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Sum rules of polarized photon structure functions revisited

NNLO corrections to the first moment of $g_1^\gamma(x, Q^2, P^2)$

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T. Ueda (Yokohama N.U.)
T. Uematsu (Kyoto U)
K. Sasaki (Yokohama N.U.)

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1. Introduction -Motivation-

- Deep inelastic scatt. of polarized lepton on nucleon target

  $\Rightarrow$ $g_1^{p(n)}(x, Q^2)$, $g_2^{p(n)}(x, Q^2)$
  Polarized nucleon structure functions

  $\Rightarrow$ Δq^i, ΔG
  Polarized parton distributions inside of the nucleon

  ⊙ necessary for the analysis of polarized semi-inclusive reactions

  ⊙ information on the spin structures of nucleon

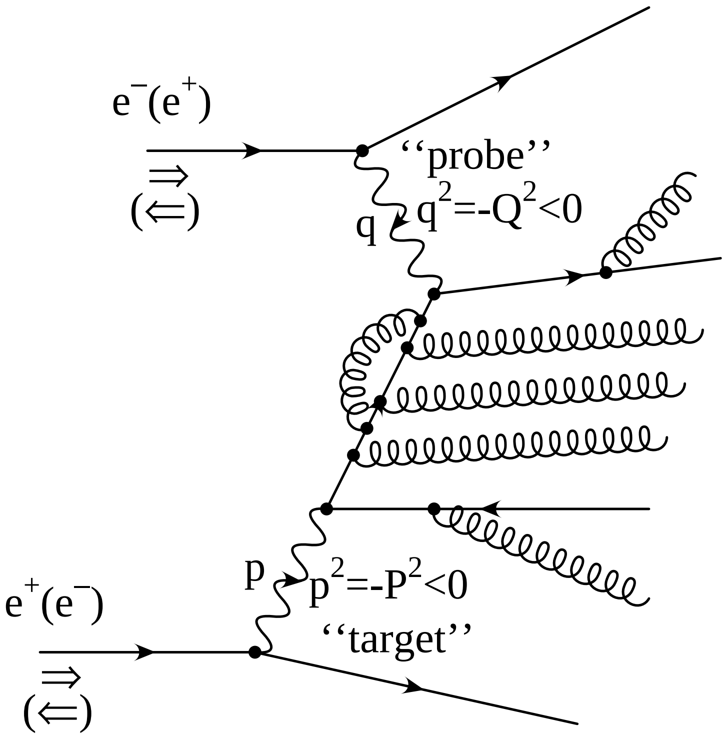
  ⊙ factorization-scheme dependent

  · rather difficult to see the features of factorization schemes

◇ Recent interests in photon spin structure functions

$$g_1^\gamma(x,Q^2,P^2), \quad g_2^\gamma(x,Q^2,P^2)$$

- Pol. e^+e^- collision in ILC or other future linear colliders
  $\Rightarrow \quad g_1^\gamma, \quad g_2^\gamma$



- Study of the polarized photon structure functions
  $\Rightarrow$ Spin structure of the photon
  $\Rightarrow$ would provide a good testing ground for factorization scheme dependence of polarized parton distributions

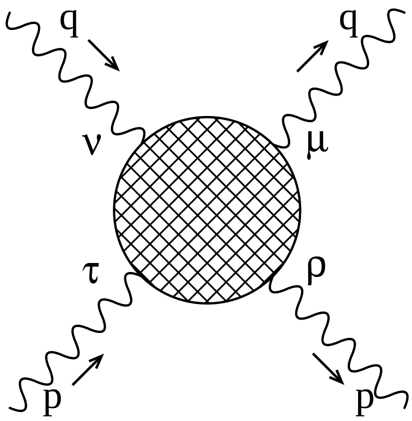
- **Theoretical prediction of the 1st moment**

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2)$$

$\Rightarrow$ **related to axial anomaly**

- **Study of $g_2^\gamma(x, Q^2, P^2)$**

$\Rightarrow$ **would provide information on twist-3 effects**



Kinematics

$$W_{\mu\nu}(p,q) \equiv \int d^4x e^{iqx}\langle\gamma(p)|J_\mu(x)J_\nu(0)|\gamma(p)\rangle$$
$$= \epsilon^{*\rho}\Big[W^S_{\mu\nu\rho\tau} + iW^A_{\mu\nu\rho\tau}\Big]\epsilon^\tau$$

where

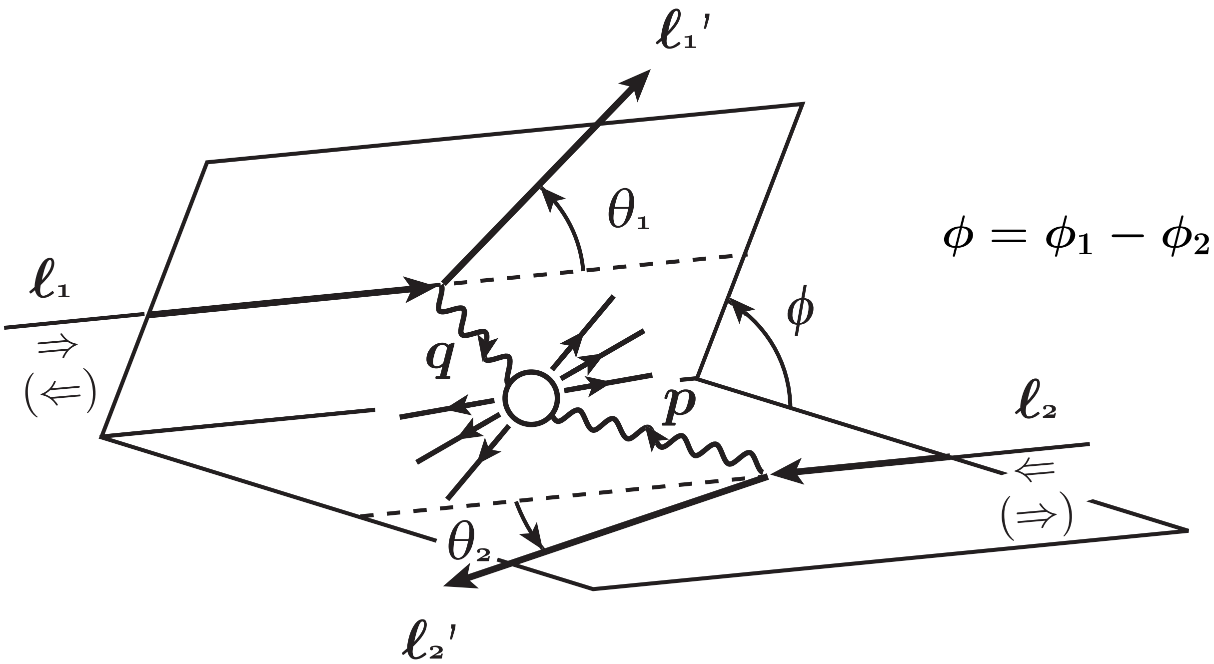
$$W^A_{\mu\nu\rho\tau} = \frac{1}{(p\cdot q)^2}[(I_-)_{\mu\nu\rho\tau}\ g_1^\gamma - (J_-)_{\mu\nu\rho\tau}\ g_2^\gamma]$$

with

$$(I_-)_{\mu\nu\rho\tau} \equiv p\cdot q\,\epsilon_{\mu\nu\lambda\sigma}\epsilon_{\rho\tau}{}^{\sigma\beta}q^\lambda p_\beta$$

$$(J_-)_{\mu\nu\rho\tau} \equiv \epsilon_{\mu\nu\lambda\sigma}\epsilon_{\rho\tau\alpha\beta}q^\lambda p^\sigma q^\alpha p^\beta - p\cdot q\,\epsilon_{\mu\nu\lambda\sigma}\epsilon_{\rho\tau}{}^{\sigma\beta}q^\lambda p_\beta$$

Two-photon process $e^+e^- \to e^+e^- +$ hadrons



$$\begin{aligned} e^{\pm}(l_1)e^{\mp}(l_2) &\longrightarrow e^{\pm}(l_1')e^{\mp}(l_2')\gamma(q)\gamma(p) \\ &\longrightarrow e^{\pm}(l_1')e^{\mp}(l_2') + \text{hadrons}, \end{aligned}$$

For colliding beams

$$\begin{aligned} l_1 &= (E, 0, 0, E)\ , \qquad l_2 = (E, 0, 0, -E)\ , \\ l_1' &= (E_1', E_1'\sin\theta_1\cos\phi_1, E_1'\sin\theta_1\sin\phi_1, E_1'\cos\theta_1)\ , \\ l_2' &= (E_2', E_2'\sin\theta_2\cos\phi_2, E_2'\sin\theta_2\sin\phi_2, -E_2'\cos\theta_2) \end{aligned}$$

$$\begin{aligned} d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow} &= \frac{E_1'E_2'dE_1'dE_2'd\cos\theta_1 d\cos\theta_2 d\phi}{\pi E^2}\ \frac{\alpha^3}{p^2q^2(p\cdot q)} \\ &\times \Bigg[\Big\{(E+E_1')(E+E_2') \\ &\quad +(E+E_1'\cos\theta_1)(E+E_2'\cos\theta_2) \\ &\quad -E_1'E_2'\sin\theta_1\sin\theta_2\cos\phi\Big\}g_1^{\gamma} \\ &\quad +\frac{4}{p\cdot q}E^2E_1'E_2'\Big\{(1-\cos\theta_1)(1-\cos\theta_2) \\ &\quad -2\sin\theta_1\sin\theta_2\cos\phi\Big\}g_2^{\gamma}\Bigg] \end{aligned}$$

where $\phi = \phi_1 - \phi_2$

$$\begin{aligned}
q^2 &= -2EE_1'(1 - \cos\theta_1) \\
p^2 &= -2EE_2'(1 - \cos\theta_2) \\
p \cdot q &= (E - E_1')(E - E_2') \\
&\quad +(E - E_1'\cos\theta_1)(E - E_2'\cos\theta_2) \\
&\quad -E_1'E_2'\sin\theta_1\sin\theta_2\cos\phi \; .
\end{aligned}$$

2. Photon spin structure function g_1^γ

- **Real photon target** $(P^2 = 0)$

$$g_1^\gamma(x,Q^2) = g_1^\gamma(x,Q^2)|_{\rm pert.} + g_1^\gamma(x,Q^2)|_{\rm non-pert.}$$

$g_1^\gamma(x,Q^2)|_{\rm pert.}$ calculable in pQCD

- · LO : K.S. ; Phys.Rev. D22(1980)
- · NLO: Stratman & Vogelsang; Phys. Lett. B386 (1996)

- **Virtual photon target** $(\Lambda^2 \ll P^2 \ll Q^2)$

$g_1^\gamma(x,Q^2,P^2)$ Λ : QCD scale parameter

∘ pQCD gives the whole information (so far, up to NLO) on

- · $F_2^\gamma(x,Q^2,P^2)$: Uematsu & Walsh; NPB199 (1982)
- · $g_1^\gamma(x,Q^2,P^2)$: Uematsu & K.S.; PRD59 (1999)
- · Parton distributions in the virtual photon

$$\Delta q_S^\gamma, \quad \Delta G^\gamma, \quad \Delta q_{NS}^\gamma$$

- · A good testing ground to see the scheme-dependence of parton distributions

Uematsu & K.S. ; Phys.Lett. B473 (2000)

Eur. Phys. J. C20 (2001)

3. Sum Rule for $g_1^\gamma(x, Q^2, P^2)$

- For real photon ($P^2 = 0$)

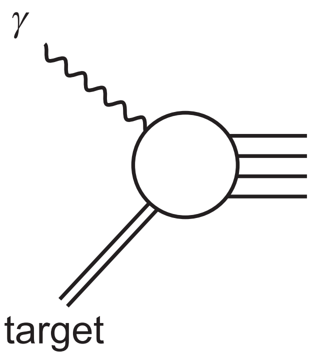
$$\int_0^1 dx g_1^\gamma(x, Q^2) = 0 \quad \text{for} \quad \forall Q^2$$

satisfied in all orders in QED and QCD

Efremov-Teryaev; Phys.Lett.B240 (1990)
Bass; Int.J. of Modern Physics A7 (1992)
Narison-Shore-Veneziano; Nucl.Phys.B391(1993)
Bass-Brodsky-Schmidt; Phys. Lett.B437 (1998)

- ○ Gerasimov-Drell-Hearn sum rule

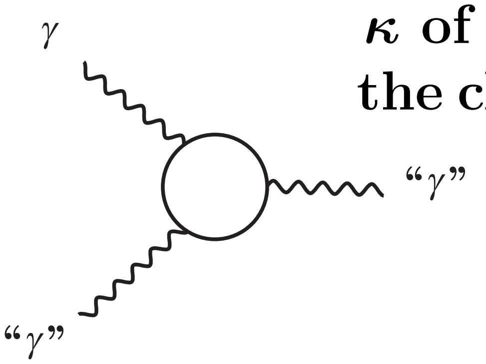
Causality, Unitarity, Lorentz and Electro-magnetic gauge inv.
No-subtraction hypothesis



$$\int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu}\Big[\sigma_A(\nu) - \sigma_P(\nu)\Big] = -\frac{4\pi^2\alpha S\kappa^2}{m^2}$$

S: target spin
κ: anomalous magnetic moment

- ○ When the target is virtual "γ" with Q^2



κ of "γ" $= 0$ Furry's theorem
the change of variable: $\nu \rightarrow x = Q^2/(2\nu)$

- **What about the virtual photon case**
  for $Q^2 \gg P^2 \gg \Lambda^2$

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2) \qquad ?$$

  - **Box diagram contributions**

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2) = -\frac{3\alpha}{\pi}\sum_{i=1}^{n_f} e_i^4 + \mathcal{O}(\alpha_s)$$

$\mathcal{O}(\alpha_s)$ QCD Corrections

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2)$$
$$= -\frac{3\alpha}{\pi}\Big[\sum_{i=1}^{n_f} e_i^4\left(1-\frac{\alpha_s(Q^2)}{\pi}\right)$$
$$-\frac{2}{\beta_0}(\sum_{i=1}^{n_f} e_i^2)^2\left(\frac{\alpha_s(P^2)}{\pi}-\frac{\alpha_s(Q^2)}{\pi}\right)\Big]$$
$$+\mathcal{O}(\alpha_s^2)$$

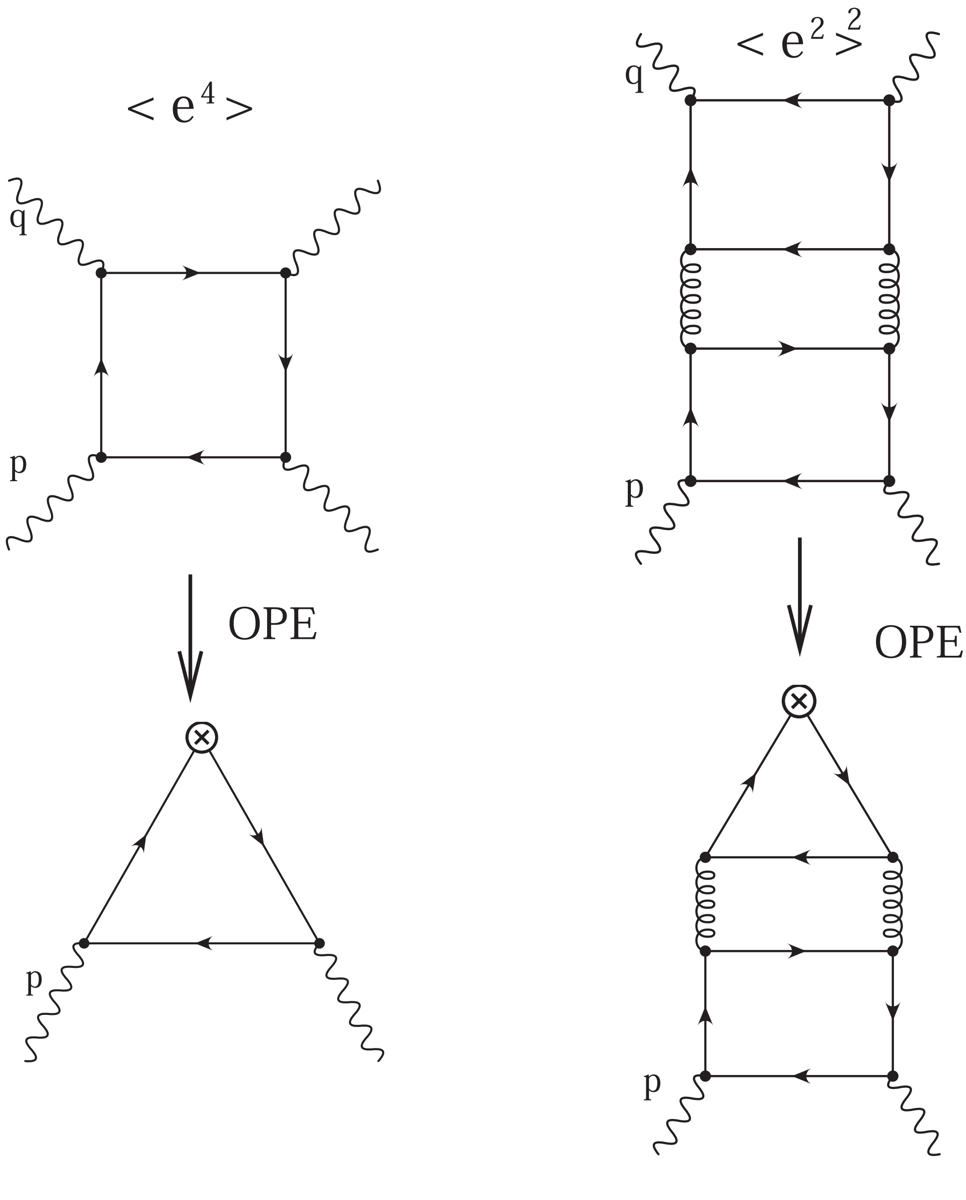
for

$Q^2 \gg P^2 \gg \Lambda^2$ $\quad n_f$: ♯ of active flavors

Narison-Shore-Veneziano; Nucl. Phys. B391 (1993)
Uematsu-K.S. ; Phys. Rev. D59 (1999)
Shore ; Nucl. Phys. B712 (2005)

- **The first term $\Leftarrow$ QED axial anomaly**
- **The second term $\Leftarrow$ QCD axial anomaly**

[$g_1^\gamma(x, Q^2, P^2)$ and the Axial Anomaly]



$$\langle e^4 \rangle = \frac{1}{n_f} \sum_i e_i^4 \,, \qquad \langle e^2 \rangle = \frac{1}{n_f} \sum_i e_i^2$$

4. $\mathcal{O}(\alpha_s^2)$ QCD Corrections

⊙ For OPE of two electromagnetic currents
only gauge-invariant operators need be included J.C. Collins

⊙ No gauge-invariant $n = 1$ gluon nor photon operators

⊙ Only quark operators (axial currents) need be considered

$$R_S^\sigma = \overline{\psi}\gamma^\sigma\gamma_5\, 1\psi \qquad \text{:flavor singlet}$$
$$R_{NS}^\sigma = \overline{\psi}\gamma^\sigma\gamma_5(Q_{ch}^2 - 1)\psi \qquad \text{:flavor non-singlet}$$

⊙ The first moment

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2)$$
$$= C_S(Q^2/\mu^2, \bar{g}(\mu^2), \alpha)\langle\gamma(p)|R_S(\mu^2)|\gamma(p)\rangle$$
$$+ C_{NS}(Q^2/\mu^2, \bar{g}(\mu^2), \alpha)\langle\gamma(p)|R_{NS}(\mu^2)|\gamma(p)\rangle$$

⊙ For $-p^2 = P^2 \gg \Lambda^2$, $\langle\gamma(p)|R_i(\mu^2)|\gamma(p)\rangle$ can be calculated perturbatively $(i = S, NS)$

⊙ $\langle\gamma(p)|R_i(\mu^2)|\gamma(p)\rangle|_{\mu^2=P^2} = \frac{\alpha}{4\pi}A_i$

$$A_i = A_i^{(0)} + \frac{\overline{g}^2(P^2)}{16\pi^2}A_i^{(1)} + \Big(\frac{\overline{g}^4(P^2)}{16\pi^2}\Big)^2 A_i^{(2)} + \cdots .$$

- Adler-Bell-Jackiw anomaly

$$A_S^{(0)} = -12n_f\langle e^2\rangle$$
$$A_{NS}^{(0)} = -12n_f(\langle e^4\rangle - \langle e^2\rangle^2)$$

- Nonrenormalization theorem for the triangle anomaly   Adler and Bardeen

$$A_S^{(1)} = A_{NS}^{(1)} = A_S^{(2)} = A_{NS}^{(2)} = 0$$

⊙ Coefficient functions

$$C_i(Q^2/P^2, \bar{g}(P^2), \alpha) = \exp\left[\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)} dg' \frac{\gamma_i(g')}{\beta(g')}\right] C_i(1, \bar{g}(Q^2), \alpha)$$

∘ NS quark axial current is conserved in the massless limit $\Rightarrow$ $\gamma_{NS}(g) = 0$

⊙ Anomalous dimension $\gamma_S(g)$ of the singlet axial current

$$\gamma_S(g)=\gamma_S^{(0)}\frac{g^2}{16\pi^2}+\gamma_S^{(1)}(\frac{g^2}{16\pi^2})^2+\gamma_S^{(2)}(\frac{g^2}{16\pi^2})^3+$$

- $\gamma_S^{(0)} = 0$
- $\gamma_S^{(1)} = 12C_F n_f$ Kodaira (1980)
- $\gamma_S^{(2)} = \left(\frac{284}{3}C_F C_A - 36C_F^2\right)n_f - \frac{8}{3}C_F n_f^2$

$\overline{MS}$ scheme Larin; Phys. Lett. B303 (1993)

- γ_5 is defined as $\gamma_\mu\gamma_5 = \frac{1}{6}\epsilon_{\mu\rho\sigma\tau}\gamma^\rho\gamma^\sigma\gamma^\tau$
- The extra finite renormalization is needed to keep the exact 1-loop Adler-Bardeen form

$$\partial^\mu J_\mu^5 = \frac{\alpha_s}{4\pi}\frac{n_f}{2}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

⊙ Nonsinglet coefficient function

$$C_{NS}(1,\bar{g}(Q^2),\alpha) = 1 - \frac{3}{4}C_F\frac{\alpha_s(Q^2)}{\pi} + C_F\Big(\frac{21}{32}C_F - \frac{23}{16}C_A + \frac{1}{4}n_f\Big)\Big(\frac{\alpha_s(Q^2)}{\pi}\Big)^2 +$$

$\overline{MS}$ Larin and Vermaseren; Phys. Lett. B259 (1991)

⊙ Singlet coefficient function

$$C_S(1,\bar{g}(Q^2),\alpha)/ < e^2 >= 1 - 3C_F\frac{\alpha_s(Q^2)}{4\pi} + \Big[\frac{21}{2}C_F^2 - 23C_F C_A + (8\zeta_3 + \frac{13}{3})C_F n_f\Big]\Big(\frac{\alpha_s(Q^2)}{4\pi}\Big)^2+$$

$\overline{MS}$ Larin; Phys. Lett. B334 (1994)

⊙ $\mathcal{O}(\alpha_s^2)$ QCD Corrections

$$
\begin{aligned}
&\int_0^1 dx g_1^\gamma(x, Q^2, P^2) \\
&= -\frac{3\alpha}{\pi}\Bigg\{\sum_i^{n_f} e_i^4\left[1-\frac{\alpha_s(Q^2)}{\pi}\right] \\
&\qquad -\frac{2}{\beta_0}\Big(\sum_i^{n_f} e_i^2\Big)^2\left[\frac{\alpha_s(P^2)}{\pi}-\frac{\alpha_s(Q^2)}{\pi}\right] \\
&\qquad +\frac{2}{\beta_0}\Big(\sum_i^{n_f} e_i^2\Big)^2\frac{\alpha_s(Q^2)}{\pi}\left[\frac{\alpha_s(P^2)}{\pi}-\frac{\alpha_s(Q^2)}{\pi}\right] \\
&\qquad +\frac{1}{4\beta_0}\Big(\frac{\beta_1}{\beta_0}-\frac{59}{3}+\frac{2}{9}n_f\Big)\Big(\sum_i^{n_f} e_i^2\Big)^2 \\
&\qquad\qquad\qquad\qquad \times\left[\frac{\alpha_s^2(P^2)}{\pi^2}-\frac{\alpha_s^2(Q^2)}{\pi^2}\right] \\
&\qquad +\frac{2n_f}{\beta_0^2}\Big(\sum_i^{n_f} e_i^2\Big)^2\left[\frac{\alpha_s(P^2)}{\pi}-\frac{\alpha_s(Q^2)}{\pi}\right]^2 \\
&\qquad -\Big(\frac{55}{12}-\frac{1}{3}n_f\Big)\sum_i^{n_f} e_i^4\frac{\alpha_s^2(Q^2)}{\pi^2} \\
&\qquad +\Big(\frac{2}{3}\zeta_3+\frac{1}{36}\Big)\Big(\sum_i^{n_f} e_i^2\Big)^2\frac{\alpha_s^2(Q^2)}{\pi^2}\Bigg\},
\end{aligned}
$$

⊙ For $n_f = 4$

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2)$$
$$= -\frac{3\alpha}{\pi}\Bigg\{0.4198 - 0.1235\frac{\alpha_s(Q^2)}{\pi} - 0.2963\frac{\alpha_s(P^2)}{\pi}$$
$$-0.02731\Big(\frac{\alpha_s(Q^2)}{\pi}\Big)^2 + 0.01185\frac{\alpha_s(Q^2)}{\pi}\frac{\alpha_s(P^2)}{\pi}$$
$$-0.3251\Big(\frac{\alpha_s(P^2)}{\pi}\Big)^2\Bigg\}$$

⊙ Sizes of NLO and NNLO corrections

Taking $\alpha_s(Q^2 = 30\text{GeV}^2) = 0.2047$
$\alpha_s(P^2 = 1\text{GeV}^2) = 0.4996$

N_f	LO	NLO	NNLO	NNLO/(LO+NLO)
3	1	-0.107	-0.0181	-0.0203
4	1	-0.131	-0.0196	-0.0226
5	1	-0.150	-0.0216	-0.0254

5. QCD parton model point of view

We expect that we get the same result

- Partition function for quark to gluon: $P_{Gq}(x)$
- The first moment: $\int_0^1 dx P_{Gq}(x)$

  $\Longleftrightarrow$ anomalous dimension: $\gamma_{Gq}^{n=1} \neq 0$

$$\gamma_{Gq}^{(0)n=1} = -6C_F \ , \quad \gamma_{Gq}^{(1)n=1} = 18C_F^2 - \frac{142}{3}C_A C_F + \frac{4}{3}C_F n_f$$

- Once quark has distribution, gluon also has distribution.

  So the first moment of polarized gluon coefficient $C_G^{n=1}(1, \bar{g}(Q^2))$ should be 0 in $\overline{MS}$ scheme

- In fact, for

$$C_G^{n=1}(1, \bar{g}^2) = \langle e^2 \rangle \left\{ \frac{\bar{g}^2}{16\pi^2} B_G^{(1),n=1} + \left(\frac{\bar{g}^2}{16\pi^2} \right)^2 B_G^{(2),n=1} + \cdots \right\}$$

we have

$$B_G^{(1),n=1}|_{\overline{MS}} = 0, \qquad B_G^{(2),n=1}|_{\overline{MS}} = 0$$

Zijlstra and van Neerven; Nucl. Phys. B417 (1994)

We expect $B_G^{(i),n=1}|_{\overline{MS}} = 0$ in all orders in pQCD

- Gluon distribution should not affect the first moment of quark distribution

  So we expect $\gamma_{qG}^{n=1}|_{\overline{MS}} = 0$ in all orders in pQCD

  We have $\gamma_{qG}^{(0)n=1} = 0$, $\gamma_{qG}^{(1)n=1}|_{\overline{MS}} = 0$

6. Summary and Discussion

- The future experiments at ILC will give us an interesting information on the polarized photon structure functions g_1^γ and g_2^γ
- The sum rule for g_1^γ is interesting because it is related to the axial anomaly
- g_1^γ for real photon target has a remarkable sum rule

$$\int_0^1 dx g_1^\gamma(x, Q^2) = 0$$

satisfied in all orders in QED and QCD

- The sum rule becomes non-zero when the target photon is off-shell
- In the kinematical region $\Lambda^2 \ll P^2 \ll Q^2$, NNLO corrections ($\mathcal{O}(\alpha\alpha_s^2)$) was studied

Taking $\alpha_s(Q^2\!=\!30\text{GeV}^2)\!=\!0.2047$
$\alpha_s(P^2\!=\!1\text{GeV}^2)\!=\!0.4996$

N_f	LO	NLO	NNLO	NNLO/(LO+NLO)
3	1	-0.107	-0.0181	-0.0203
4	1	-0.131	-0.0196	-0.0226
5	1	-0.150	-0.0216	-0.0254

- The axial anomaly plays an important role for the sum rule
- Considering the same sum rule from the QCD parton model point of view,

  we expect

  – The first moment of polarized gluon coefficient function $C_G^{n=1}(1, \overline{g}(Q^2))|_{\overline{MS}}$ should be 0 in all orders in pQCD

  – $\gamma_{qG}^{n=1}|_{\overline{MS}} = 0$ in all orders in pQCD