}H_{0,0}-4H_{0,0}\zeta_2-\frac{23}{12}H_{0,0,0}+5H_{0,0,0,0}+\frac{2}{3}H_3\\
&-24H_1\zeta_3-16H_{1,-2,0}+\frac{67}{9}H_{1,0}-2H_{1,0}\zeta_2+\frac{31}{3}H_{1,0,0}+11H_{1,0,0,0}+8H_{1,1,0,0}-8H_{1,3}+H_4\\
&+\frac{67}{9}H_2-2H_2\zeta_2+\frac{11}{3}H_{2,0}+5H_{2,0,0}+H_{3,0}\Big]+p_{\rm qq}(-x)\Big[\frac{1}{4}\zeta_2^{\,2}-\frac{67}{9}\zeta_2+\frac{31}{4}\zeta_3+5H_{-3,0}\\
&-32H_{-2}\zeta_2-4H_{-2,-1,0}-\frac{31}{6}H_{-2,0}+21H_{-2,0,0}+30H_{-2,2}-\frac{31}{3}H_{-1}\zeta_2-42H_{-1}\zeta_3+\frac{9}{4}H_0\\
&-4H_{-1,-2,0}+56H_{-1,-1}\zeta_2-36H_{-1,-1,0,0}-56H_{-1,-1,2}-\frac{134}{9}H_{-1,0}-42H_{-1,0}\zeta_2-H_{3,0}\\
&+32H_{-1,3}-\frac{31}{6}H_{-1,0,0}+17H_{-1,0,0,0}+\frac{31}{3}H_{-1,2}+2H_{-1,2,0}+\frac{13}{12}H_0\zeta_2+\frac{29}{2}H_0\zeta_3+\frac{67}{9}H_{0,0}\\
&+13H_{0,0}\zeta_2+\frac{89}{12}H_{0,0,0}-5H_{0,0,0,0}-7H_2\zeta_2-\frac{31}{6}H_3-10H_4\Big]+(1-x)\Big[\frac{133}{36}+4H_{0,0,0,0}\\
&-\frac{167}{4}\zeta_3-2H_0\zeta_3-2H_{-3,0}+H_{-2}\zeta_2+2H_{-2,-1,0}-3H_{-2,0,0}+\frac{77}{4}H_{0,0,0}-\frac{209}{6}H_1-7H_1\zeta_2\\
&+4H_{1,0,0}+\frac{14}{3}H_{1,0}\Big]+(1+x)\Big[\frac{43}{2}\zeta_2-3\zeta_2^{\,2}+\frac{25}{2}H_{-2,0}-31H_{-1}\zeta_2-14H_{-1,-1,0}-\frac{13}{3}H_{-1,0}\\
&+24H_{-1,2}+23H_{-1,0,0}+\frac{55}{2}H_0\zeta_2+5H_{0,0}\zeta_2+\frac{1457}{48}H_0-\frac{1025}{36}H_{0,0}-\frac{155}{6}H_2+H_2\zeta_2-15H_3\\
&+2H_{2,0,0}-3H_4\Big]-5\zeta_2-\frac{1}{2}\zeta_2^{\,2}+50\zeta_3-2H_{-3,0}-7H_{-2,0}-H_0\zeta_3-\frac{37}{2}H_0\zeta_2-\frac{242}{9}H_0\\
&-2H_{0,0}\zeta_2+\frac{185}{6}H_{0,0}-22H_{0,0,0}-4H_{0,0,0,0}+\frac{28}{3}H_2+6H_3+\delta(1-x)\Big[\frac{151}{64}+\zeta_2\zeta_3-\frac{205}{24}\zeta_2\\
&-\frac{247}{60}\zeta_2^{\,2}+\frac{211}{12}\zeta_3+\frac{15}{2}\zeta_5\Big]\Big)+16C_A^{\,2}C_F\Big(p_{\rm qq}(x)\Big[\frac{245}{48}-\frac{67}{18}\zeta_2+\frac{12}{5}\zeta_2^{\,2}+\frac{1}{2}\zeta_3+\frac{1043}{216}H_0\\
&+H_{-3,0}+4H_{-2,-1,0}-\frac{3}{2}H_{-2,0}-H_{-2,0,0}+2H_{-2,2}-\frac{31}{12}H_0\zeta_2+4H_0\zeta_3+\frac{389}{72}H_{0,0}-2H_{2,0,0}\\
&-H_{0,0,0,0}+9H_1\zeta_3+6H_{1,-2,0}-H_{1,0}\zeta_2-\frac{11}{4}H_{1,0,0}-3H_{1,0,0,0}-4H_{1,1,0,0}+4H_{1,3}+\frac{31}{12}H_{0,0,0}\\
&+\frac{11}{12}H_3+H_4\Big]+p_{\rm qq}(-x)\Big[\frac{67}{18}\zeta_2-\zeta_2^{\,2}-\frac{11}{4}\zeta_3-H_{-3,0}+8H_{-2}\zeta_2+\frac{11}{6}H_{-2,0}-4H_{-2,0,0}\\
&-3H_{-1,0,0,0}+\frac{11}{3}H_{-1}\zeta_2+12H_{-1}\zeta_3-16H_{-1,-1}\zeta_2+8H_{-1,-1,0,0}+16H_{-1,-1,2}+\frac{67}{9}H_{-1,0}
\end{aligned}
$$

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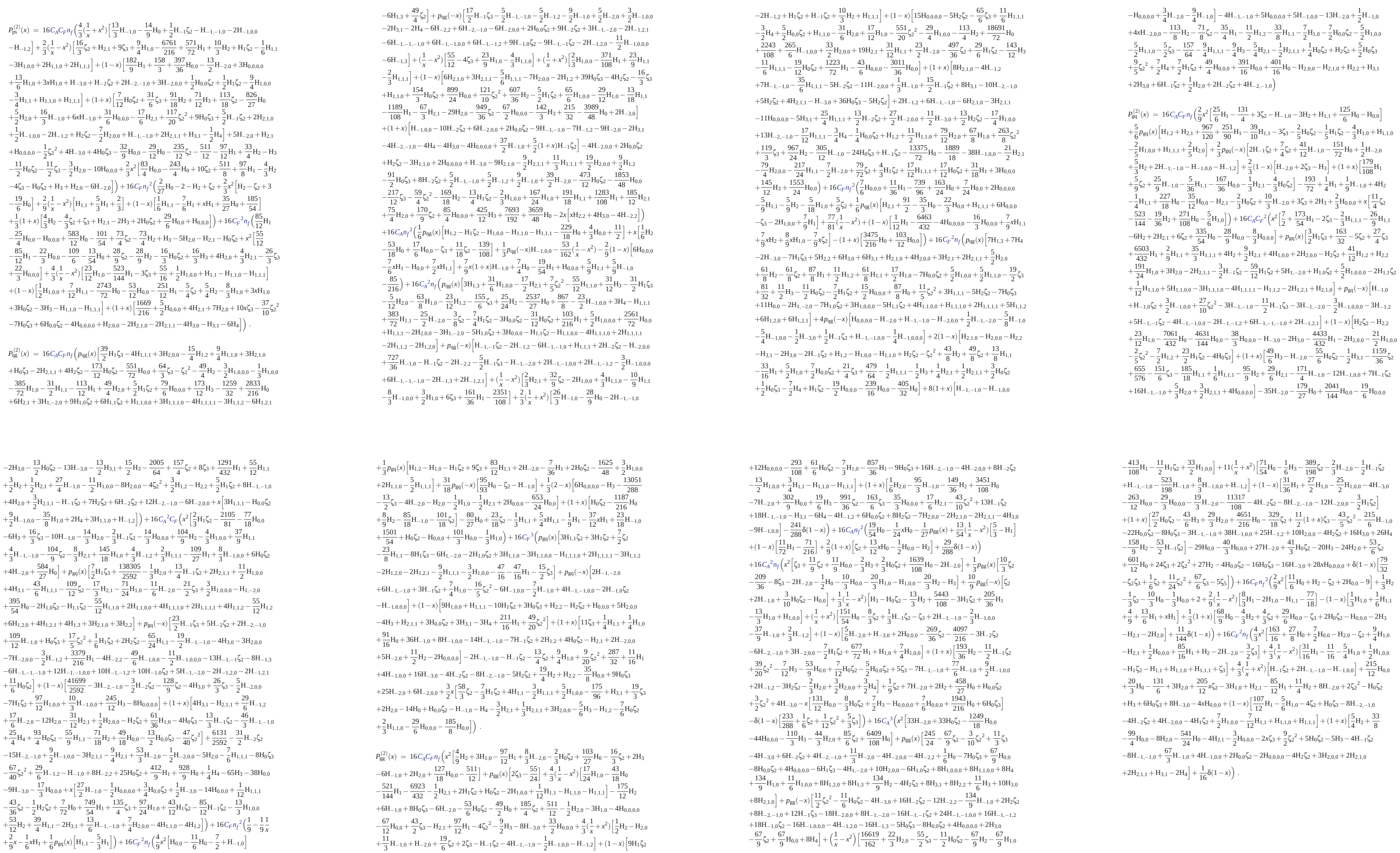
\begin{aligned}
&-8H_{-2,2}+11H_{-1,0}\zeta_2+\frac{11}{6}H_{-1,0,0}-\frac{11}{3}H_{-1,2}-8H_{-1,3}-\frac{3}{4}H_0-\frac{1}{6}H_0\zeta_2-4H_0\zeta_3-\frac{67}{18}H_{0,0}\\
&-3H_{0,0}\zeta_2-\frac{31}{12}H_{0,0,0}+H_{0,0,0,0}+2H_2\zeta_2+\frac{11}{6}H_3+2H_4\Big]+(1-x)\Big[\frac{1883}{108}-\frac{1}{2}H_{0,0,0,0}+11H_1\\
&-H_{-2,-1,0}+\frac{1}{2}H_{-3,0}-\frac{1}{2}H_{-2}\zeta_2+\frac{1}{2}H_{-2,0,0}+\frac{523}{36}H_0+H_0\zeta_3-\frac{13}{3}H_{0,0}-\frac{5}{2}H_{0,0,0}+2H_1\zeta_2\\
&-2H_{1,0,0}\Big]+(1+x)\Big[8H_{-1}\zeta_2+4H_{-1,-1,0}+\frac{8}{3}H_{-1,0}-5H_{-1,0,0}-6H_{-1,2}-\frac{13}{3}\zeta_2+\frac{3}{8}\zeta_2^{\,2}\\
&-\frac{43}{4}\zeta_3-\frac{5}{2}H_{-2,0}-\frac{11}{2}H_0\zeta_2-\frac{1}{2}H_2\zeta_2-\frac{5}{4}H_{0,0}\zeta_2+7H_2-\frac{1}{4}H_{2,0,0}+3H_3+\frac{3}{4}H_4\Big]+\frac{1}{2}H_{0,0}\zeta_2\\
&+\frac{1}{4}\zeta_2^{\,2}-\frac{8}{3}\zeta_2+\frac{17}{2}\zeta_3+H_{-2,0}-\frac{19}{2}H_0+\frac{5}{2}H_0\zeta_2-H_0\zeta_3+\frac{13}{3}H_{0,0}+\frac{5}{2}H_{0,0,0}+\frac{1}{2}H_{0,0,0,0}\\
&-\delta(1-x)\Big[\frac{1657}{576}-\frac{281}{27}\zeta_2+\frac{1}{8}\zeta_2^{\,2}+\frac{97}{9}\zeta_3-\frac{5}{2}\zeta_5\Big]\Big)+16C_Fn_f^2\Big(\frac{1}{18}p_{\rm qq}(x)\Big[H_{0,0}-\frac{1}{3}+\frac{5}{3}H_0\Big]\\
&+(1-x)\Big[\frac{13}{54}+\frac{1}{9}H_0\Big]-\delta(1-x)\Big[\frac{17}{144}-\frac{5}{27}\zeta_2+\frac{1}{9}\zeta_3\Big]\Big)+16C_F^{\,2}n_f\Big(\frac{1}{3}p_{\rm qq}(x)\Big[5\zeta_3-4H_{1,0,0}\\
&-\frac{55}{16}+\frac{5}{8}H_0+H_0\zeta_2+\frac{3}{2}H_{0,0}-H_{0,0,0}-\frac{10}{3}H_{1,0}-\frac{10}{3}H_2-2H_{2,0}-2H_3\Big]+\frac{2}{3}p_{\rm qq}(-x)\Big[\frac{5}{3}\zeta_2\\
&-\frac{3}{2}\zeta_3+H_{-2,0}+2H_{-1}\zeta_2+\frac{10}{3}H_{-1,0}+H_{-1,0,0}-2H_{-1,2}-\frac{1}{2}H_0\zeta_2-\frac{5}{3}H_{0,0}-H_{0,0,0}+H_3\Big]\\
&-(1-x)\Big[\frac{10}{9}+\frac{19}{18}H_{0,0}-\frac{4}{3}H_1+\frac{2}{3}H_{1,0}+\frac{4}{3}H_2\Big]+(1+x)\Big[\frac{4}{3}H_{-1,0}-\frac{25}{24}H_0+\frac{1}{2}H_{0,0,0}\Big]+\frac{2}{9}H_0\\
&+\frac{7}{9}H_{0,0}+\frac{4}{3}H_2-\delta(1-x)\Big[\frac{23}{16}-\frac{5}{12}\zeta_2-\frac{29}{30}\zeta_2^{\,2}+\frac{17}{6}\zeta_3\Big]\Big)+16C_F^{\,3}\Big(p_{\rm qq}(x)\Big[\frac{9}{10}\zeta_2^{\,2}-2H_{-3,0}\\
&+6H_{-2}\zeta_2+12H_{-2,-1,0}-6H_{-2,0,0}-\frac{3}{16}H_0-\frac{3}{2}H_0\zeta_2+H_0\zeta_3+\frac{13}{8}H_{0,0}-2H_{0,0,0,0}+8H_{1,3}\\
&+12H_1\zeta_3+8H_{1,-2,0}-6H_{1,0,0}-4H_{1,0,0,0}+4H_{1,2,0}-3H_{2,0}+2H_{2,0,0}+4H_{2,1,0}+4H_{2,2}\\
&+4H_{3,0}+4H_{3,1}+2H_4\Big]+p_{\rm qq}(-x)\Big[\frac{7}{2}\zeta_2^{\,2}-\frac{9}{2}\zeta_3-6H_{-3,0}+32H_{-2}\zeta_2+8H_{-2,-1,0}+3H_{-2,0}\\
&-26H_{-2,0,0}-28H_{-2,2}+6H_{-1}\zeta_2+36H_{-1}\zeta_3+8H_{-1,-2,0}-48H_{-1,-1}\zeta_2+40H_{-1,-1,0,0}\\
&+48H_{-1,-1,2}+40H_{-1,0}\zeta_2+3H_{-1,0,0}-22H_{-1,0,0,0}-6H_{-1,2}-4H_{-1,2,0}-32H_{-1,3}-\frac{3}{2}H_0\\
&-\frac{3}{2}H_0\zeta_2-13H_0\zeta_3-14H_{0,0}\zeta_2-\frac{9}{2}H_{0,0,0}+6H_{0,0,0,0}+6H_2\zeta_2+3H_3+2H_{3,0}+12H_4\Big]\\
&+(1-x)\Big[2H_{-3,0}-\frac{31}{8}+4H_{-2,0,0}+H_{0,0}\zeta_2-3H_{0,0,0,0}+35H_1+6H_1\zeta_2-H_{1,0}+\frac{5}{2}H_{2,0}\Big]\\
&+(1+x)\Big[\frac{37}{10}\zeta_2^{\,2}-\frac{93}{4}\zeta_2-\frac{81}{2}\zeta_3-15H_{-2,0}+30H_{-1}\zeta_2+12H_{-1,-1,0}-2H_{-1,0}-26H_{-1,0,0}\\
&-24H_{-1,2}-\frac{539}{16}H_0-28H_0\zeta_2+\frac{191}{8}H_{0,0}+20H_{0,0,0}+\frac{85}{4}H_2-3H_{2,0,0}-2H_{3,0}+13H_3\\
&-H_4\Big]+4\zeta_2+33\zeta_3+4H_{-3,0}+10H_{-2,0}+\frac{67}{2}H_0+6H_0\zeta_3+19H_0\zeta_2-25H_{0,0}-17H_{0,0,0}\\
&-2H_2-H_{2,0}-4H_3+\delta(1-x)\Big[\frac{29}{32}-2\zeta_2\zeta_3+\frac{9}{8}\zeta_2+\frac{18}{5}\zeta_2^{\,2}+\frac{17}{4}\zeta_3-15\zeta_5\Big]\Big)\,.
\end{aligned}
$$

$$
\begin{aligned}
P_{\rm ns}^{(2)-}(x) \;=\;& P_{\rm ns}^{(2)+}(x)+16C_AC_F\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\rm qq}(-x)\Big[\frac{134}{9}\zeta_2-4\zeta_2^{\,2}-11\zeta_3-4H_{-3,0}\\
&+32H_{-2}\zeta_2+\frac{22}{3}H_{-2,0}-16H_{-2,0,0}-32H_{-2,2}+\frac{44}{3}H_{-1}\zeta_2+48H_{-1}\zeta_3-64H_{-1,-1}\zeta_2\\
&+32H_{-1,-1,0,0}+64H_{-1,-1,2}+\frac{268}{9}H_{-1,0}+44H_{-1,0}\zeta_2+\frac{22}{3}H_{-1,0,0}-12H_{-1,0,0,0}-\frac{44}{3}H_{-1,2}\\
&-32H_{-1,3}-3H_0-\frac{2}{3}H_0\zeta_2-16H_0\zeta_3-\frac{134}{9}H_{0,0}-12H_{0,0}\zeta_2-\frac{31}{3}H_{0,0,0}+4H_{0,0,0,0}+8H_2\zeta_2\\
&+\frac{22}{3}H_3+8H_4\Big]+(1-x)\Big[\frac{367}{18}+\frac{1}{2}\zeta_2^{\,2}+2H_{-3,0}-2H_{-2}\zeta_2-4H_{-2,-1,0}-10H_{-2,0}-2H_{0,0}\\
&+2H_{-2,0,0}+2H_0\zeta_3+H_{0,0}\zeta_2-H_{0,0,0,0}+8H_1\zeta_2+\frac{140}{3}H_1\Big]+(1+x)\Big[32H_{-1}\zeta_2-18\zeta_2\\
&-23\zeta_3+\frac{26}{3}H_{-1,0}-16H_{-1,0,0}-32H_{-1,2}-\frac{481}{18}H_0-29H_0\zeta_2+5H_{0,0,0}+24H_3+\frac{70}{3}H_2\Big]\\
&-2\zeta_2-2\zeta_3+32H_0+14H_0\zeta_2+2H_{0,0,0}-16H_3\Big)+16C_Fn_f\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\rm qq}(-x)\Big[2\zeta_3\\
&-\frac{20}{9}\zeta_2-\frac{4}{3}H_{-2,0}-\frac{8}{3}H_{-1}\zeta_2-\frac{40}{9}H_{-1,0}-\frac{4}{3}H_{-1,0,0}+\frac{8}{3}H_{-1,2}+\frac{2}{3}H_0\zeta_2+\frac{20}{9}H_{0,0}+\frac{4}{3}H_{0,0,0}\\
&-\frac{4}{3}H_3\Big]+(1-x)\Big[\frac{61}{9}-\frac{8}{3}H_1\Big]+(1+x)\Big[2H_{0,0}-\frac{8}{3}H_{-1,0}+\frac{41}{9}H_0-\frac{4}{3}H_2\Big]\Big)\\
&+16C_F^{\,2}\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\rm qq}(-x)\Big[9\zeta_3-7\zeta_2^{\,2}+12H_{-3,0}-64H_{-2}\zeta_2-16H_{-2,-1,0}-6H_{-2,0}\\
&+52H_{-2,0,0}+56H_{-2,2}-12H_{-1}\zeta_2-72H_{-1}\zeta_3-16H_{-1,-2,0}+96H_{-1,-1}\zeta_2-80H_{-1,-1,0,0}\\
&-96H_{-1,-1,2}-80H_{-1,0}\zeta_2-6H_{-1,0,0}+44H_{-1,0,0,0}+12H_{-1,2}+8H_{-1,2,0}+64H_{-1,3}+3H_0\\
&+3H_0\zeta_2+26H_0\zeta_3+28H_{0,0}\zeta_2+9H_{0,0,0}-12H_{0,0,0,0}-12H_2\zeta_2-6H_3-4H_{3,0}-24H_4\Big]\\
&-(1-x)\Big[15+8H_{-3,0}+8H_{-2,0,0}+61H_0+6H_0\zeta_3+2H_{0,0}\zeta_2-6H_{0,0,0,0}+12H_1\zeta_2+60H_1\\
&+8H_{1,0}\Big]+(1+x)\Big[24\zeta_2+57\zeta_3+10H_{-2,0}-48H_{-1}\zeta_2-4H_{-1,0}+40H_{-1,0,0}+48H_{-1,2}\\
&+59H_0\zeta_2-22H_{0,0}-35H_{0,0,0}-22H_2-4H_{2,0}-44H_3\Big]+8\zeta_2-42\zeta_3-4H_{-2,0}+42H_0\\
&-38H_0\zeta_2+14H_{0,0}-16H_2+26H_{0,0,0}+24H_3\Big)\,.
\end{aligned}
$$

$$
\begin{aligned}
P_{\rm ns}^{(2)s}(x) \;=\;& 16n_f\frac{d^{abc}d_{abc}}{n_c}\Big(\frac{1}{2}(1-x)\Big[\frac{50}{3}+\frac{41}{12}\zeta_2-\frac{5}{4}\zeta_2^{\,2}-H_{-3,0}+H_{-2}\zeta_2-H_{-2,0,0}+\frac{9}{4}H_3\\
&+2H_{-2,-1,0}+\frac{3}{2}H_{0,0}\zeta_2-\frac{1}{2}H_1\zeta_2-\frac{3}{4}H_{1,0,0}+\frac{91}{12}H_1\Big]+\frac{1}{2}(1+x)\Big[H_{-1,-1,0}-\frac{3}{2}H_{-1}\zeta_2+\frac{3}{4}H_0\\
&-\frac{13}{6}H_{-1,0}+\frac{1}{2}H_{-1,0,0}+2H_{-1,2}-\frac{3}{2}H_{-2,0}+\frac{9}{4}H_0\zeta_2+\frac{29}{12}H_{0,0}+\frac{41}{12}H_2-H_2\zeta_2-\frac{1}{2}H_{2,0,0}\\
&+\frac{3}{2}H_4\Big]-\frac{1}{3}\Big(\frac{1}{x}+x^2\Big)\Big[3H_{-1}\zeta_2+2H_{-1,-1,0}-2H_{-1,0,0}-2H_{-1,2}+H_1\zeta_2\Big]+\frac{1}{3}x^2\Big[5\zeta_3-2H_3\\
&+2H_{-2,0}+4H_0\zeta_2-2H_{0,0,0}+2H_1\zeta_2\Big]+\frac{91}{24}H_0+\zeta_3-\frac{9}{2}\zeta_2+\zeta_2^{\,2}-H_0\zeta_3-H_0\zeta_2-2H_{0,0}\zeta_2\\
&+\frac{3}{8}H_{0,0}-\frac{1}{4}H_{0,0,0}+\frac{1}{2}H_{0,0,0,0}+H_{-2,0}-H_3\Big)\,.
\end{aligned}
$$

NNLO singlet splitting functions

S.M., Vermaseren, Vogt '04



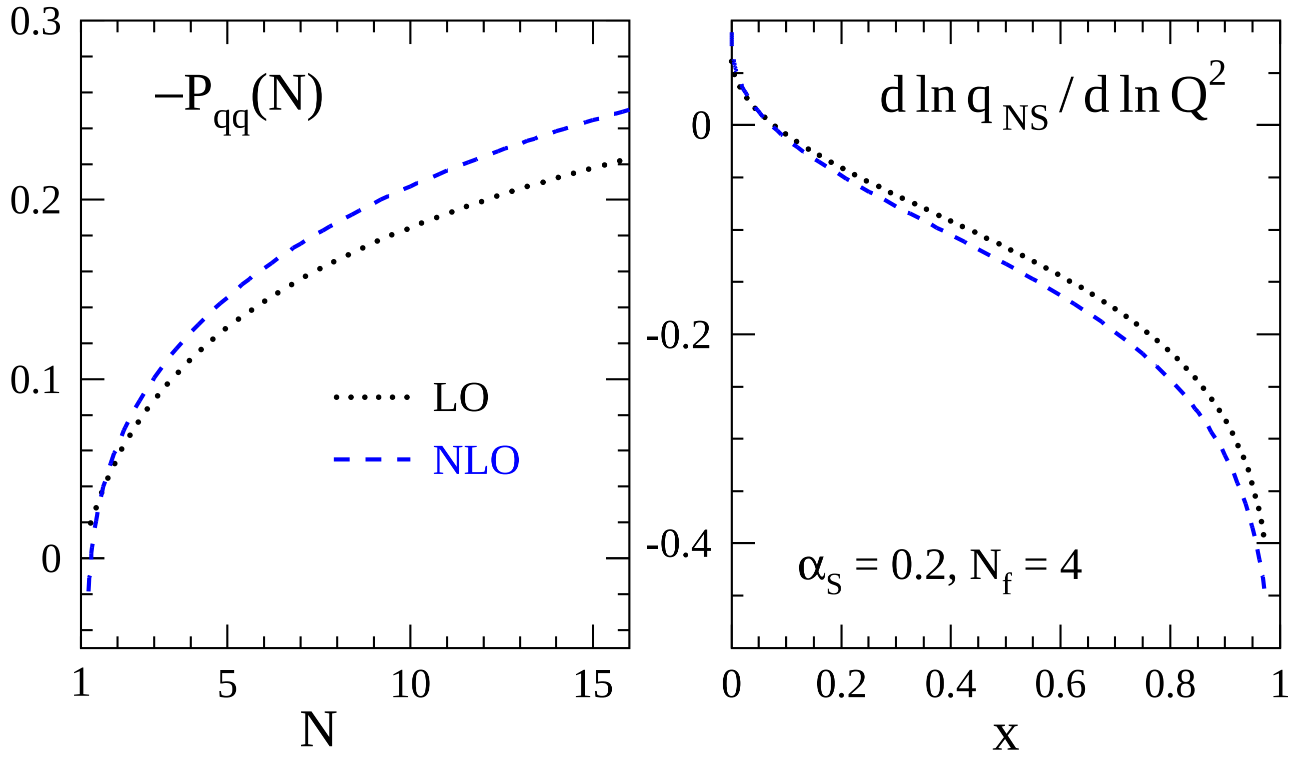
Coefficient functions in DIS at three loops

S.M., Vermaseren, Vogt ‘05

- Exact (analytical) results for coefficient functions of F_2 and F_L fill $\mathcal{O}(100)$ pages (normalsize fonts)

Parton evolution at large N/ large x

- Recall: $A^N = \int_0^1 dx\, x^{N-1} A(x)$, non-singlet $u+\bar{u} - (d+\bar{d})$ etc.



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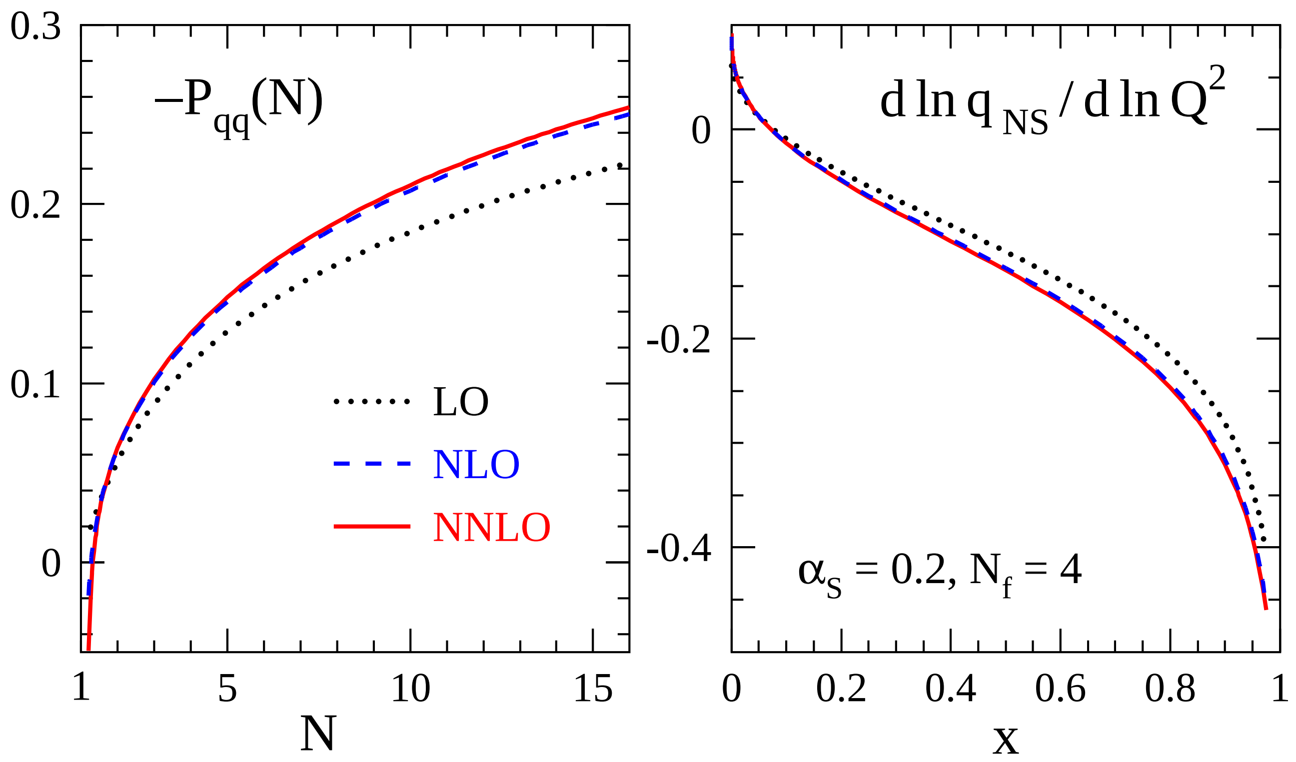

- Perturbative expansion very benign: expect < 1% beyond NNLO

The large x-limit: $x \to 1$

- Large x-limit for diagonal splitting functions $P^{(2)}_{\rm aa}$, $\rm a = q, g$

$$P^{(2)}_{\rm aa, \to 1}(x) \; = \; \frac{A^{\rm a}_3}{(1-x)_+} \; + \; B^{\rm a}_3 \, \delta(1-x) \; + \; C^{\rm a}_3 \; \ln(1-x) \; + \; \mathcal{O}(1)$$

one-loop $\quad A^{\rm q}_1 \; = \; 4\, C_F$

two-loop $\quad A^{\rm q}_2 \; = \; 8\, C_F C_A \left(\frac{67}{18} - \zeta_2 \right) - \frac{5}{9}\, C_F n_f$ Kodaira, Trentadue ‘80

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- $A^{\rm a}_3$ important for **threshold resummation** in soft/collinear limit

$$A^{\rm q}_3 \; = \; 16\, C_F C_A^2 \left(\frac{245}{24} - \frac{67}{9}\,\zeta_2 + \frac{11}{6}\,\zeta_3 + \frac{11}{5}\,\zeta_2^2\right) \; + \; 16\, C_F^2 n_f \left(-\frac{55}{24} + 2\,\zeta_3\right)$$
$$+ \; 16\, C_F C_A n_f \left(-\frac{209}{108} + \frac{10}{9}\,\zeta_2 - \frac{7}{3}\,\zeta_3\right) \; + \; 16\, C_F n_f^2 \left(-\frac{1}{27}\right)$$

- Maximally non-Abelian colour structure, $A^{\rm g}_3 = \frac{C_A}{C_F} A^{\rm q}_3$ Korchemsky ’89

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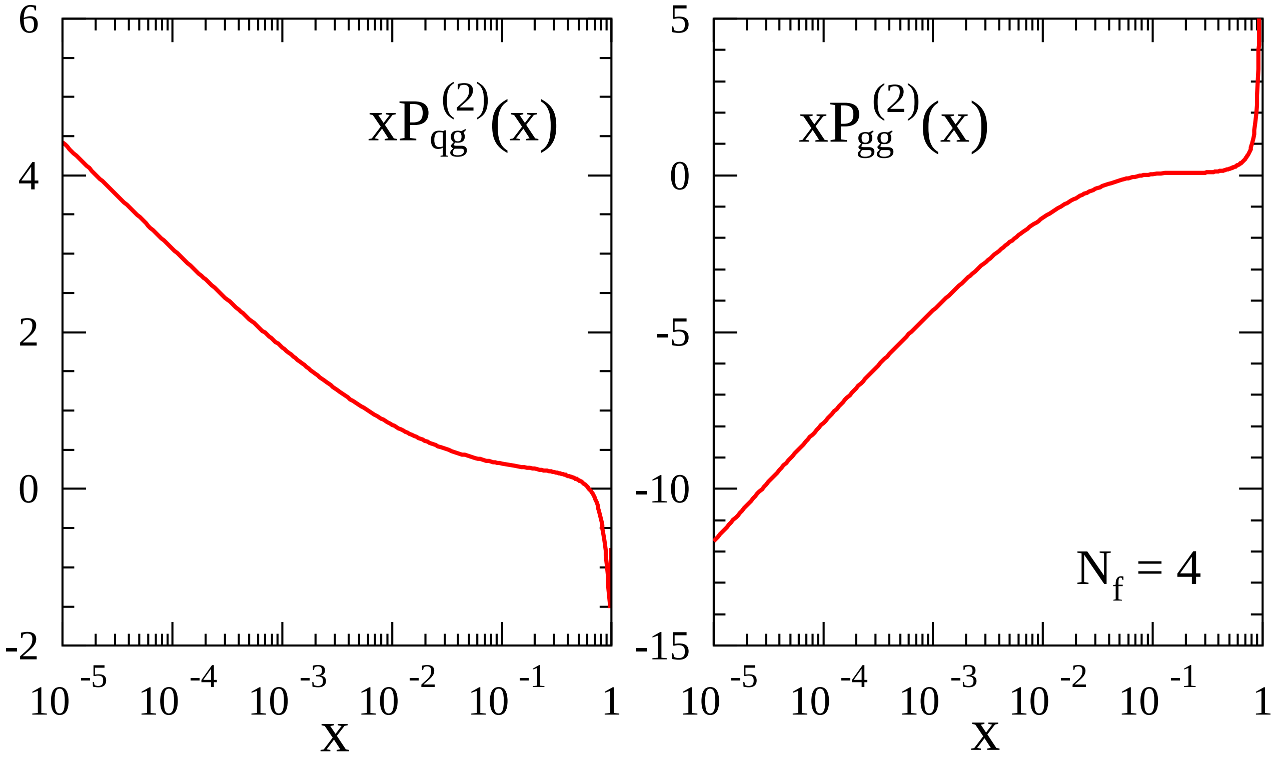
- Subleading logarithms (unexplored structure)

$$C^{\rm a}_1 = 0\,, \quad C^{\rm a}_2 = (A^{\rm a}_1)^2\,, \quad C^{\rm a}_3 = 2\,A^{\rm a}_1 A^{\rm a}_2$$

Three-loop splitting functions at small x

- Small momentum fractions x : $g \to i$ splitting $P_{\rm ig}$ most important



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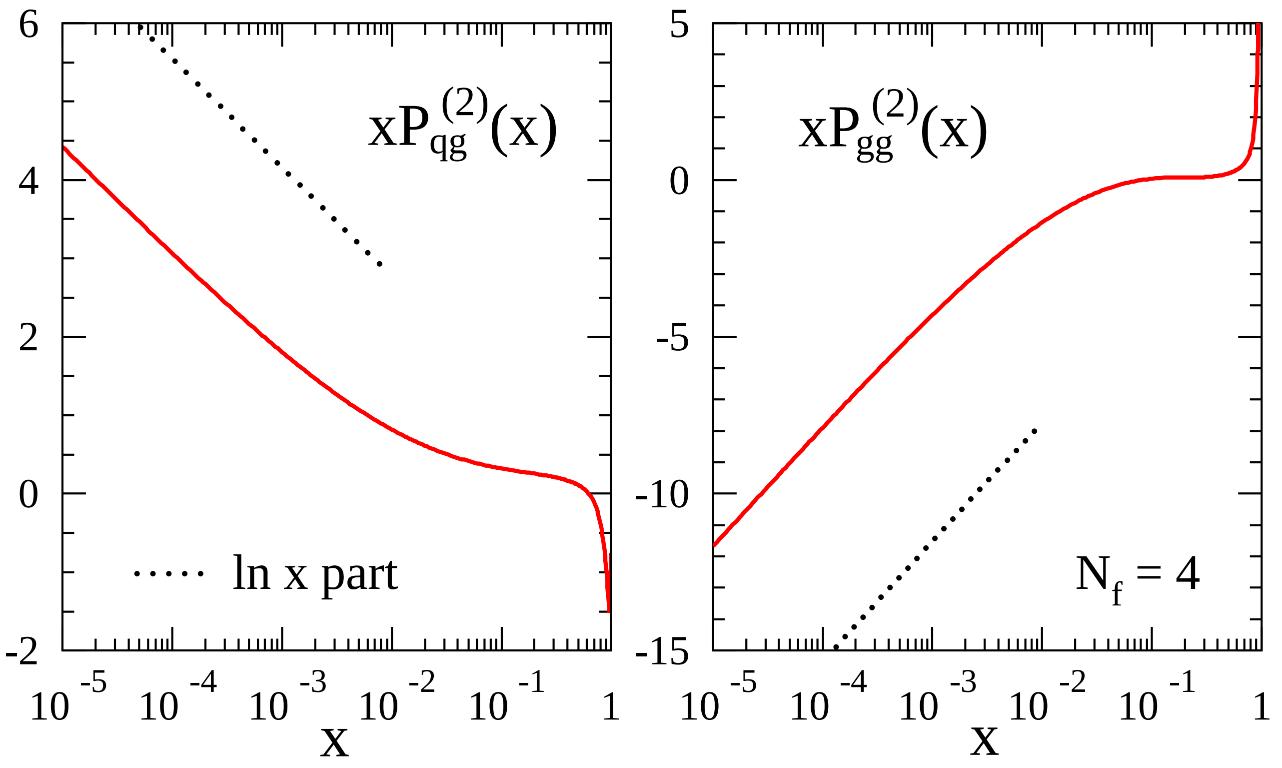

- Leading $x \to 0$-terms (BFKL) confirmed but insufficient at colliders

The small x-limit: $x \to 0$

- **Singlet**
  - Structure of singlet splitting functions at small x

$$P^{(2)}_{\mathrm{ab},\to 0}(x) \;=\; E_1^{\mathrm{ab}}\,\frac{\ln x}{x} \;+\; E_2^{\mathrm{ab}}\,\frac{1}{x} \;+\; \mathcal{O}(\ln^{\,4} x)$$

$$E_1^{\mathrm{gg}} \;=\; \Big(\frac{6320}{27} - \frac{176}{3}\,\zeta_2 - 32\,\zeta_3\Big)C_A^3 + \Big(\frac{1136}{27} - \frac{32}{3}\,\zeta_2\Big)C_A^2 n_f - \Big(\frac{1376}{27} - \frac{64}{3}\,\zeta_2\Big)C_A C_F n_f$$

$$\cong\; 2675.85 + 157.269\,n_f$$

$$E_2^{\mathrm{gg}} \;\cong\; 14214.2 + 182.958\,n_f - 2.79835\,n_f^2$$

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  - predicted by leading logarithmic BFKL equation

    Kuraev, Lipatov, Fadin ‘77; Balitsky, L.N. Lipatov ‘78; Jaroszewicz ‘82

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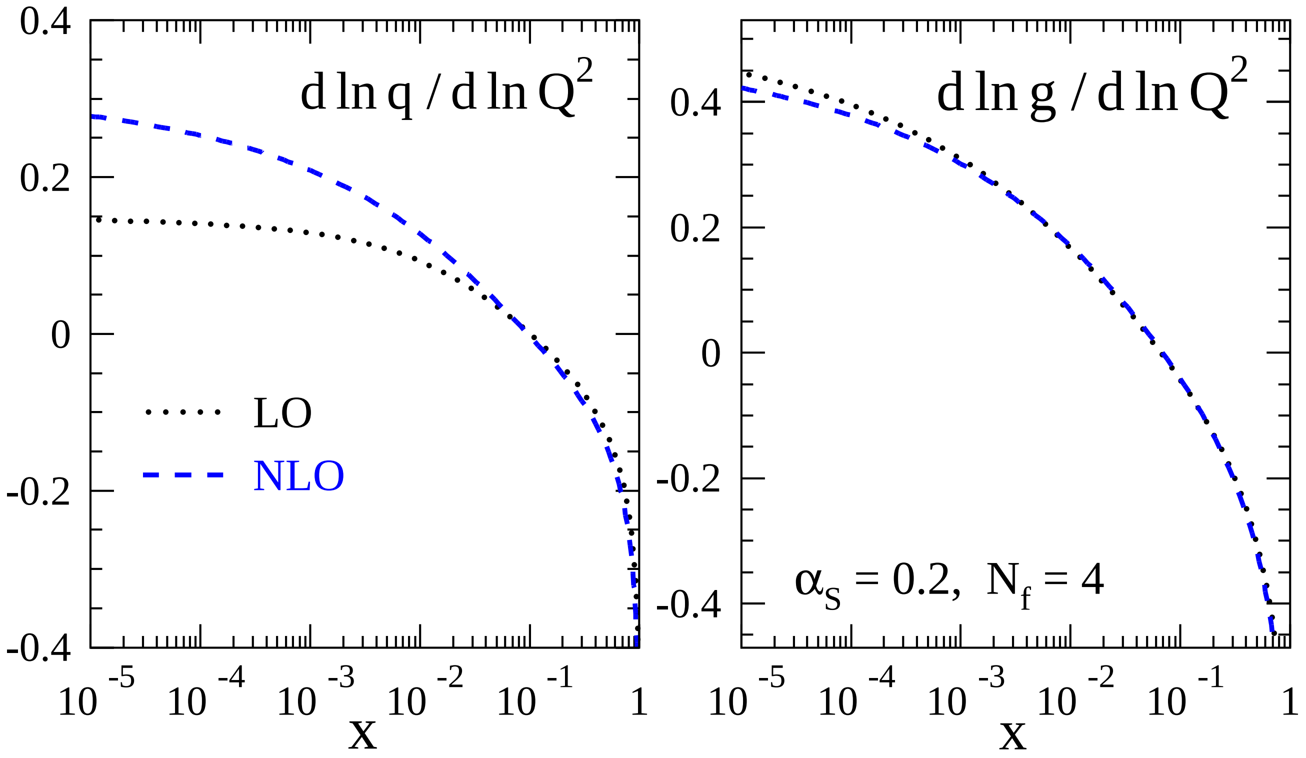
- Coefficients for quark case Catani, Hautmann ‘94

Evolution of parton distributions at small x

- Scale derivatives of quark and gluon distributions at $Q^2 \approx 30$ GeV^2



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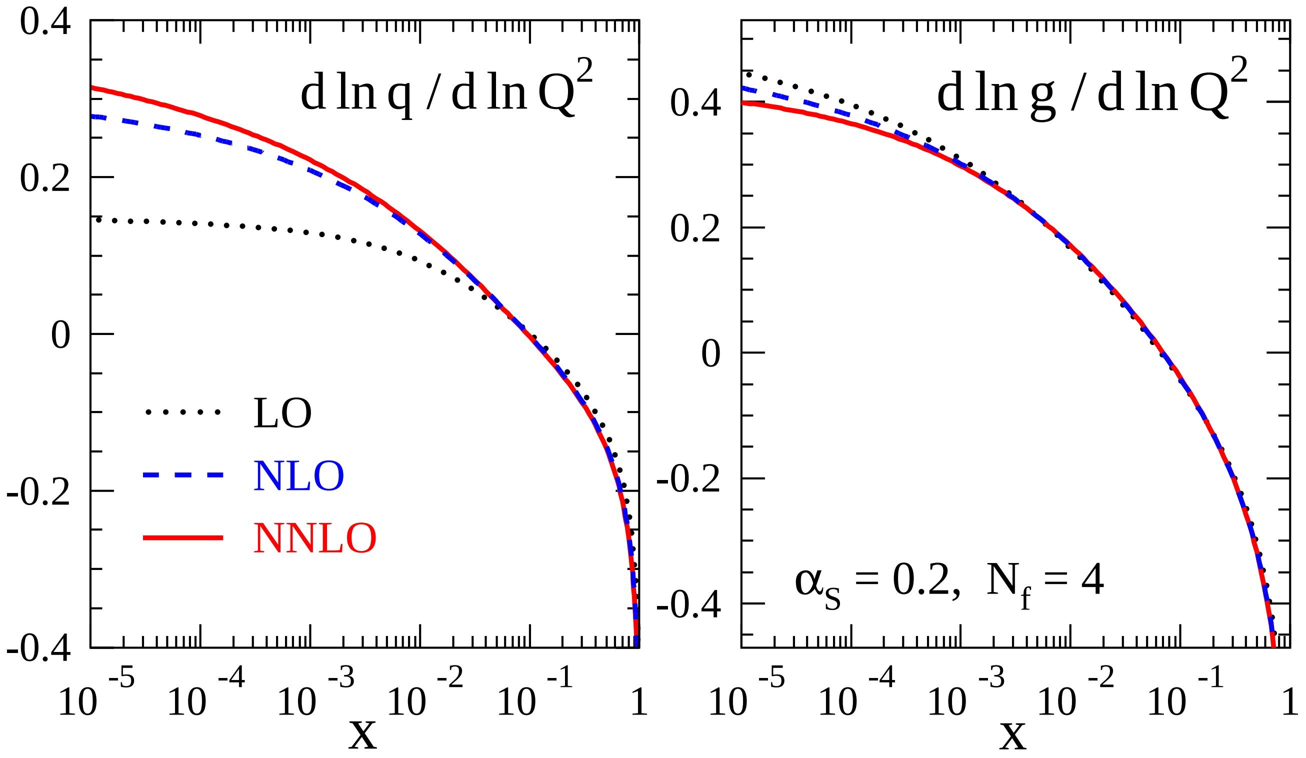
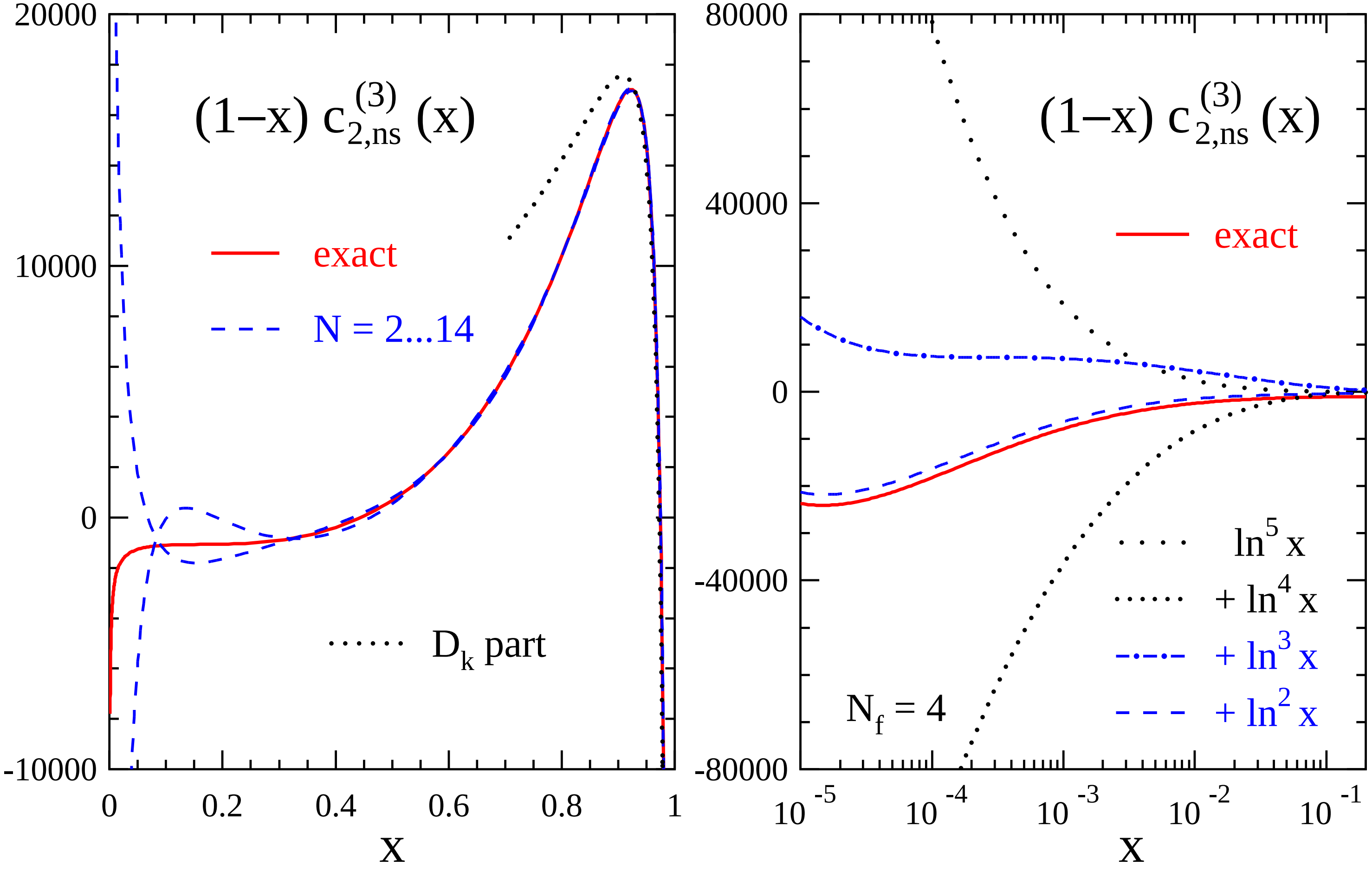


- Expansion very stable except for very small momenta $x \lesssim 10^{-4}$

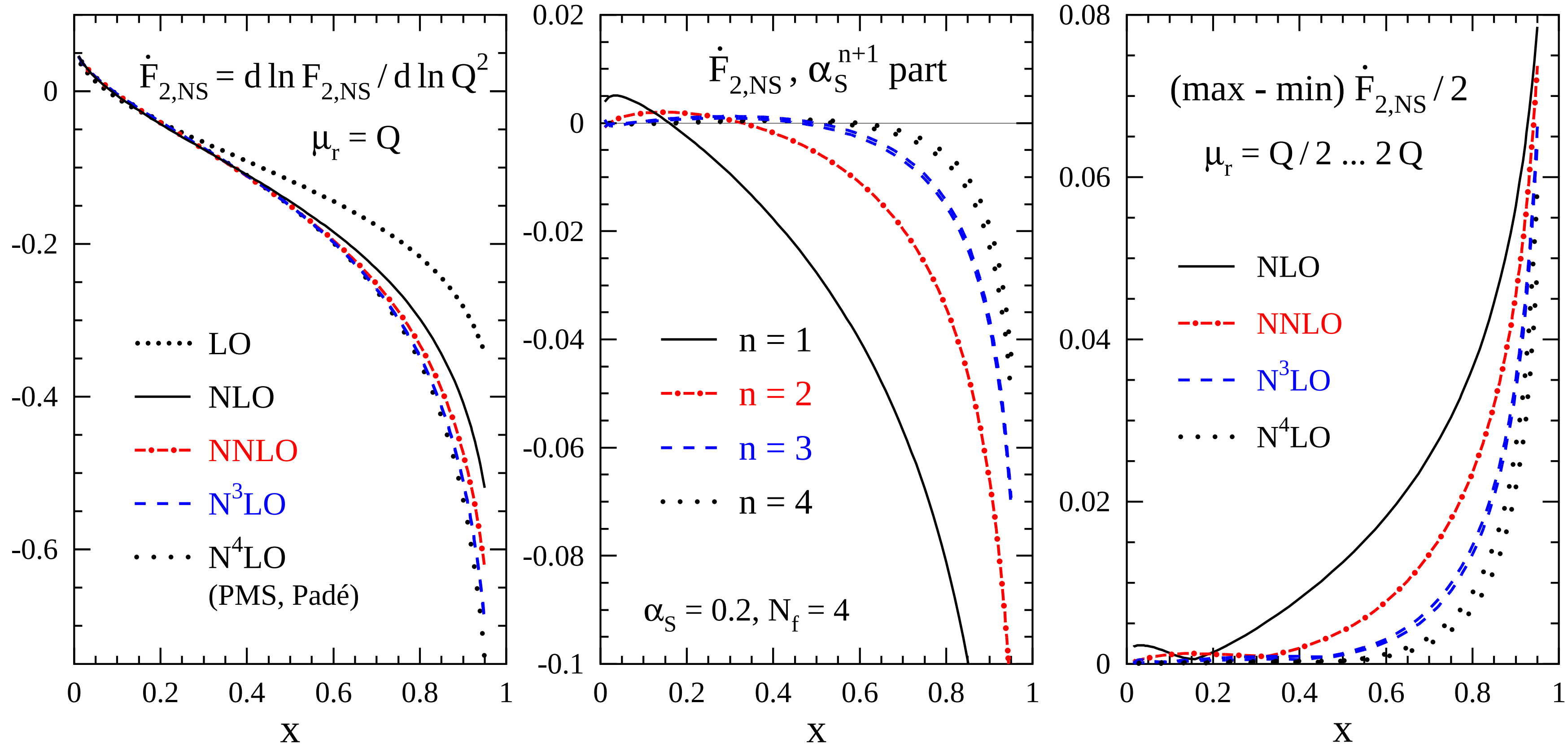
Three-loop coefficient functions for F_2 (non-singlet)



- Large x-limit at n$^{\mathrm{th}}$-order $\alpha_s^n \, \dfrac{\ln^{2n-1}(1-x)}{(1-x)_+}$
- Small-x limit insufficient for accurate description

The structure function F_2 (non-singlet)

- Large-x convergence of perturbative series
  - approx. N^{3}LO structure functions



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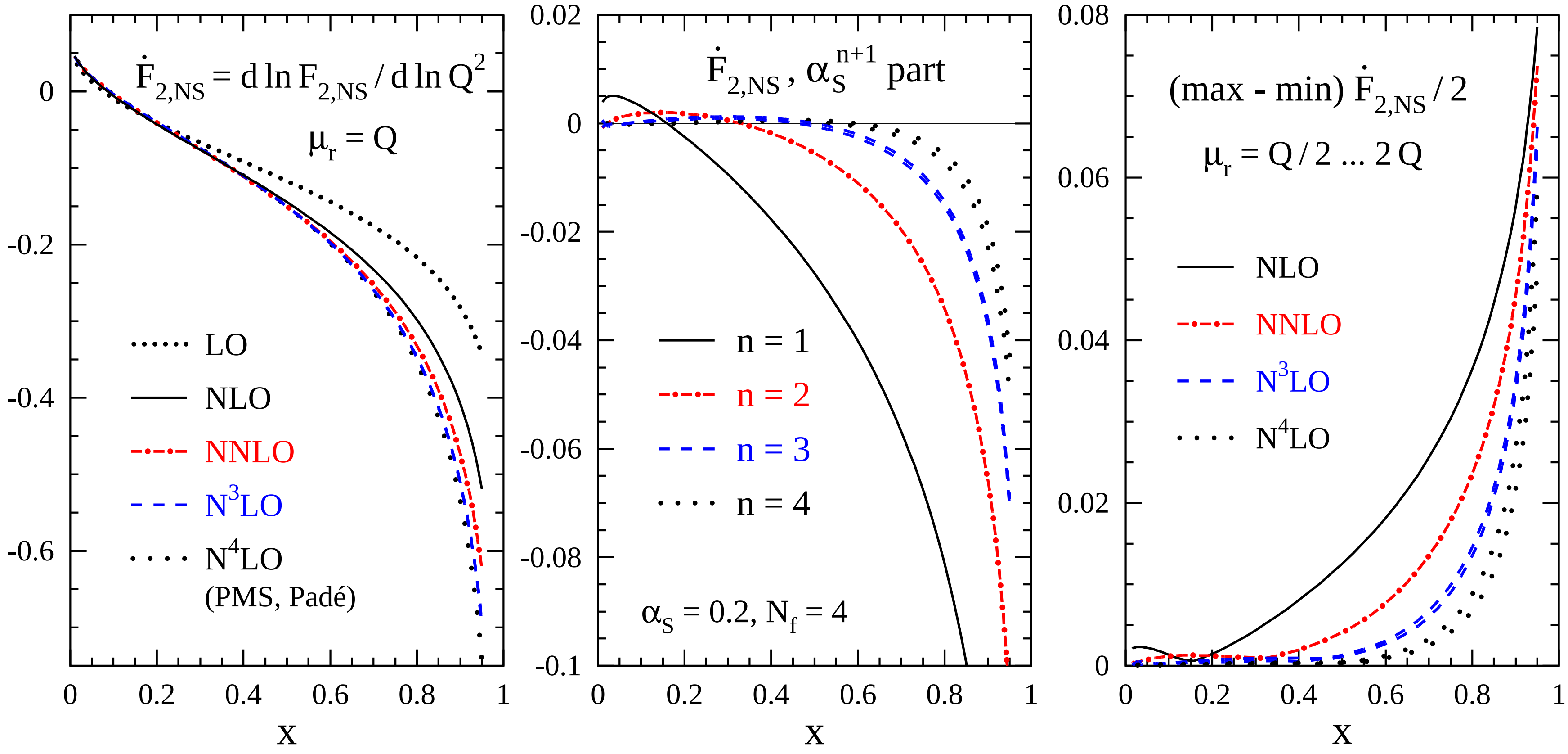

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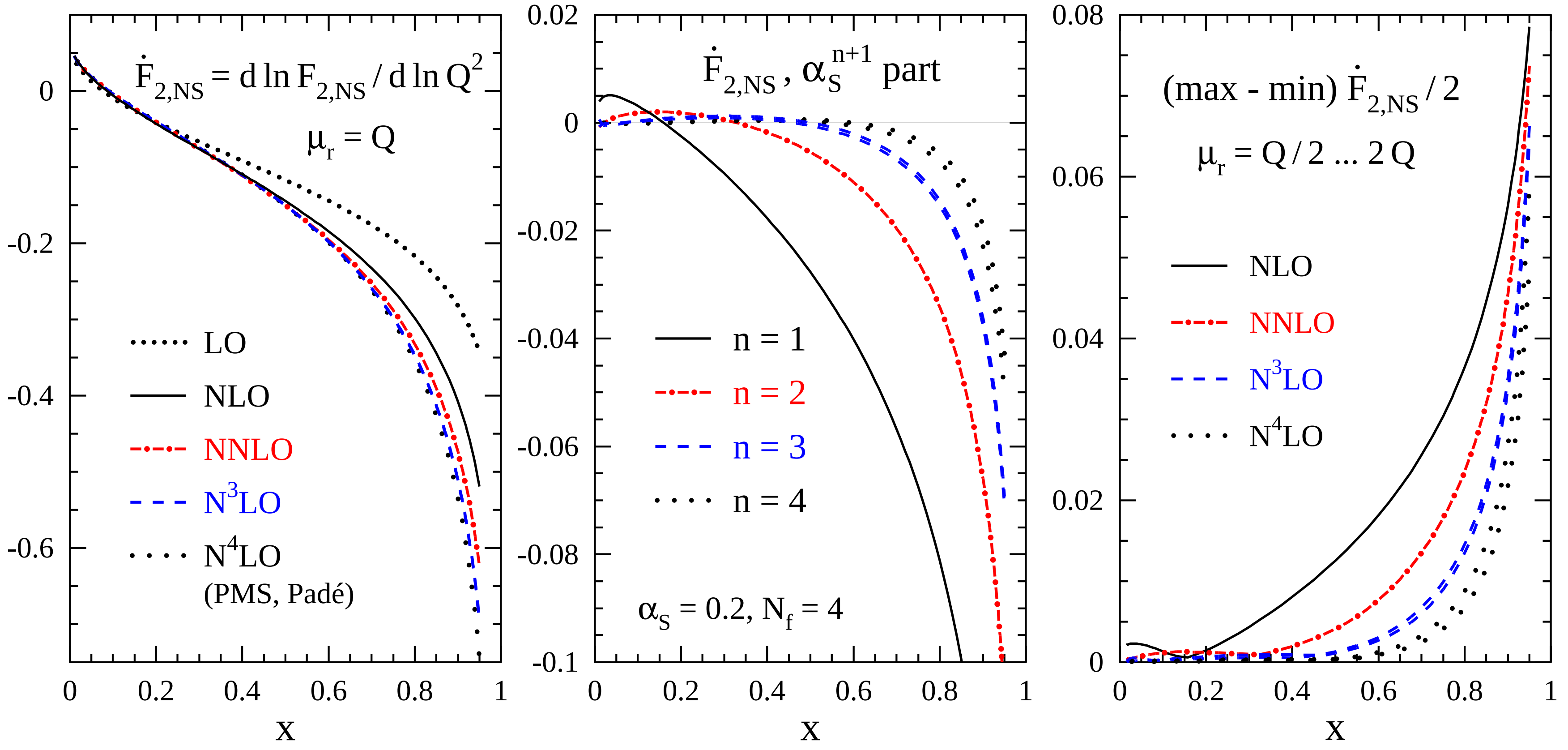

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- At very large x: soft-gluon resummation S.M., Vermaseren, Vogt '05

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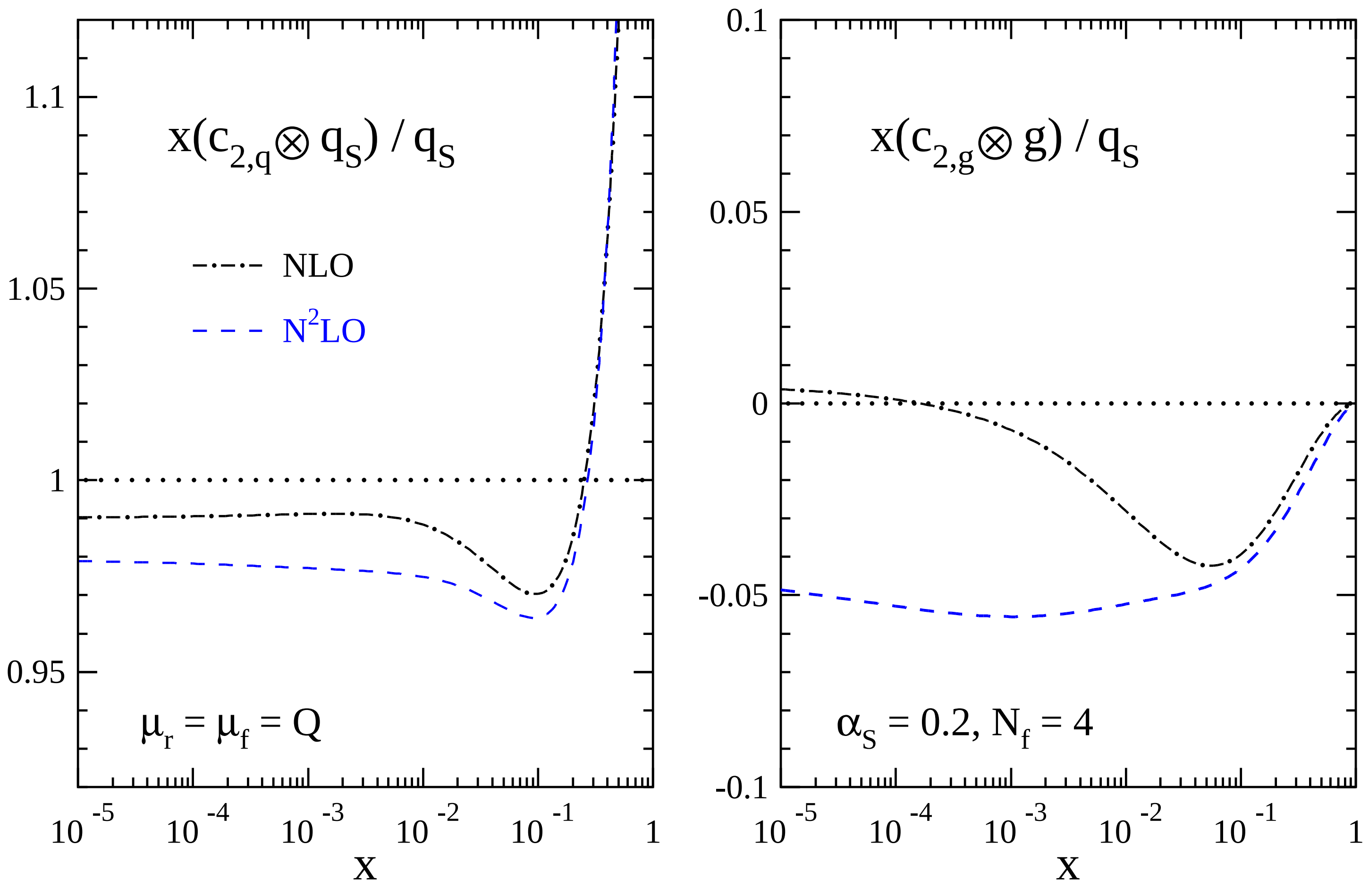

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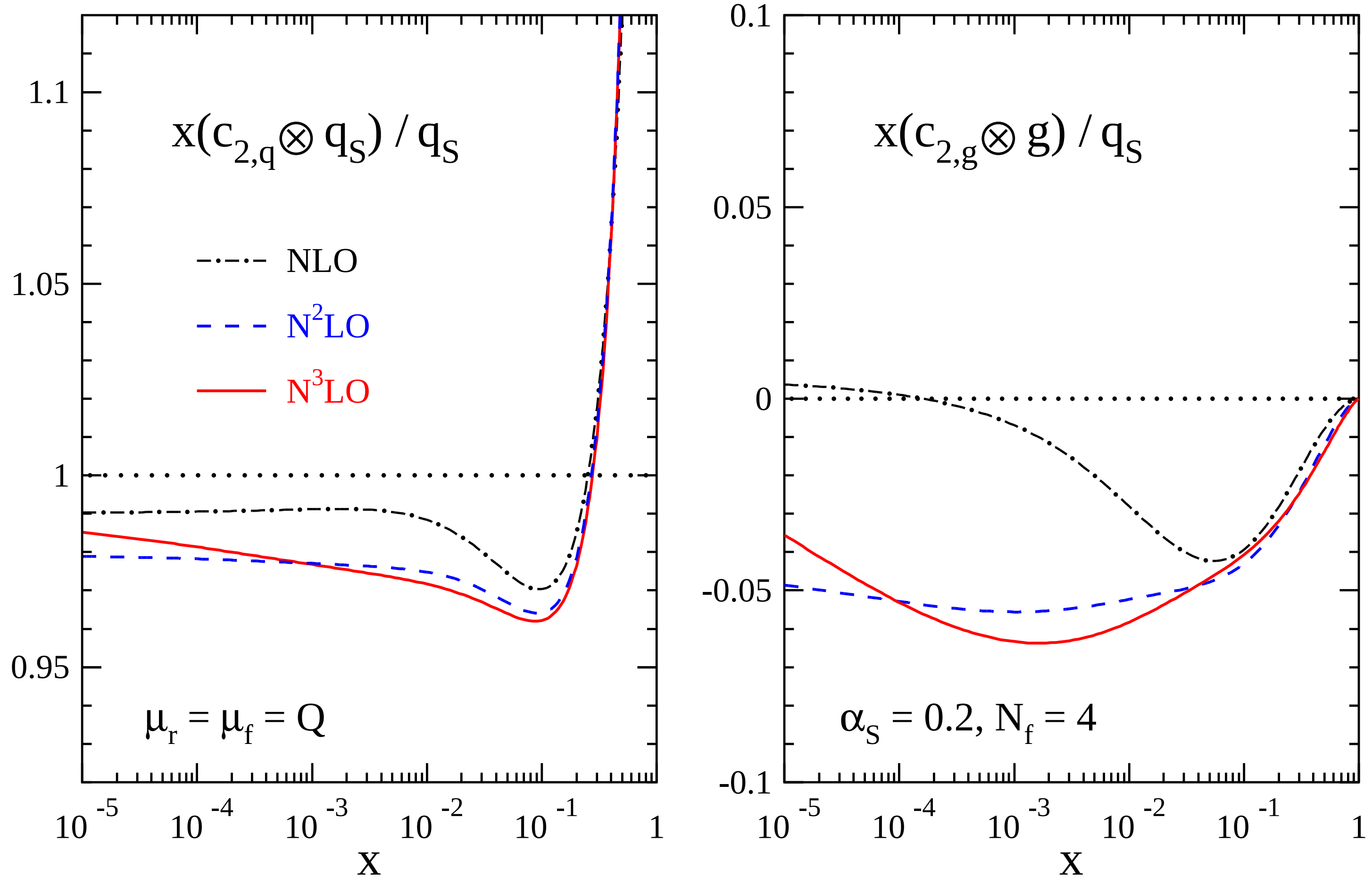
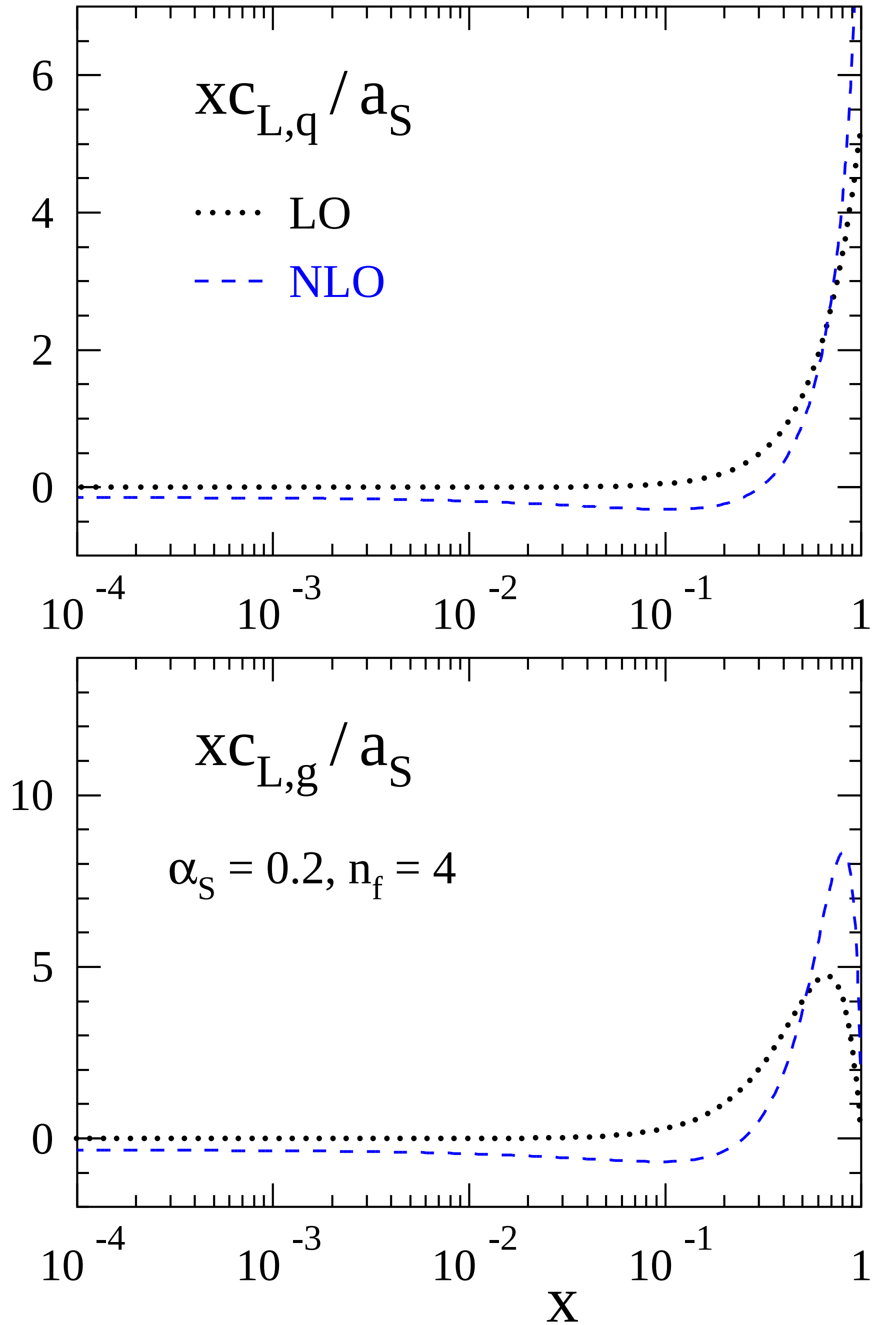


- Perturbative expansion to N^{3}LO of the quark and gluon contribution
- Perturbative stability of F_2

Three-loop coefficient functions for F_L



- Perturbative expansion of singlet-quark and gluon coefficient functions $c_{L,\mathrm{q}}$ and $c_{L,\mathrm{g}}$ for F_L with $n_f = 4$ and $\alpha_s(\mu^2) = 0.2$ (results divided by $a_{\mathrm{s}} = \alpha_s/(4\pi)$)
- LO and NLO contributions remarkably small

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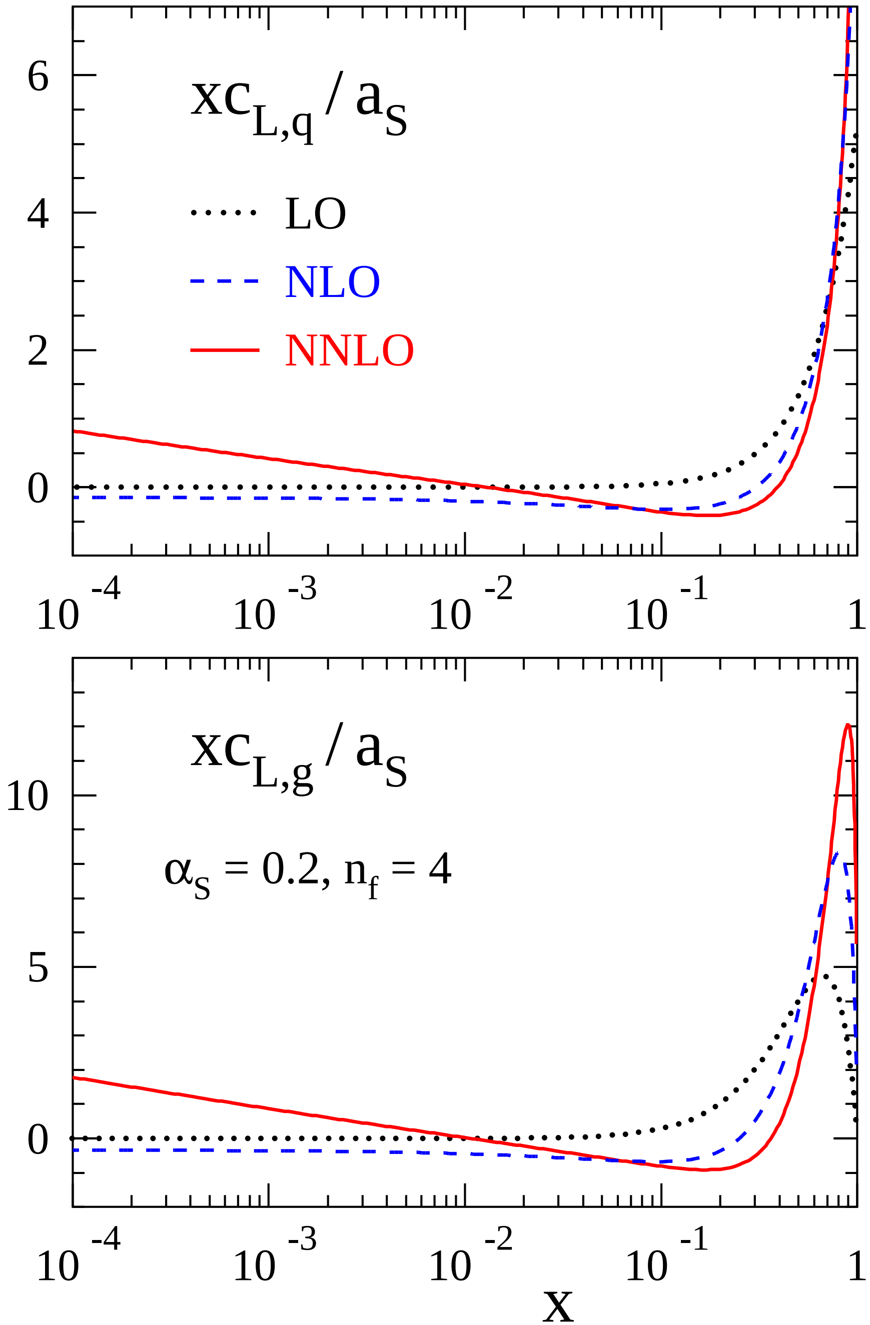

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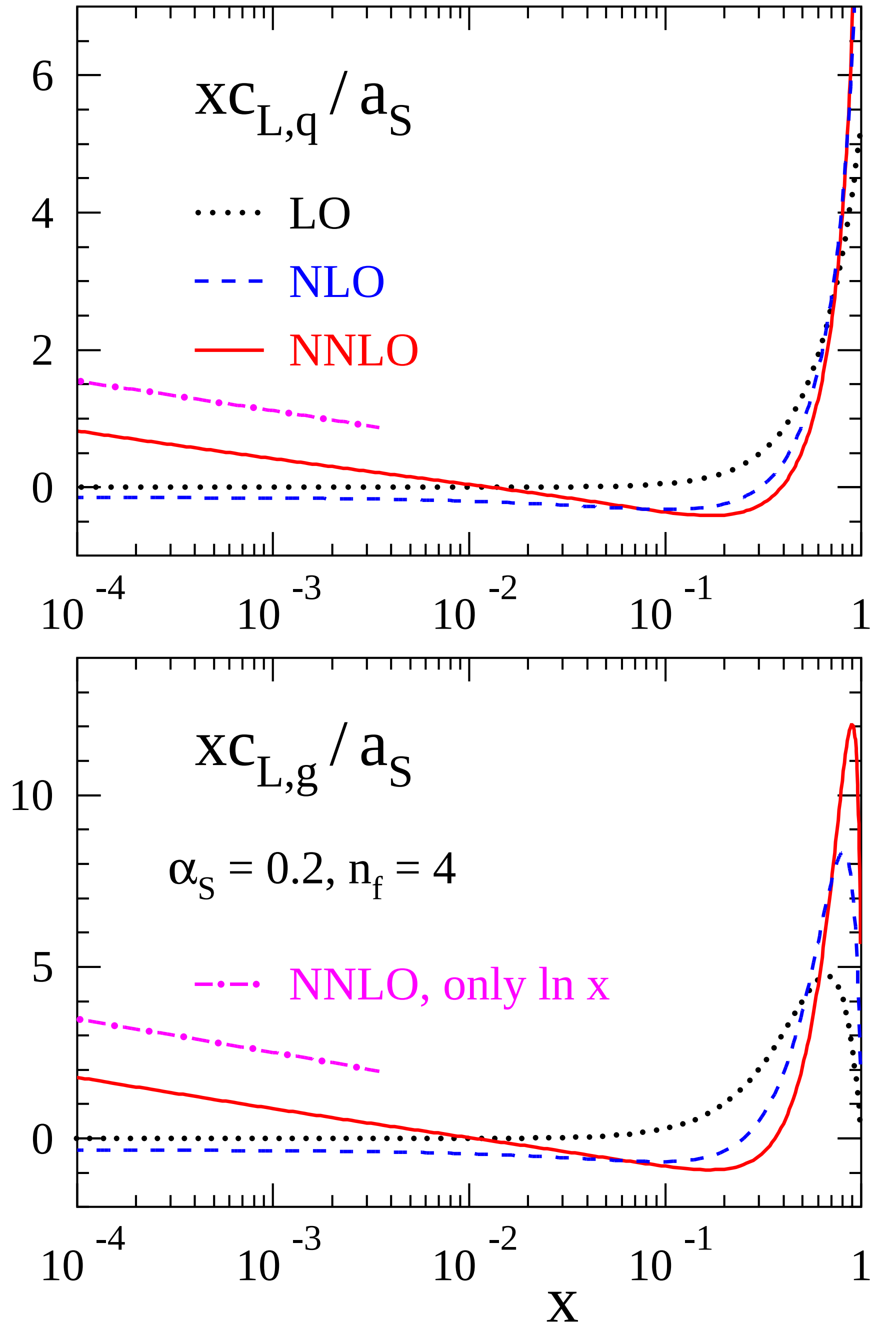

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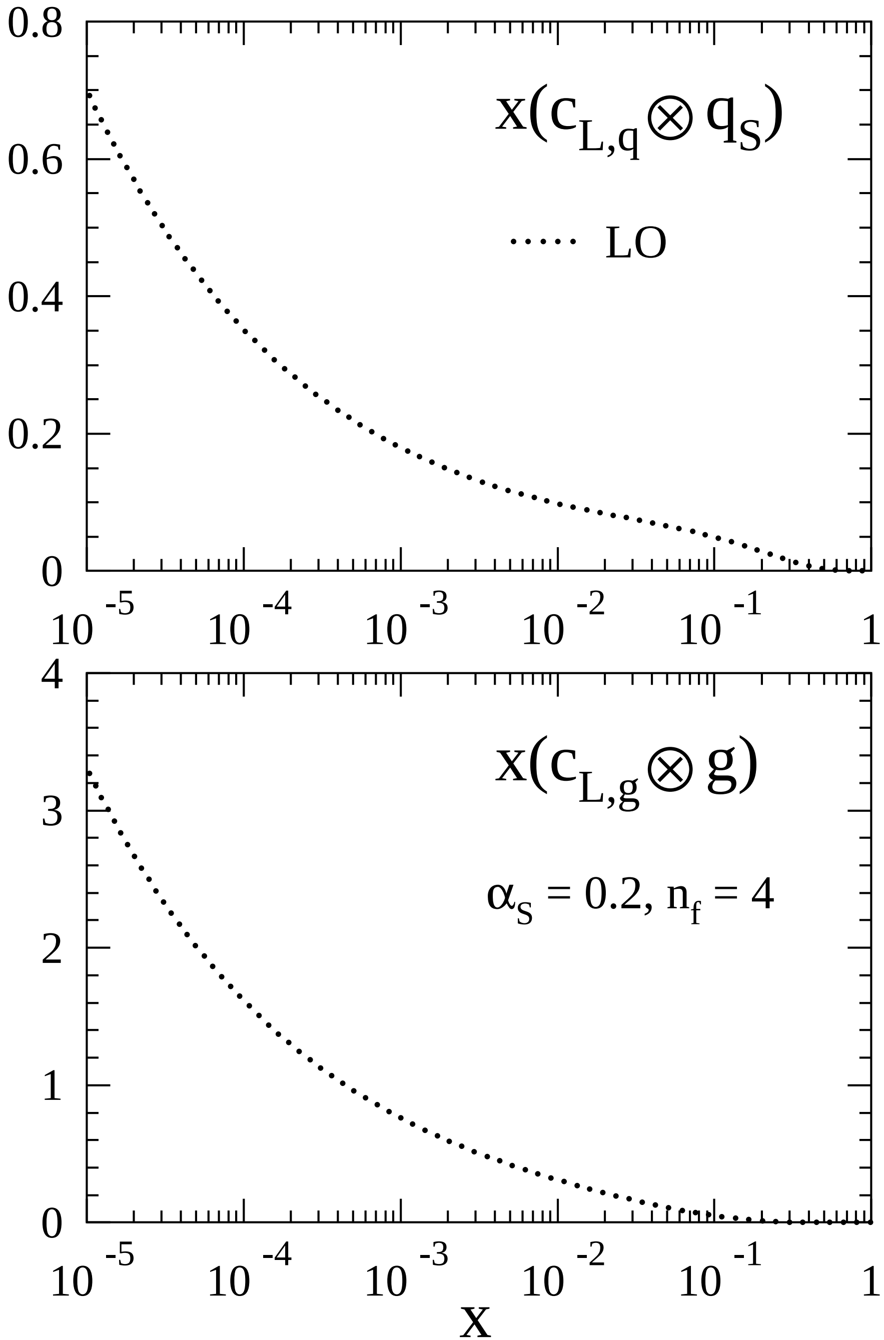
- Leading small-x term at NNLO results $xc^{(3)}_{L,a} \sim \ln x$

The longitudinal structure function



- Perturbative expansion of singlet-quark and gluon contributions to F_L for $n_f = 4$ and $\alpha_s(\mu^2) = 0.2$ (results divided by $\langle e^2 \rangle$)
- Parametrization of singlet distributions (order independent)

$$xq_{\rm s}(x, \mu_0^2) =$$
$$0.6\, x^{-0.3} (1-x)^{3.5}\, (1 + 5.0\, x^{0.8})$$
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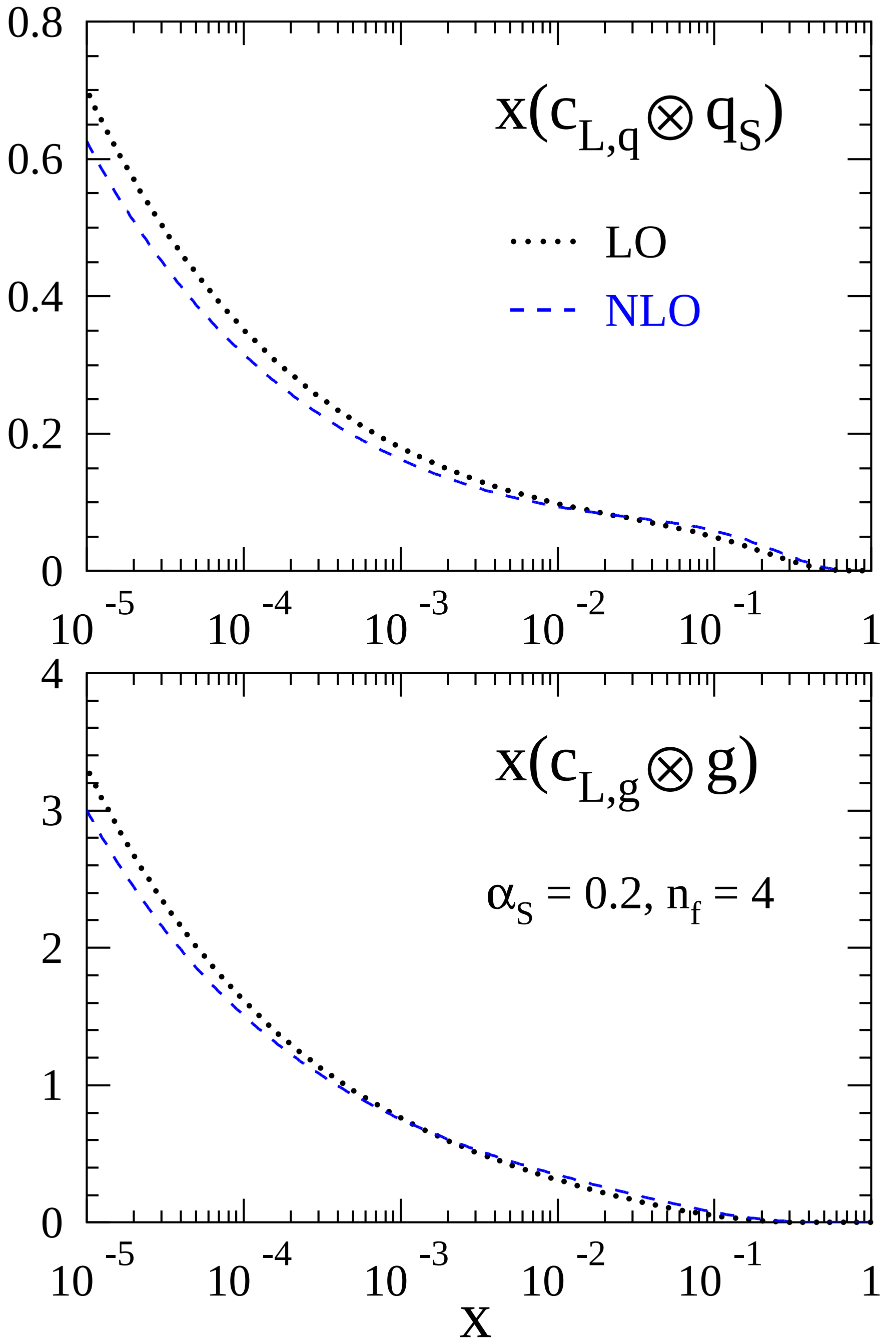

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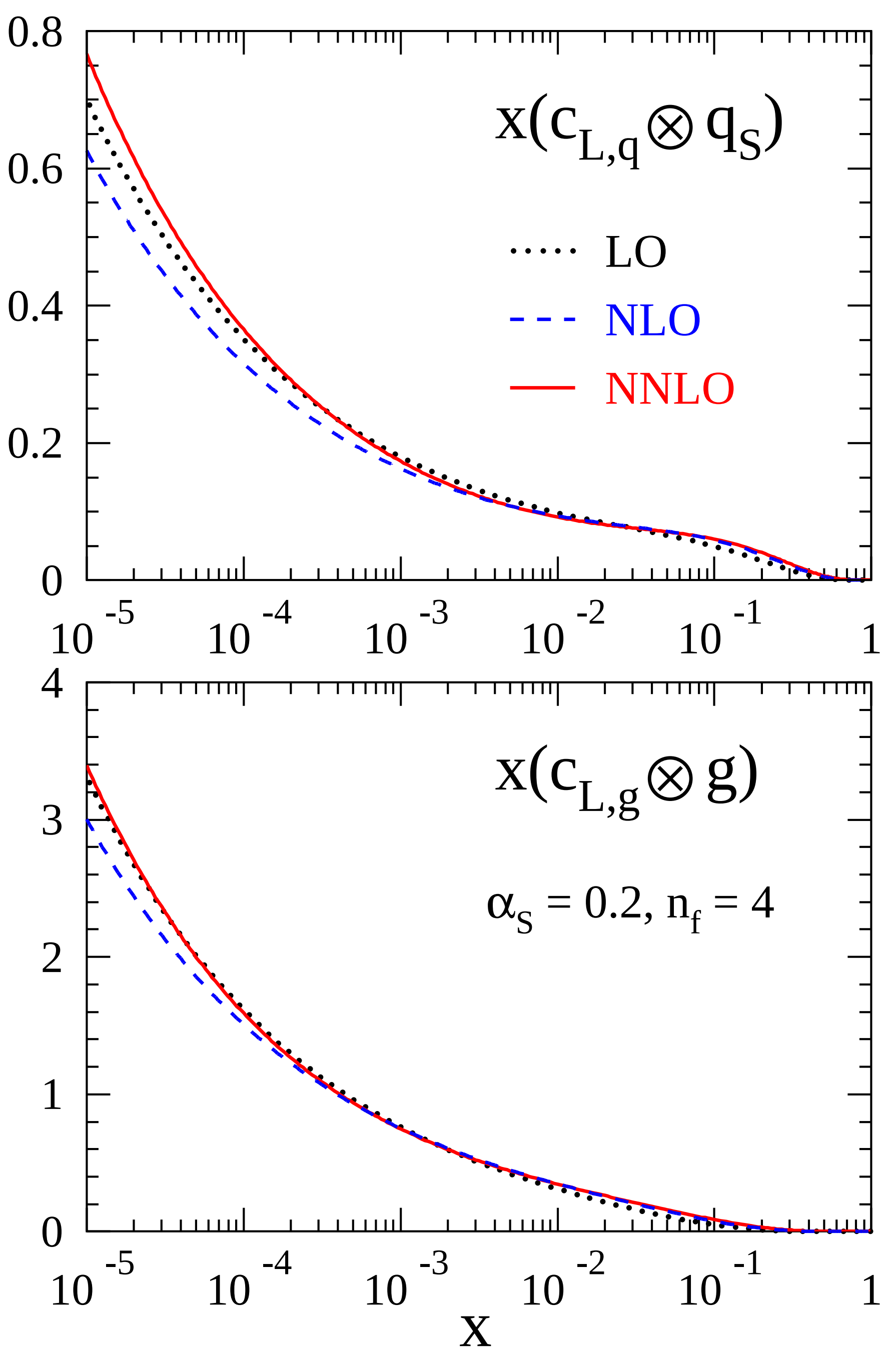

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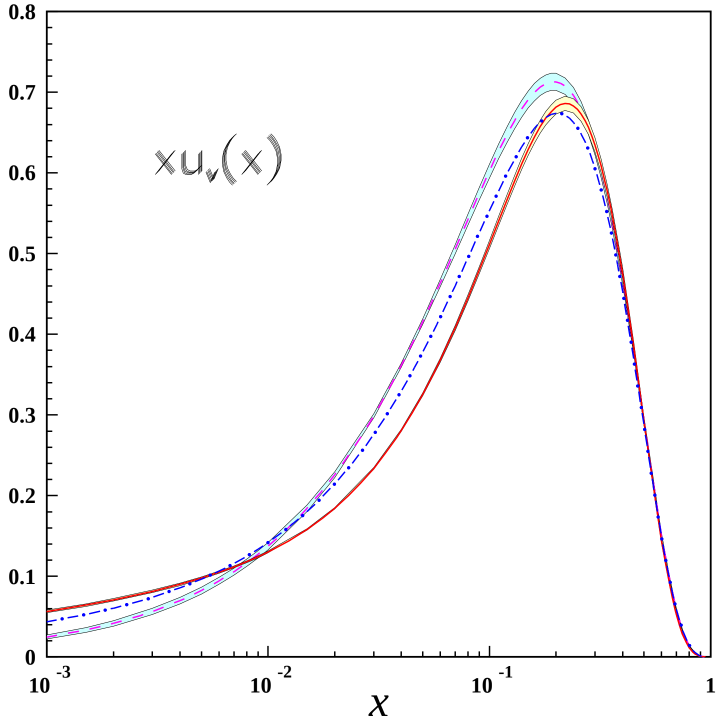
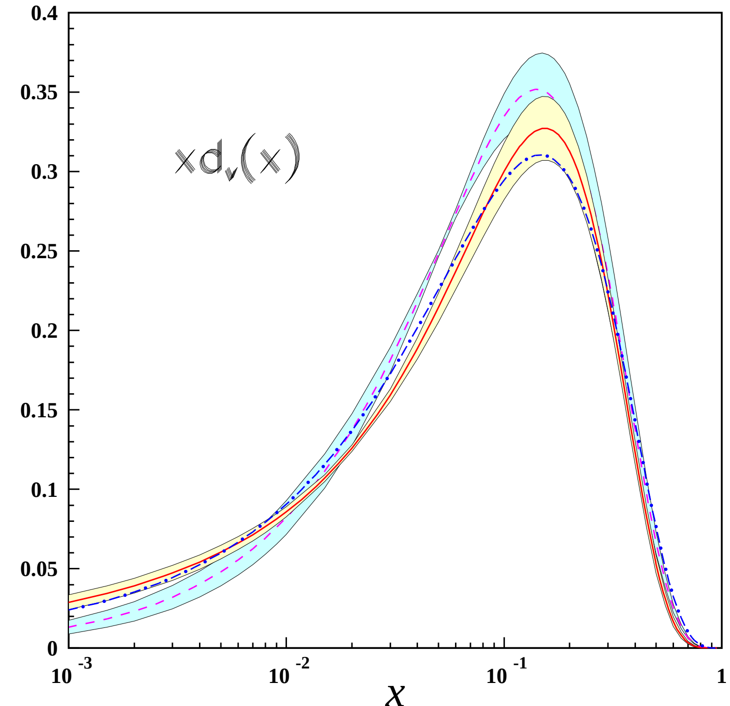
NNLO evolution and data analyses





- Statistical error bands for xu_v and xd_v at 1σ at $Q_0^2 = 4.0 \text{ GeV}^2$

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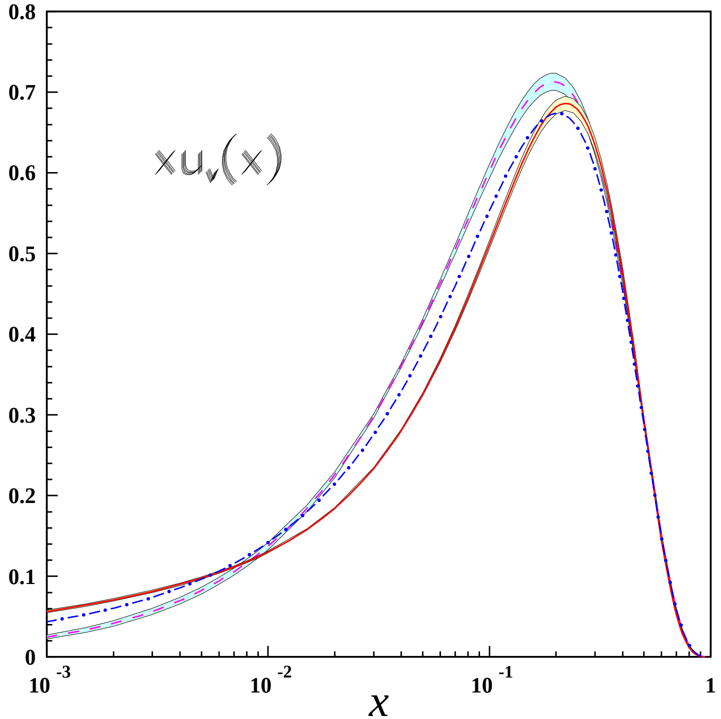

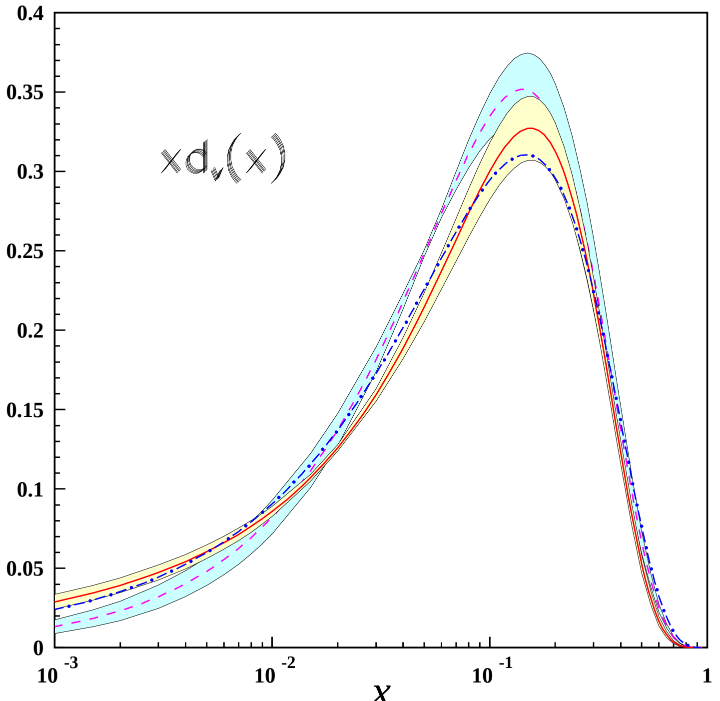

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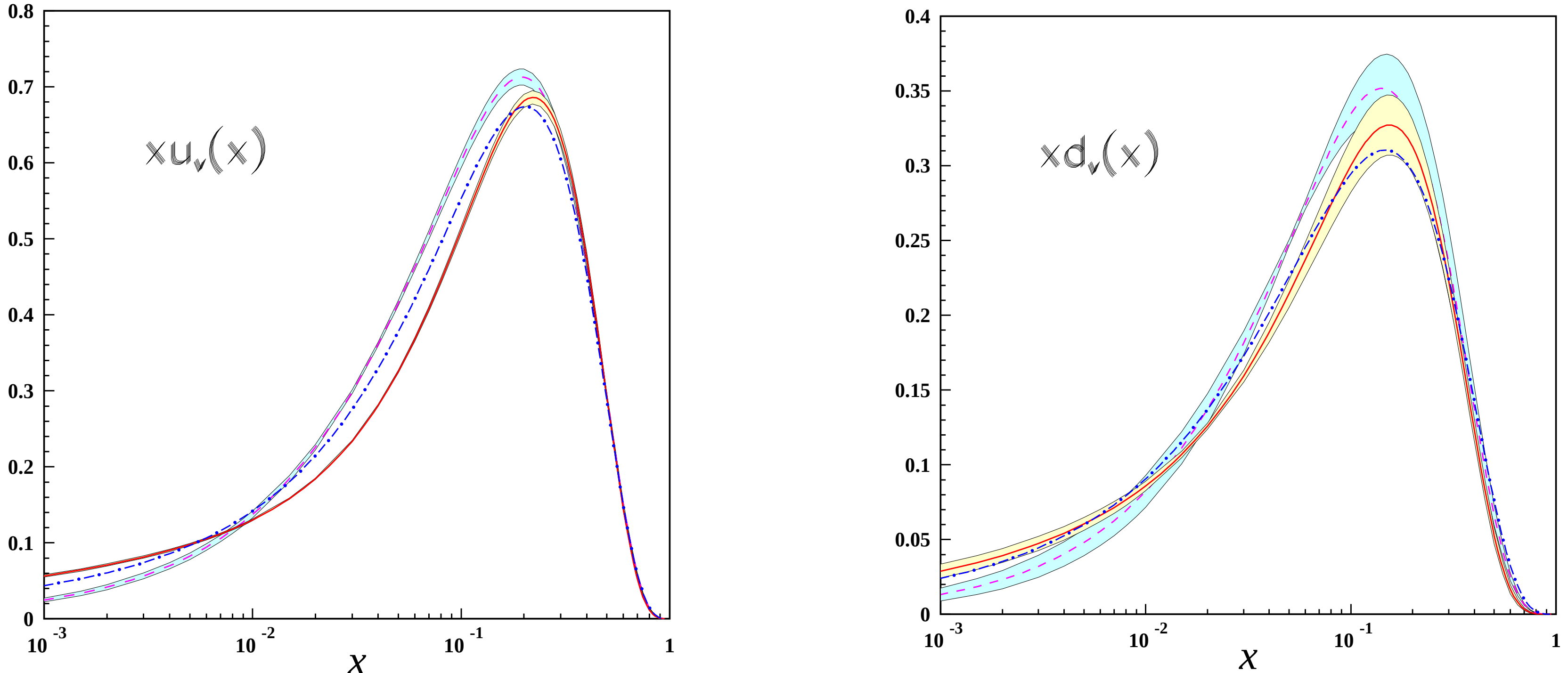

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- Further topics
  CERN/DESY Workshop HERA and the LHC '04-'05; `www.desy.de/heralhc`
  - precision parton distributions
  - impact on LHC parton luminosity
  - ...

Applications

$N = 4$ Super Yang-Mills theory

- Maximally supersymmetric Yang-Mills theory in four dimensions (MYSM)
  - renewed interest from AdS/CFT and from twistor space methods
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- MSYM identification for colour coefficients $C_A = C_F = n_c$
  - n_f-terms do not contribute at highest transcendentality

Anomalous dimensions of MYSM

- Universal anomalous dimension in MYSM to three loops
  Kotikov, Lipatov, Onishchenko, Velizhanin ‘04
  - at loops l-loops harmonic sums of weight $w = 2l - 1$

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is

$$\gamma(j) \equiv \gamma_{uni}(j) \; = \; \hat{a}\gamma_{uni}^{(0)}(j) + \hat{a}^2\gamma_{uni}^{(1)}(j) + \hat{a}^3\gamma_{uni}^{(2)}(j) + ..., \qquad \hat{a} = \frac{\alpha N_c}{4\pi}\,, \tag{9}$$

where

$$\frac{1}{4}\,\gamma_{uni}^{(0)}(j+2) \; = \; -S_1, \tag{10}$$

$$\frac{1}{8}\,\gamma_{uni}^{(1)}(j+2) \; = \; \Big(S_3 + \overline{S}_{-3}\Big) - 2\,\overline{S}_{-2,1} + 2\,S_1\Big(S_2 + \overline{S}_{-2}\Big), \tag{11}$$

$$\begin{aligned} \frac{1}{32}\,\gamma_{uni}^{(2)}(j+2) \; = \; & 2\,\overline{S}_{-3}\,S_2 - S_5 - 2\,\overline{S}_{-2}\,S_3 - 3\,\overline{S}_{-5} + 24\,\overline{S}_{-2,1,1,1} \\ & +6\Big(\overline{S}_{-4,1} + \overline{S}_{-3,2} + \overline{S}_{-2,3}\Big) - 12\Big(\overline{S}_{-3,1,1} + \overline{S}_{-2,1,2} + \overline{S}_{-2,2,1}\Big) \\ & -\Big(S_2 + 2\,S_1^2\Big)\Big(3\,\overline{S}_{-3} + S_3 - 2\,\overline{S}_{-2,1}\Big) - S_1\Big(8\,\overline{S}_{-4} + \overline{S}_{-2}^2 \\ & +4\,S_2\,\overline{S}_{-2} + 2\,S_2^2 + 3\,S_4 - 12\,\overline{S}_{-3,1} - 10\,\overline{S}_{-2,2} + 16\,\overline{S}_{-2,1,1}\Big) \end{aligned} \tag{12}$$

and $S_a \equiv S_a(j),\; S_{a,b} \equiv S_{a,b}(j),\; S_{a,b,c} \equiv S_{a,b,c}(j)$ are harmonic sums

$j \quad 1 \qquad\qquad j \quad 1$

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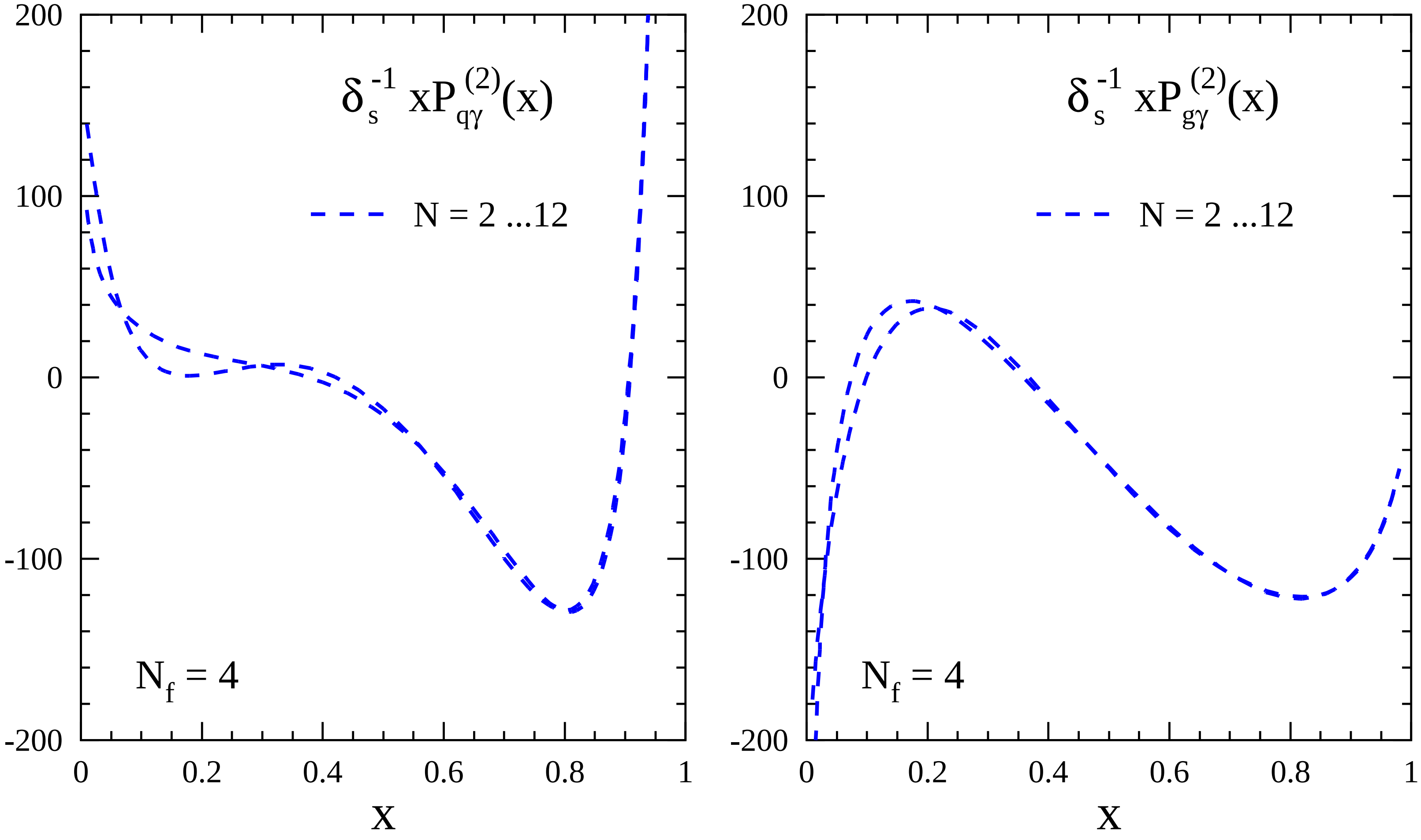
- Result agrees with predictions based on integrability for planar three-loop contribution to dilatation operator
  Beisert, Kristjansen, Staudacher ‘03
- Additional check up to spin $N = 70$ from a Bethe ansatz for S-matrix of spin chain Staudacher ‘04

Order $\alpha\alpha_s^2$ photon-parton splitting functions

S.M., Vermaseren, Vogt unpublished



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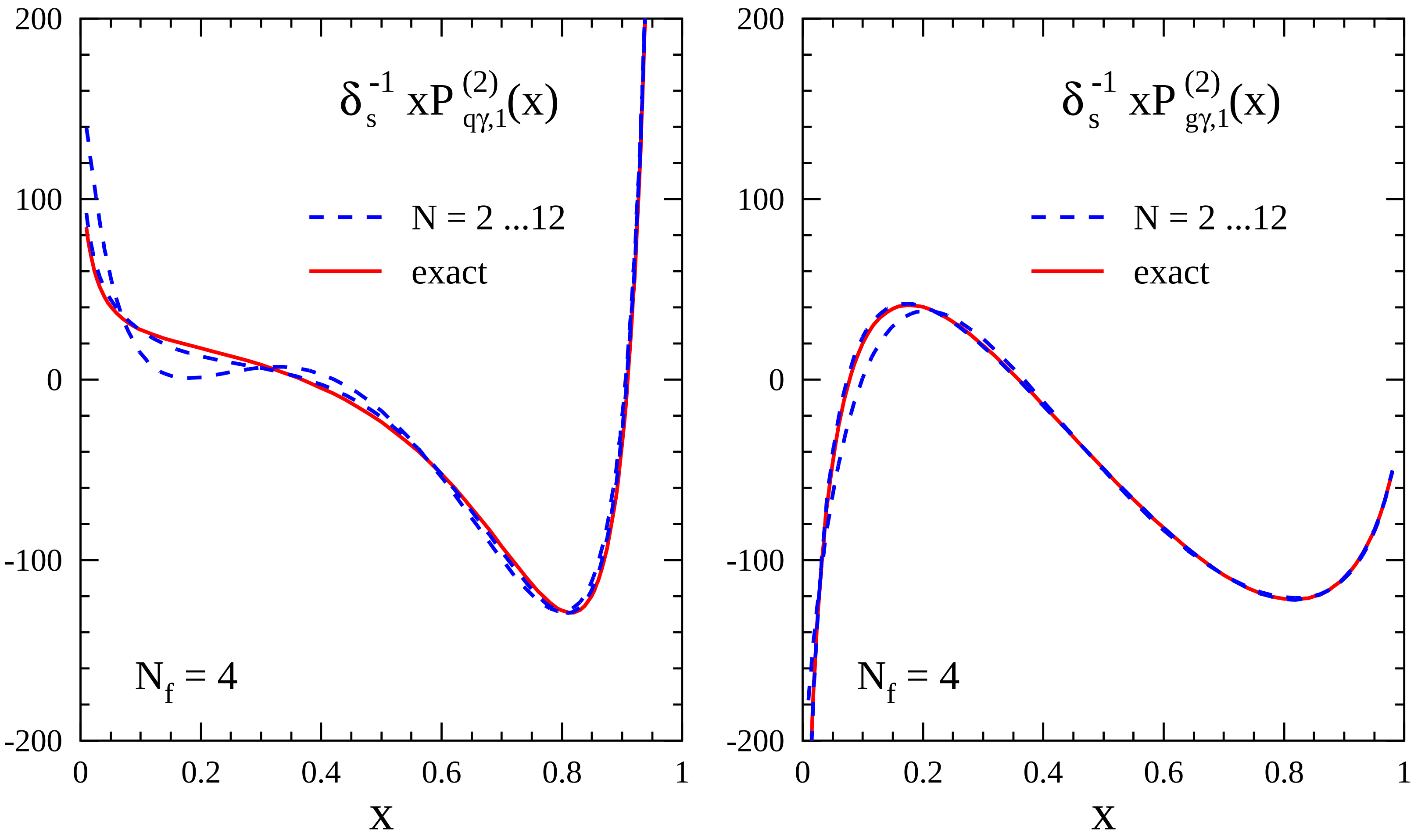

- Large x: approximation based on fixed-N results sufficiently reliable

Photon-quark splitting functions up to NNLO

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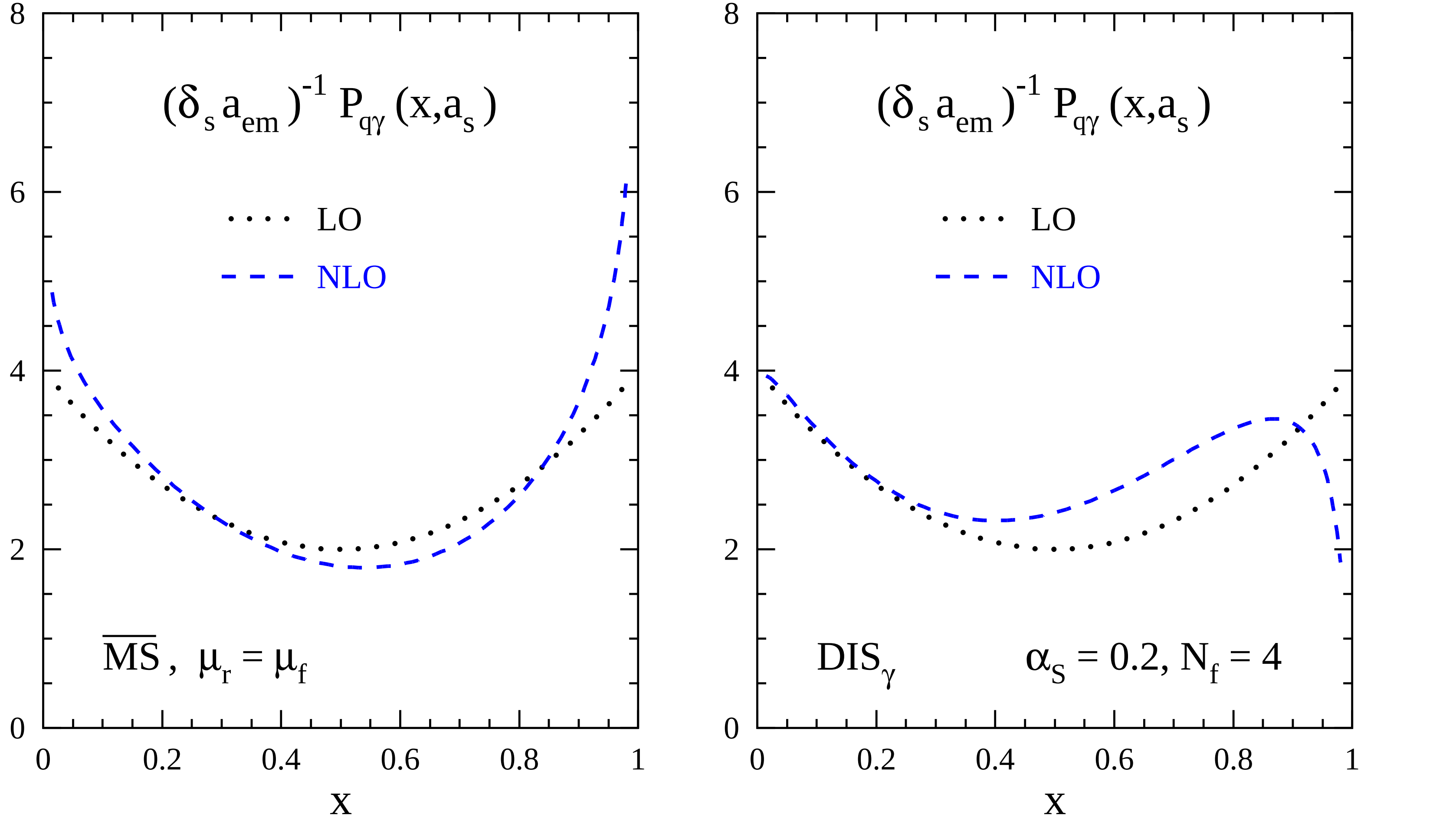

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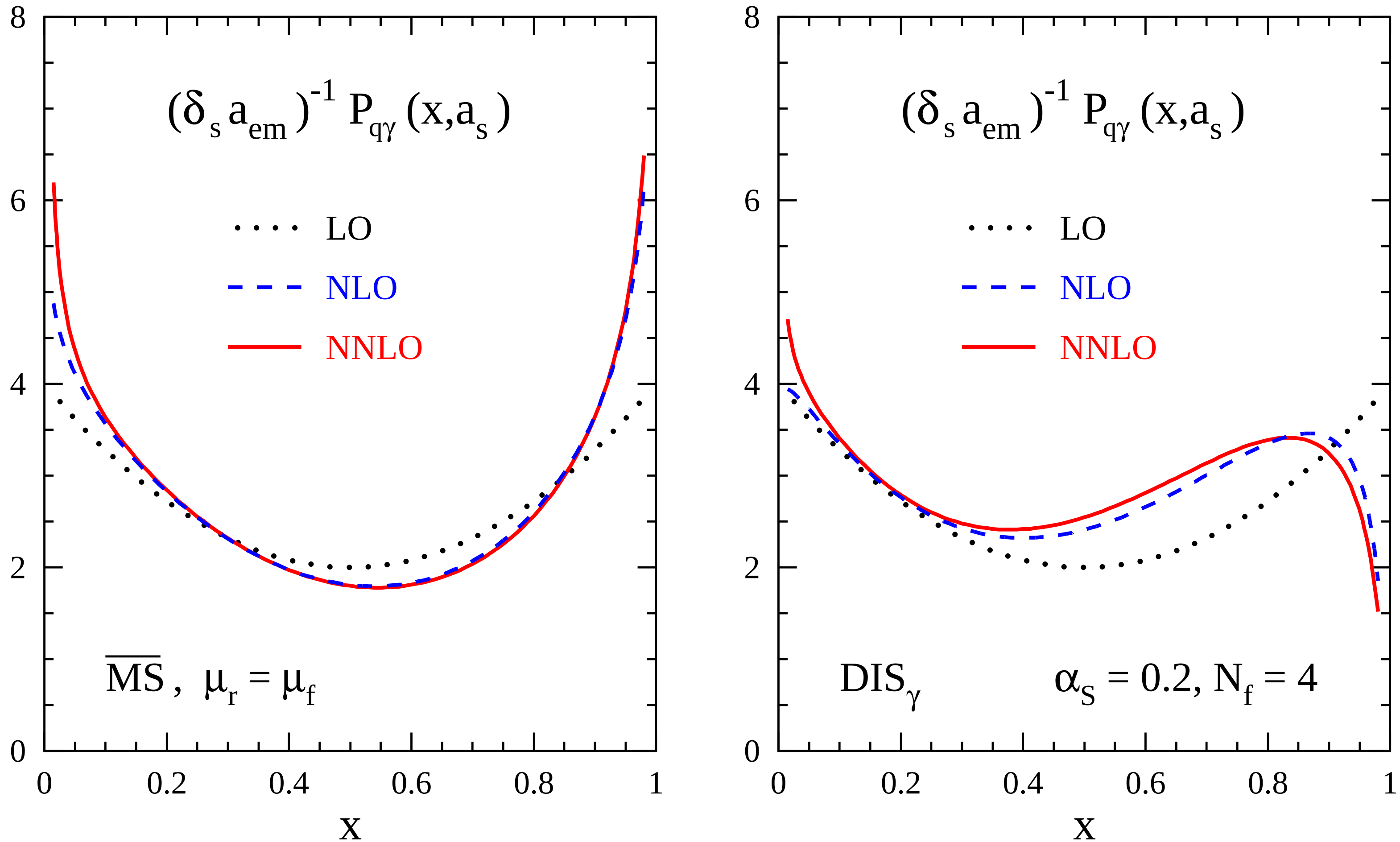

- Large x: corrections small in both $\overline{\mathrm{MS}}$ and DIS_γ factorization

Higgs production in soft limit

- N^3LO results for Higgs production in gluon-gluon fusion with $x = M^2/s$
  S.M., Vogt '05

$$
\begin{aligned}
c_3^{\text{Higgs}} \quad = \quad & \frac{\ln(1-x)^5}{(1-x)_+}\left\{512 C_A^3\right\} + \cdots + \\
& \frac{1}{(1-x)_+}\left\{C_A^3\left[-\frac{594058}{729} + \frac{137008}{81}\zeta_2 + \frac{143056}{27}\zeta_3 + \frac{4048}{15}\zeta_2^2\right.\right. \\
& -\frac{23200}{3}\zeta_2\zeta_3 + 11904\,\zeta_5\Big] + C_A^2 n_f\Big[\frac{125252}{729} - \frac{34768}{81}\zeta_2 - \frac{7600}{9}\zeta_3 \\
& -\frac{544}{15}\zeta_2^2\Big] + C_A C_F n_f\Big[\frac{3422}{27} - 32\,\zeta_2 - \frac{608}{9}\zeta_3 - \frac{64}{5}\zeta_2^2\Big] \\
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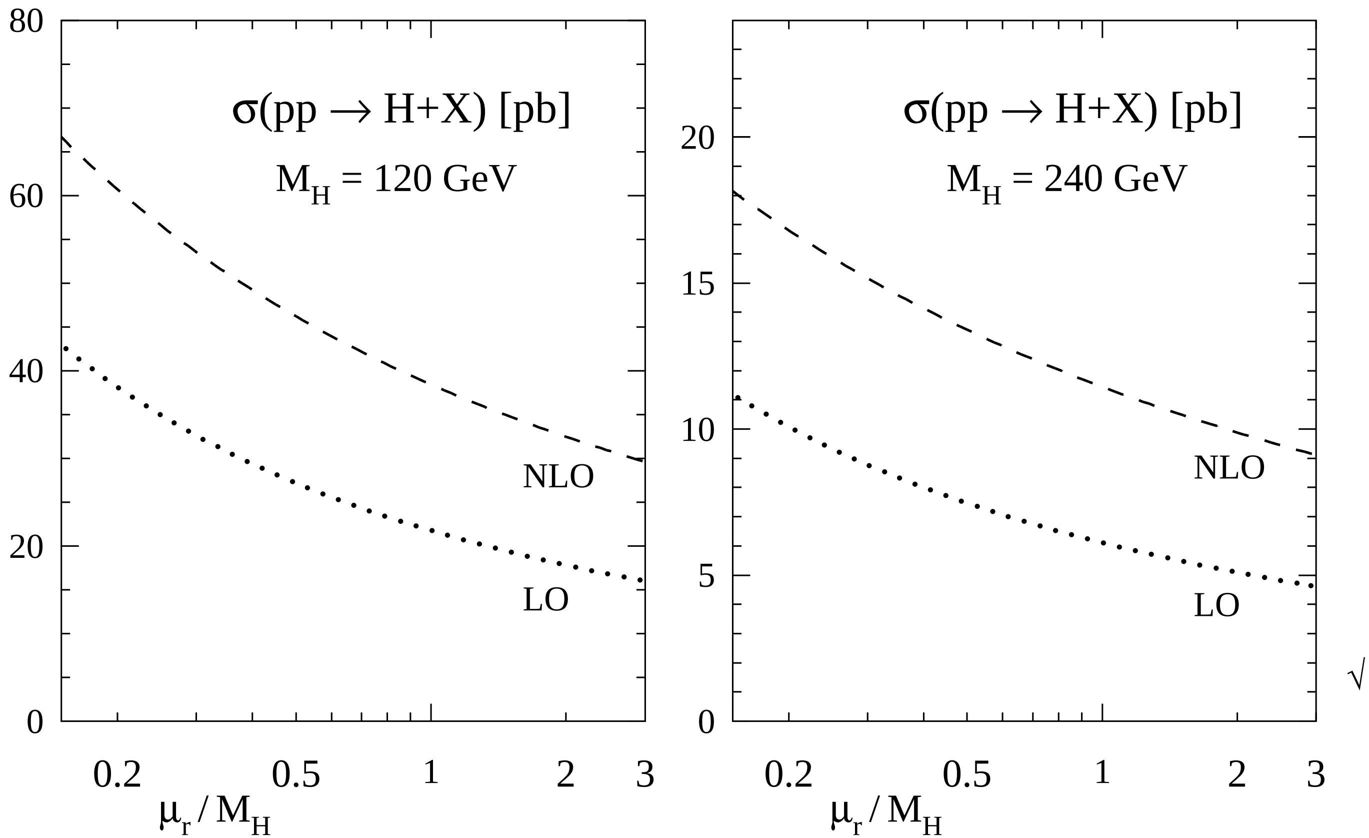
- N^{3}LO results for Drell-Yan process in $q\bar{q}$-annihilation with $x = M^2/s$
  and independent cross check on $\dfrac{1}{(1-x)_+}$-term Laenen, Magnea '05

Cross section Higgs production (cont'd)



- Variation of cross section at LHC with renormalizaion scale for different Higgs masses: $M_H = 120$GeV (left) and $M_H = 240$GeV (right)

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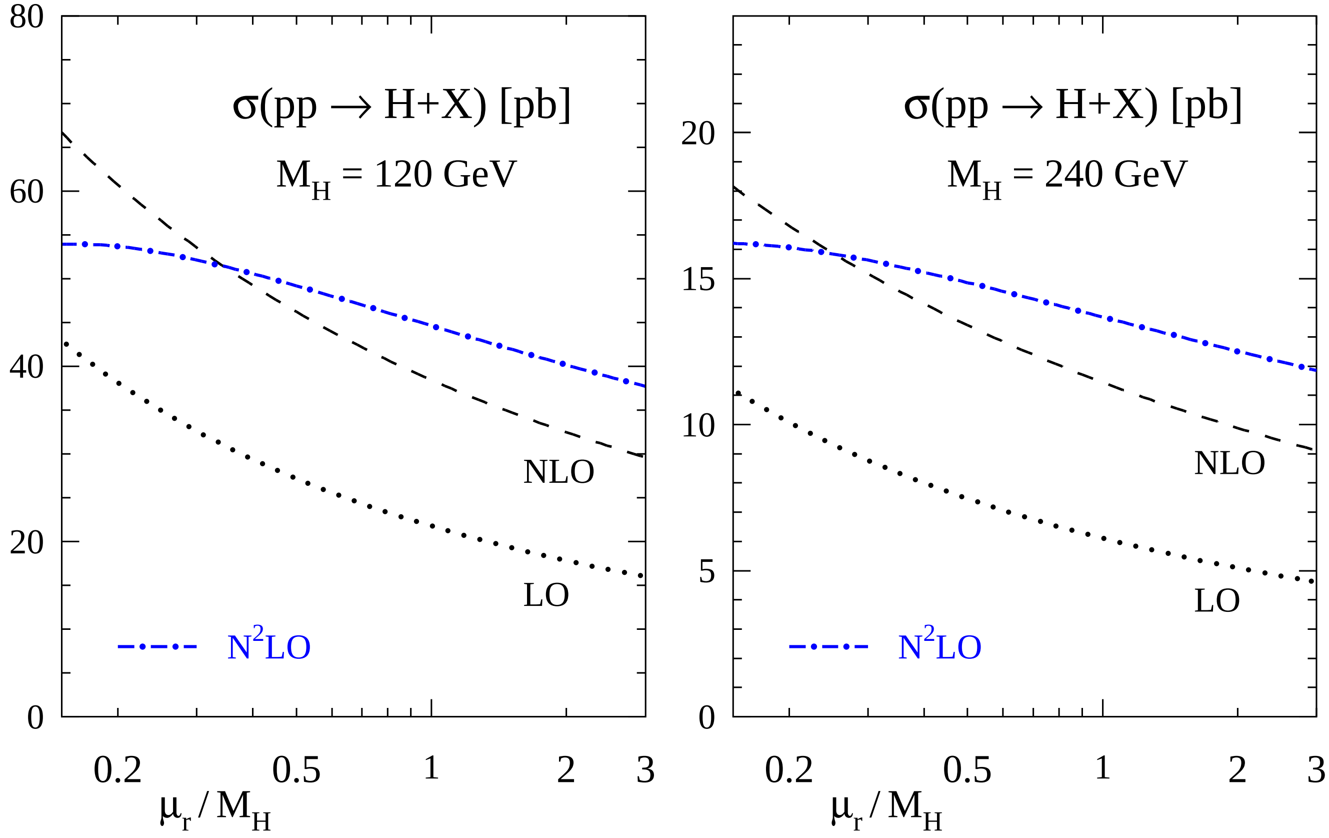

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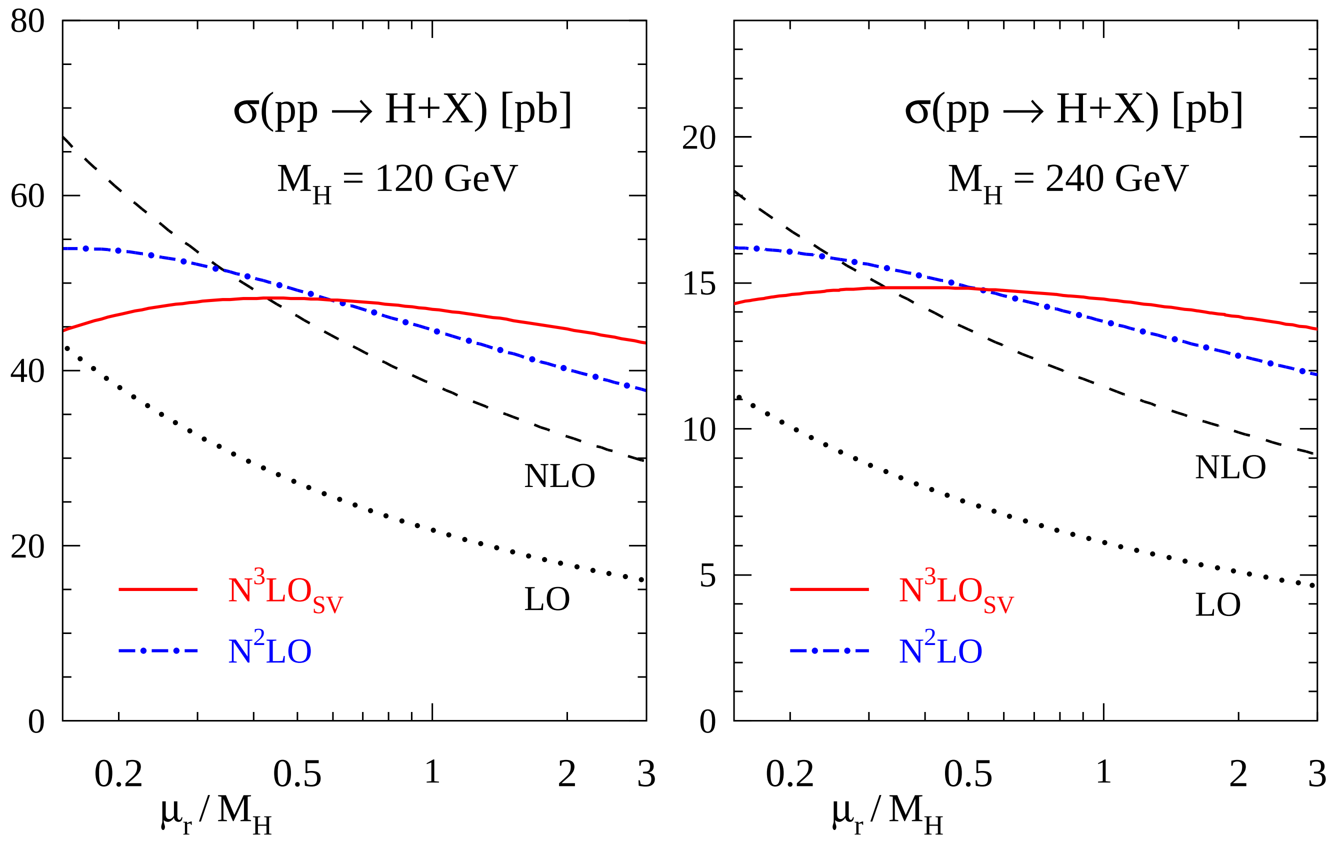

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Summary and Outlook

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  - first (complete) three-loop calculation for single-scale quantities

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