NLO QCD Corrections to Drell-Yan in TeV-scale Gravity Models

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- Graviton mediated Drell-Yan
- QCD Factorisation scale ambiguity
- NLO corrections to new physics
- Conclusions

In collaboration with

Willy van Neerven, Prakash Mathews and K. Sridhar
Perturbative QCD provides a framework to compute infrared safe observables at high energies. They are "often" sensitive to:

1) Renormalisation scale
2) Factorisation scale
3) Missing higher order contributions (stability of perturbation)
4) Non-perturbative quantities that enter, such as parton density functions

Solution to (1) to (3) to compute higher order QCD corrections.

P_1 + P_2 ! l_1 + l is of course one of the most important processes to discover "new physics" at high energy colliders such as TeV scale gravity models (Large Extra-Dimensional theories).

Higher order QCD corrections increases the reliability of the predictions of the theory.
Snap shot of my talk

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Large Extra Dimensions

Models of "Extra Dimensions" are now studied as serious contenders for "Physics Beyond SM" (BSM). They provide an alternate view of the "hierarchy" between the EW (\(~ 1 \text{ TeV}\)) and the Planck scale (\(10^{16} \text{ TeV}\)).
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Kaluza-Klein Picture
Kaluza-Klein Modes

- Extra dimensions being compact, gravitational field will be periodic function in the extra dimension.
- In 4-dim it would correspond to nearly mass degenerate tower of KK modes $m_{ \bar{n}}^2 \sim \bar{n}^2 / R^2$
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Massless graviton and KK modes couple with SM fields with coupling $\frac{1}{M_{Pl}} \sim \frac{R^2}{d}$
Gravity-QCD Coupling

Gravitational interaction with SM fields:

\[ S = S_{SM} - \frac{\kappa}{2} \int d^n x \, T_{\mu\nu}(x) \, G^{\mu\nu}(x) \]

strength of interaction \( \kappa \sim \sqrt{G_N} \sim M_P^{-1} \)

Energy momentum tensor:

\[ T^{QCD}_{\mu\nu} = -g_{\mu\nu} \, L_{QCD} - F^a_{\mu\rho} F^{a\rho}_{\nu} - g_{\mu\nu} \, \frac{1}{\xi} \, \partial^\rho \left( A_\rho \partial^\sigma A_\sigma \right) \]

\[ + \left[ \left( \frac{i}{4} \overline{\psi} \left[ \gamma_\mu \left( \overrightarrow{\partial} \nu - ig T^a A^a_\nu \right) - \gamma_\mu \left( \overleftarrow{\partial} \nu + ig T^a A^a_\nu \right) \right] \psi \right. \]

\[ + \left. \frac{1}{\xi} A_\nu \partial_\mu (\partial^\sigma A^a_\sigma) + \partial_\mu \bar{\omega} \left( \partial_\nu \omega^a - gf^{abc} A^c_\nu \omega^b \right) \right] + (\mu \leftrightarrow \nu) \]

\[ \begin{align*}
A^a_\mu & \quad \text{Gauge fields} \\
\psi & \quad \text{Fermionic fields} \\
\omega^a & \quad \text{Ghost fields} \\
G_{\mu\nu} & \quad \text{Graviton Fields}
\end{align*} \]

Gravitons couple to anything and everything
Feynman Rules

- QED
  \[ e^- \rightarrow G_{\mu\nu} \quad e^+ \rightarrow \gamma \]
  \[ e^- \rightarrow \gamma \quad e^+ \rightarrow G_{\mu\nu} \]

- QCD
  \[ q \rightarrow G_{\mu\nu} \quad g \rightarrow G_{\mu\nu} \]
  \[ \bar{q} \rightarrow G_{\mu\nu} \quad g \rightarrow G_{\mu\nu} \]
  \[ g \rightarrow G_{\mu\nu} \quad g \rightarrow G_{\mu\nu} \]

Kaluza-Klein suppression in ADD

$$\kappa \approx 10^{-16}$$
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\[ \kappa \approx 10^{-16} \]

- Summation of KK modes:

\[ \sum_n \frac{1}{Q^2 - m_n^2 + i\epsilon} = \frac{16\pi}{\kappa^2} \left( \frac{Q^2}{M_S^2} \right)^{d-2} \left( \frac{1}{M_S^4} \right) \int \left( \frac{M_S}{Q} \right) \]

Kaluza-Klein suppression in ADD

\[ q \xrightarrow{\sum_n G^{n}} l^- \]
\[ \bar{q} \xrightarrow{l^+} \]

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- (Coupling)\(^2\) (Sum over the KK mode) leads to

\[ (\kappa)^2 \sum_n \frac{1}{Q^2 - m_{\tilde{n}}^2 + i\epsilon} \rightarrow 16\pi \left( \frac{Q^2}{M_S^2} \right)^{\frac{d-2}{2}} \frac{1}{M_S^4} I \left( \frac{M_S}{Q} \right) \]
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\[ \frac{1}{M_S^{2+d}} \approx \frac{1}{(\text{TeV})^{2+d}} \]

Planck suppression is compensated by High multiplicity of KK modes.
In the Standard Model, the partonic cross sections decreases with the energy scale ($Q$ or $p_T$ involved):

$$\hat{s} \frac{d}{dQ^2} \hat{\sigma}_{ab}^{SM}(\hat{s}, Q^2) \sim \frac{1}{Q^2}$$

In the Gravity mediated processes, the partonic cross sections increase monotonically with the energy scale involved:

$$\hat{s} \frac{d}{dQ^2} \hat{\sigma}_{ab}^{\text{Gravity}}(\hat{s}, Q^2)$$

Gravity mediated cross sections can show up at high $Q$.

The processes where the virtual/real KK gravitons contribute significantly:

1. Di-lepton or Drell-Yan production at large invariant mass $Q$
2. Di-photon or Di-boson production at large $Q, p_T$
3. Observables with missing energy

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Phenomenology with Extra-Dimension
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Drell-Yan Process

\[ P_1(p_1) + P_2(p_2) \rightarrow [\gamma, Z, G] + \text{hadronic states}(X) \]
\[ \leftrightarrow l^+(k_1) + l^-(k_2) \quad (k_1 + k_2)^2 = Q^2 \]
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### Contributing Subprocess

#### Leading Order:

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#### Born contributions
QCD improved Parton Model

\[ P_1 + P_2 \rightarrow l^+ l^- + X \quad m_h^2 = (l^+ + l^-)^2 \]
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- Non-perturbative in nature and process independent.
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- \( \hat{\sigma}_{ab} \) are the partonic cross sections.
- Perturbatively calculable.
Factorisation Theorem (Parton Model)

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

\[ 2S \, d\sigma^{P_1P_2} (\tau, m_h^2) = \sum_{ab} \int_{\tau}^{1} \frac{dx}{x} \Phi_{ab} (x, \mu_F) \, 2\hat{s} \, d\hat{\sigma}^{ab} \left( \frac{\tau}{x}, m_h^2, \mu_F \right) \]
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d\hat{\sigma}^{ab} (z, m_h^2, \mu_F) = \sum_{i=0}^{\infty} \left( \frac{\alpha_s (\mu_R)}{4\pi} \right)^i d\hat{\sigma}^{ab,(i)} (z, m_h^2, \mu_F, \mu_R)
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- The Renormalisation group invariance:

\[ \frac{d}{d\mu} \sigma^{P_1 P_2} (\tau, m_h^2) = 0, \quad \mu = \mu_F, \mu_R \]
Altarelli-Parisi/Renormalisation Group Equations

Renormalised parton density:

$$f_a(z; F) = \frac{1}{Z_1 x} \int_z^1 \frac{dz}{z} P_{ab}(z; F) f_b(x; F)$$

Altarelli-Parisi Evolution equation:

$$F \frac{d}{dF} f_a(x; F) = Z_1 x \int_z^1 \frac{dz}{z} P_{ab}(z; F) f_b(x z; F)$$

Perturbatively Calculable:

$$P_{ab}(z; F) = s(F) 4 P_{ab}(0) + s(F) 2 P_{ab}(1) + s(F) 3 P_{ab}(2)$$

One loop (LO) was computed by "Gross, Wilczek and Politzer" (Nobel prize paper) also see "Altarelli and Parisi" and

Two loop (NLO) is computed recently (summer 2004) by "Moch, Vermaseren and Vogt"
Renormalised parton density:

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Altarelli-Parisi Evolution equation:

\[ \mu_F \frac{d}{d\mu_F} f_a(x, \mu_F) = \int_x^1 \frac{dz}{z} P_{ab}(z, \mu_F) f_b \left( \frac{x}{z}, \mu_F \right), \quad P \equiv \Gamma^{-1} \left( \mu_F \frac{d}{d\mu_F} \right) \Gamma \]
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\[ f_a(z, \mu_F) = \Gamma_{ab}(z, \mu_F, \frac{1}{\varepsilon_{IR}}) \otimes f_a^B(z) \]

Altarelli-Parisi Evolution equation:

\[ \mu_F \frac{d}{d\mu_F} f_a(x, \mu_F) = \int_x^1 \frac{dz}{z} P_{ab}(z, \mu_F) f_b \left( \frac{x}{z}, \mu_F \right), \quad P = \Gamma^{-1} \left( \mu_F \frac{d}{d\mu_F} \right) \Gamma \]

Perturbatively Calculable:

\[ P_{ab}(z, \mu_F) = \left( \frac{\alpha_s(\mu_F)}{4\pi} \right)^n P^{(n)}_{ab}(z) \]

- one loop \((LO)\)
- two loop \((NLO)\)
- three loop \((NNLO)\)

LO was computed by "Gross,Wilczek and Politzer" (Nobel prize paper also see "Altarelli and Parisi") and NNLO is computed recently (summer 2004) by "Moch,Vermaseren and Vogt"
UV Scale dependence of partonic cross section

Collinear finite partonic cross sections are calculable in perturbative QCD in powers of $\alpha_s$, bare strong coupling constant.

$$d_{ab}^{z,m_2} h; F = \sum_{i=0}^{s(R)} \frac{1}{R^4} \alpha_s^{(i)} d_{ab}^{z,m_2} h; F; 1$$

UV singularities are regularised in dimensional regularisation $\bar{n} = 4 + \epsilon$.

Ultraviolet divergences are removed by renormalisation in MS, at the Renormalisation scale $R$.

UV Renormalised partonic cross section:

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- Observables are "free" of \( \mu_R \) and \( \mu_F \).

\[ \mu \frac{d}{d\mu} \sigma^{P_1 P_2} = 0, \quad \mu = \mu_F, \mu_R \]
Scale Variation of Flux at LHC

\[
\Phi^{I}_{ab}(x, \mu_F) = \int_{x}^{1} \frac{dz}{z} f^{I}_{a}(z, \mu_F) f^{I}_{b}\left(\frac{x}{z}, \mu_F\right) \quad I = LO, NLO
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\[
\mu_0 = 700 \text{ GeV}, \quad x = \frac{Q}{\sqrt{S}}, \quad Q = 700 \text{ GeV} \quad \sqrt{S} = 14 \text{ TeV}
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- Compute Next to leading order NLO QCD corrections to LO processes
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d\hat{\sigma}_{ab}(\hat{s}, Q^2, \mu_F^2) = d\hat{\sigma}_{ab}^{(0)}(\hat{s}, Q^2, \mu_F^2) \left[ 1 + \frac{\alpha_s(\mu_R^2)}{4\pi} \Delta_{ab}^{(1)}(\hat{s}, Q^2, \mu_F^2, \mu_R^2) \right]
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QCD corrections are larger than other EW and gravity corrections.

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- All the soft and collinear divergences are regulated in dimensional regularisation \(n = 4 + \varepsilon\).

- Collinear mass factorisation is done in \(\overline{MS}\) scheme.
Virtual Corrections, $q\bar{q} \rightarrow G$

$$\tilde{\Delta}_{q\bar{q}}^{G} = \Delta_{q\bar{q}}^{(0)G} + \alpha_s \frac{2}{\varepsilon} \Gamma_{qq}^{(1)} \otimes \Delta_{q\bar{q}}^{(0)G} + \alpha_s \Delta_{q\bar{q}}^{(1)G}$$

$q + \bar{q} \rightarrow G$ (1 loop):
Real emission, $q \bar{q} \rightarrow g \ G$
Virtual Corrections, $g \bar{g} \rightarrow G$

$$\bar{\Delta}_{gg}^G = \Delta_{gg}^{(0)G} + \alpha_s \frac{2}{\varepsilon} \Gamma_{gg}^{(1)} \otimes \Delta_{gg}^{(0)G} + \alpha_s \Delta_{gg}^{(1)G}$$

$g + g \rightarrow G$ (1 loop):

- Diagram 1
- Diagram 2
- Diagram 3
- Diagram 4
Real emission, $g\ g \rightarrow g\ G$

\[ g \quad g \quad G \quad g \]

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Real emissions, $q \bar{g} \rightarrow qG$

$$\bar{\Delta}^{G}_{qg} = \alpha_s \frac{1}{\varepsilon} \left( \Gamma^{(1)}_{qg} \otimes \Delta^{(0)G}_{q\bar{q}} + \Gamma^{(1)}_{gq} \otimes \Delta^{(0)G}_{gg} \right) + \alpha_s \Delta^{(1)G}_{qg}$$

Real emission, $qg \rightarrow qG$
Invariant lepton pair mass $Q$ distributions:

$$\frac{d\sigma^I(Q)}{dQ}$$
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\frac{d\sigma^I(Q)}{dQ}
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- LHC: SM dominates for $Q < 600$ GeV but for $Q > 600$ GeV the gravity mediated processes dominate.
- TEV: for $Q > 700$ GeV the gravity mediated process becomes larger.
Invariant lepton pair mass $Q$ distributions:

\[ \frac{d\sigma^I(Q)}{dQ} \]

- **LHC**: SM dominates for $Q < 600$ GeV but for $Q > 600$ GeV the gravity mediated processes dominates
- **TEV**: for $Q > 700$ GeV the gravity mediated process becomes larger
Contributions at LHC

- SM the $q\bar{q}$ subprocess dominates (no gluon initiated process)
- Gravity mediated process $gg$ sub process initiated process dominates and substantially contributes to the cross section at large $Q^2$
K-Factor

\[ K^{(SM+GR)}(Q) = \frac{K^S M + K^G R}{1 + G^{(0)}} \]

\[ G^{(0)}(Q) = \left[ \frac{d\sigma^{SM}_{LO}(Q)}{dQ} \right]^{-1} \left[ \frac{d\sigma^{GR}_{LO}(Q)}{dQ} \right] \]

- \( G^{(0)}(Q) \) behavior is governed by a competing ‘couplings’ and PDF flux at LHC and TEV
- At high Q when Gravity contribution becomes comparable to SM, the PDF flux dictates the proceedings
K-Factor:

\[ K^I = \left( \frac{d\sigma_{LO}^I(Q)}{dQ} \right)^{-1} \left( \frac{d\sigma_{NLO}^I(Q)}{dQ} \right) \]
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\]

- **K-Factor (Q)**
  - M$_S$ = 2 TeV
  - d = 3

- **LHC:**
  - K$_{SM}$ is moderate for all values of Q while K$_{GR}$ is much larger than K$_{SM}$ at large Q $>$ 700 GeV, K$_{GR}$ dominates the K$_{SM}$ + GR.

- **TEV:**
  - K$_{SM}$ and K$_{SM}$ + GR are not very different.
K-Factor: \[
K^{I} = \left[ \frac{d\sigma^{I}_{LO}(Q)}{dQ} \right]^{-1} \left[ \frac{d\sigma^{I}_{NLO}(Q)}{dQ} \right]
\]

- LHC: \( K^{SM} \) is moderate for all values of \( Q \) while \( K^{GR} \) is much larger than \( K^{SM} \) at large \( Q \). \( Q > 700 \) GeV, \( K^{GR} \) dominates the \( K^{SM+GR} \). \( gg \) sub process contribute at LO itself via Gravity. NLO large effects due to small \( x \) terms in \( \Delta_{gg}^{(1)G} \)
- TEV: \( K^{SM} \) and \( K^{SM+GR} \) are not very different
R-Factor:

\[
R^{I}_{LO,NLO} = \left[ \frac{d\sigma^{I}_{LO,NLO}(Q, \mu = \mu_0)}{dQ} \right]^{-1} \left[ \frac{d\sigma^{I}_{LO,NLO}(Q, \mu)}{dQ} \right]_{Q=Q_0}
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\[\mu_0 = Q \quad Q_0 = 700 \text{ GeV (LHC)} \quad Q_0 = 400 \text{ GeV (TEV)}\]
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- Scale variation appreciably reduces in going from LO to NLO
- Inclusion of SM to GR also reduces scale variation
RS Scenario Results

\[ \mathcal{D}(Q^2) = \sum_{n=1}^{\infty} \frac{1}{Q^2 - M_n^2 + iM_n \Gamma_n} \equiv \frac{\lambda}{m_0^2} \]

\[ \frac{c_0^2}{m_0^2} \mathcal{D}(Q^2) = \frac{c_0^2}{m_0^4} \lambda \]

Away from the resonance region gravity contribution is negligible. K-Factor behavior can be understood from the \( K(0) \) behavior for the RS model.
\[ D(Q^2) = \sum_{n=1}^{\infty} \frac{1}{Q^2 - M_n^2 + iM_n \Gamma_n} \equiv \frac{\lambda}{m_0^2} \]

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- K-Factor behavior can be understood from the \( K^{(0)} \) behavior for the RS model.
- Scale variation reduced considerably in going from LO → NLO
- Inclusion of SM to GR also reduces scale variation
Summary

- Next to Leading Order coefficient functions for DY process in models of TeV-scale gravity are available now.

- Various distributions *viz.* $Q, x_F, Y$ distributions and $A_{FB}$ asymmetry at NLO are studied for ADD & RS models.

- Theoretical uncertainties get significantly reduced at NLO level

- Quantitative impact of the QCD corrections for searches of extra dimension at hadron colliders investigated