Monte Carlo Solutions of the QCD Evolution Equations

M. Skrzypek

in collaboration with

K. Golec-Biernat, S. Jadach, W. Płaczek, Z. Was

IFJ-PAN, Kraków, Poland

RADCOR 05

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Introduction

- LHC will start operating in two years!
- High statistics of W and Z bosons (millions) will provide opportunity for precise measurements
  - $M_W \to 15$ MeV (now 34 MeV)
  - anomalous V-boson couplings $\to 10^{-3}$ (now 10^{-2})
- Parton luminosities will be measured with 1% precision
- Are the Monte Carlo tools ready?

Desired specifications of MC tools for LHC

Observable/proc	EW	QED	QCD	MC type
M_W/W,Z	Impr. Born	$\mathcal{O}(\alpha)_{\rm exp}$ FSR!	pdf(x,pT), NLO?	events!
Anom coupl/VV	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha)$	NLO! NNLO?	events!
$\sin^2\theta_{EW}$/Z	$\mathcal{O}(\alpha, \alpha^2_{Sud.})$	$\mathcal{O}(\alpha)_{\rm exp}$ FSR!	NLO	events!
Parton $\mathcal{L}$/W,Z	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha)_{\rm exp}$ FSR?	NLO! NNLO?	events?

! $\equiv$ mandatory,

? $\equiv$ to be checked...

None of the existing TOOLS fulfills this specs

Existing EW+QCD tools for $pp \to$V and $pp \to$VV, V=W,Z

Tool	Process	EW	QCD	MC type
WGRAD	W	$\mathcal{O}(\alpha)$	pdf(x),LO	histogrs.
ZGRAD2	Z	$\mathcal{O}(\alpha)$	pdf(x),LO	histogrs.
WINHAC	W	QED FSR $\mathcal{O}(\alpha)_{\rm EEX}$	pdf(x),LO	events
HORACE	W,Z	QED FSR part.sh.	pdf(x),LO	events
SANC	W,Z	$\mathcal{O}(\alpha)$	???	events?
RESBOS	W, Z	LO	pdf(x,pT),NLO	histogrs.
DYRAD	V+(0j-1j)	LO	pdf(x),NLO	histogrs.
MCFM	V,VV	LO	pdf(x),NLO	histogrs.
DKS	WW, WZ, ZZ	LO, Anom.Coup.	pdf(x),NLO	histogrs.
dFS	Wγ, Zγ	LO, Anom.Coup.	pdf(x),NLO	histogrs.
MC@NLO	WW	LO	part.sh. NLO	events
MC@NLO	W or Z	LO	part.sh. NLO	events

Required at least $\mathcal{O}(\alpha)$ electroweak or NLO QCD, none has both!

Vocabulary

- Markovian MC algorithm
  The algorithm in which the number of emissions (determining the dimension of the phase-space integral), is generated as *the last* variable
- non-Markovian MC algorithm
  The algorithm in which the number of emissions (the dimension of the integral), is generated as one of *the first* variables.
- **Constrained MC algorithm = CMC**
  Distributions the same as in normal Markovian evolution, but final energy $x = \prod z_i$ and parton type are predefined i.e. constrained.
- **HERWIG Evolution (terminology by P. Nason), 1-loop CCFM**:
  Two ingredients: $\alpha_S(Q(1-z))$ (Amati, Basetto, Ciafaloni, Marchesini, Veneziano, NPB173, 1980) and $\varepsilon_{IR} = Q_0/Q$ where $Q_0 \sim$ 1GeV (Webber, Marchesini, NPB310, 1988).
  MS-bar DGLAP evolution $\neq$ HERWIG evolution – at the LL they differ by large NLL and Q_0/Q terms.

R&D on MC solutions of QCD Evolution in Cracow

Monte Carlo modeling of the QCD $\overline{MS}$ DGLAP evolution:

- Markovian MC (forward) precision ($\sim 10^{-3}$) solutions of the full LL DGLAP equations (massless quarks). Acta.Phys.Pol. B35 (2004).
- Markovian MC precision solutions of the full NLL DGLAP equations (massless quarks). IFJPAN-V-04-08, to appear.
- Markovian MC study of the CCFM one-loop evolution. IFJPAN-V-05-03, to appear

Constrained Monte Carlo (CMC) algorithms for DGLAP evolution:

- Constrained MC (non-Markovian) class II. Proc. Loops&Legs 2004, Nucl. Phys. Proc. Suppl. 135 (2004) and IFJPAN-V-04-06, hep-ph/0504205.
- Constrained MC (non-Markovian) class I. IFJPAN-V-04-07, hep-ph/0504263.

The long standing problem

- Markovian MC implementing the QCD/QED evolution equations is basic ingredient in all parton shower type MCs
- Unconstrained Markovian, with evolution kernels from perturbative QCD/QED, can only be used for FSR (inefficient for ISR)
- For ISR the *Backward Markovian* of Sjostrand (Phys.Lett. 157B, 1985) is a widely adopted *remedy*.
- *Backward Markovian* does not solve evolution eqs. It merely exploits their solutions coming from the *external* non-MC methods
- **Is it possible to invent an efficient MC algorithm for constrained Markovian based on *internal* MC solutions of the evolution eqs?**

Evolution Equation

$$\frac{\partial}{\partial t} D_k(t,x) = \sum_j \mathcal{P}_{kj}(t,\cdot) \otimes D_j(t,\cdot)$$

Differential equation $\longrightarrow$ integral equation:

$$e^{\Phi_k(t,t_0)} D_k(t,x) = D_k(t_0,x) + \int_{t_0}^{t} dt_1 e^{-\Phi_k(t_1,t_0)} \sum_j \mathcal{P}^{\Theta}_{kj}(t_1,\cdot) \otimes D_j(t_1,\cdot)(x)$$

where IR regulator is introduced

$$\mathcal{P}_{kj}(t,z) = -\mathcal{P}^{\delta}_{kk}(\epsilon(t))\delta_{kj}\delta(1-z) + \mathcal{P}^{\Theta}_{kj}(t,z)\Theta(1-z-\epsilon)$$

and the Sudakov formfactor appears

$$\Phi_k(t,t_0) = \int_{t_0}^{t} dt' \; \mathcal{P}^{\delta}_{kk}(\epsilon(t'))$$

Master equation for Markovian solution

$$xD_K(\tau,x) = \int_{\tau_1>t} d\tau_1 dz_1 \sum_{K_1} \bar{\omega}(\tau_1,x_1,K_1|\tau_0,x_0,K)\ \ xD_K(\tau_0,x)$$

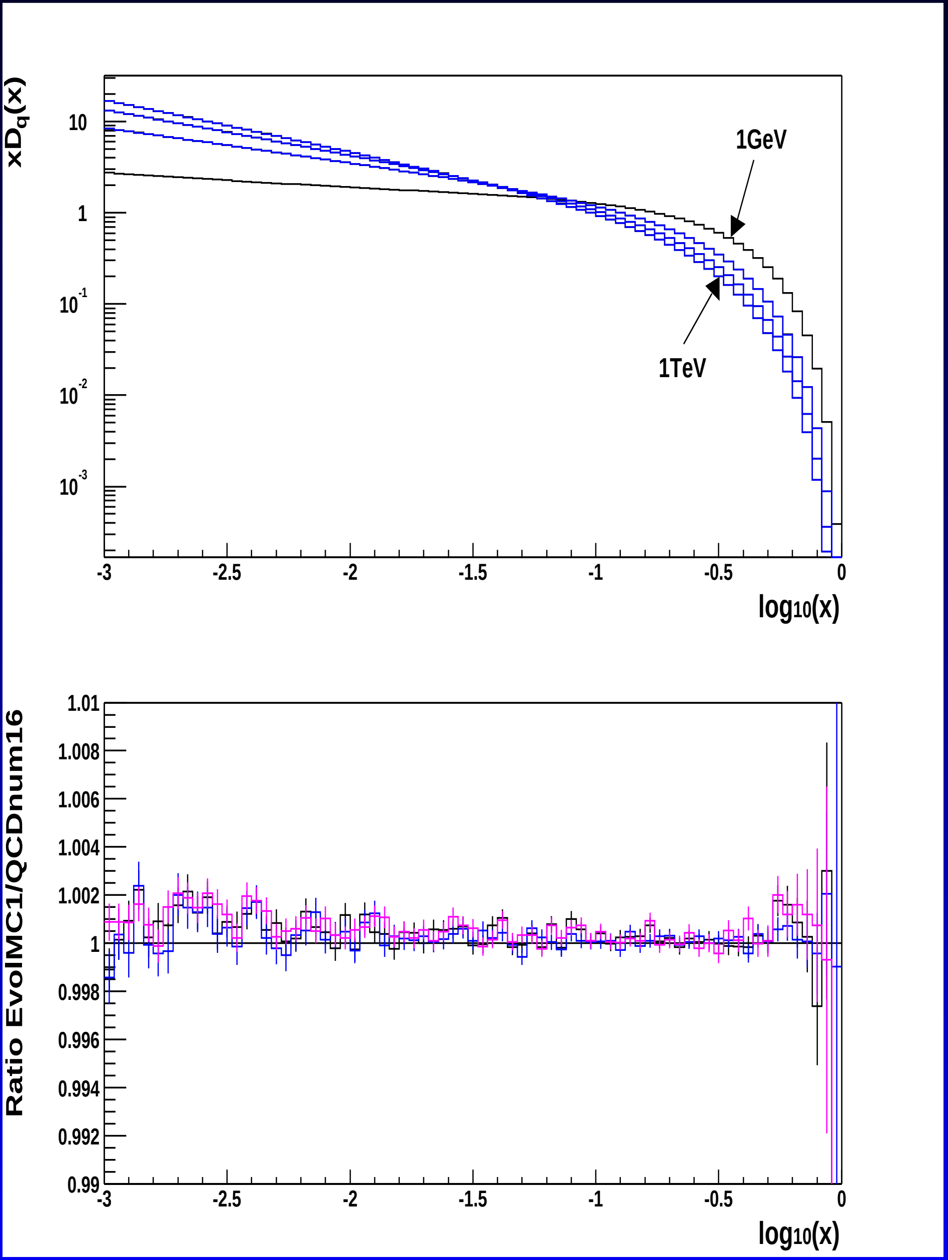
$$+\sum_{n=1}^{\infty} \int_0^1 dx_0 \int_{\tau_{n+1}>\tau} d\tau_{n+1} dz_{n+1} \sum_{K_{n+1}} \sum_{K_0\dots K_{n-1}} \prod_{i=1}^{n} \int_{\tau_i<\tau}^{t} d\tau_i dz_i$$

$$\times\, \bar{\omega}(\tau_{n+1},x_{n+1},K_{n+1}|\tau_n,x_n,K_n) \qquad \leftarrow \text{ spillover}$$

$$\times \prod_{i=1}^{n} \bar{\omega}(\tau_i,x_i,K_i|\tau_{i-1},x_{i-1},K_{i-1}) \qquad \leftarrow \text{ normal step}$$

$$\times\, \delta\Big(x - x_0 \prod_{i=1}^{n} z_i\Big)\ x_0 D_{K_0}(\tau_0,x_0)\ \bar{w}_P\ \bar{w}_\Delta \qquad \leftarrow \text{ MCweight}$$

Tests of Markovian sol.: Proton $\rightarrow$ quarks

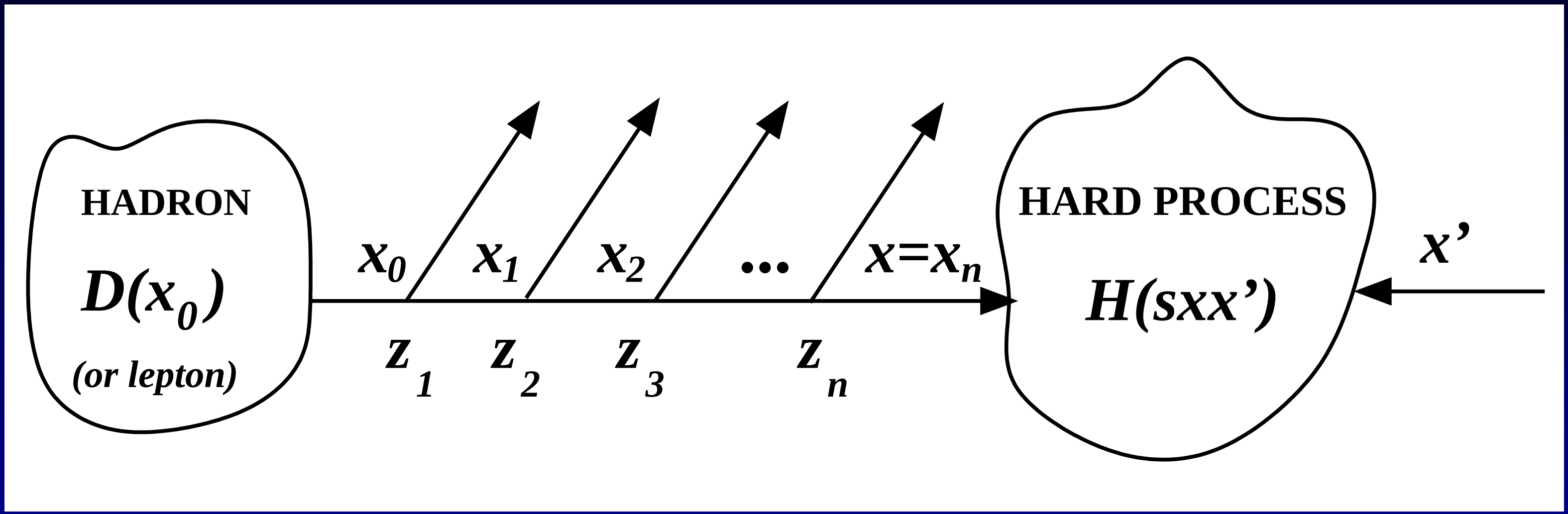


Upper plot shows quark singlet distribution $xD_G(x, Q_i)$ evolved from $Q_0 = 1\text{GeV}$ to $Q_i = 10, 100, 100\text{GeV}$ obtained from `QCDnum16` and `EvolMC1`, while lower plot shows their ratio.

The horizontal axis is $\log_{10}(x)$.

Starting distribution is complete proton at $Q = 1\text{GeV}$.

Constrained Solutions class I and II



$$\int dx_0\ D(x_0) \int \prod_i dz_i P(z_i)\ H\Big(sx_0 \prod z_i\Big)$$

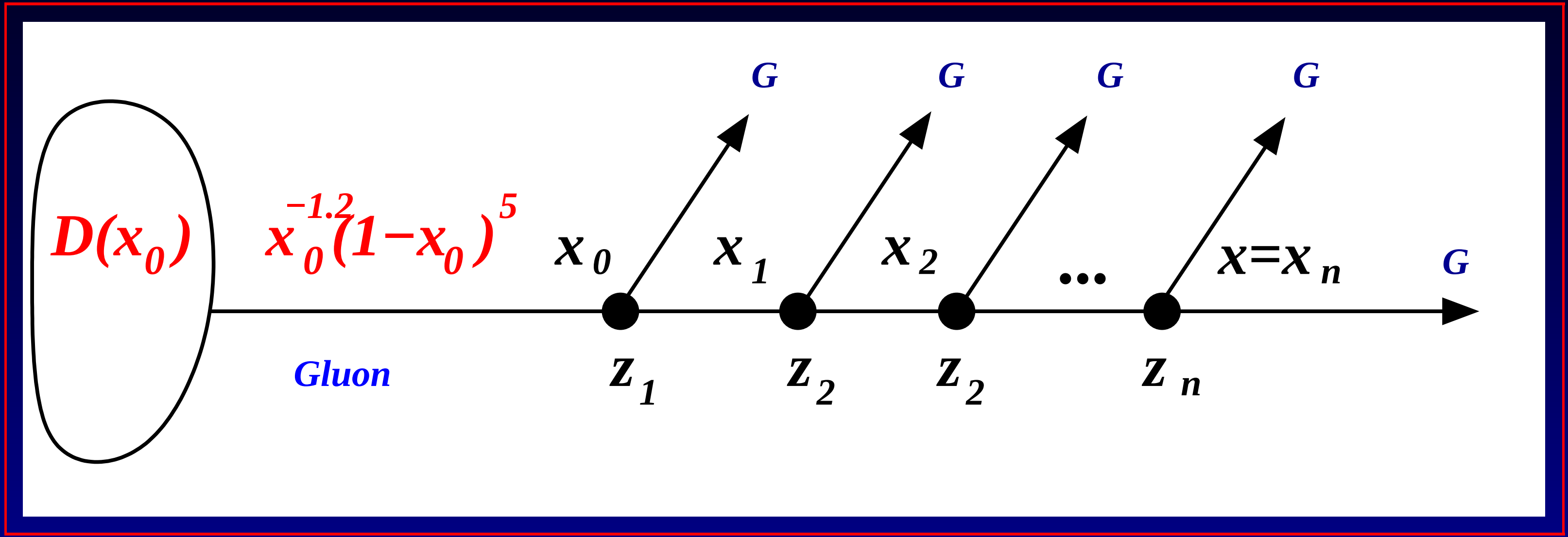
Solutions class I

$$\int dx\ dx_0\ D(x_0) H\Big(sx\Big) \int \prod_i dz_i P(z_i) \delta(x - x_0 \textstyle\prod_i z_i)$$

Solutions class II

$$\int dx\ H\Big(sx\Big) \int \prod_i \frac{dz_i}{z_i} P(z_i)\ \ D(x/\textstyle\prod_i z_i)\, \Theta(\prod z_i - x)$$

Solution IIB

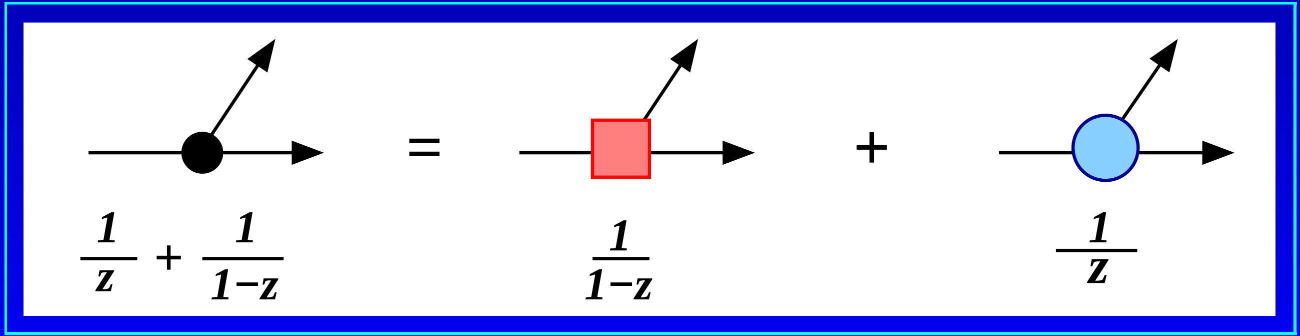


Replace $D(x_0) \to 1/x_0 = x \prod \frac{1}{z_i}$. Compensated by MC weight.

Must generate $P(z_i) = 2C_A\left(\frac{1}{z_i} + \frac{1}{1-z_i}\right)$

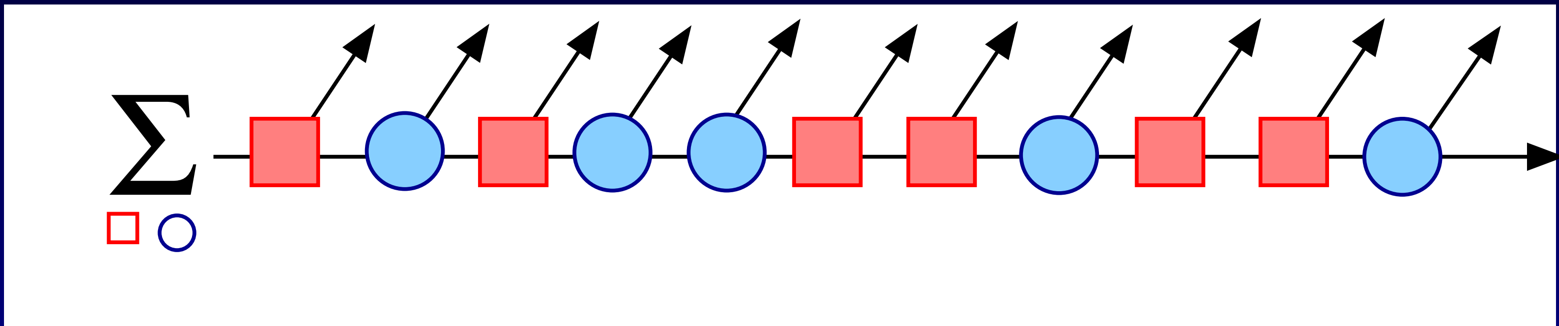
with the constraint $\prod_i z_i \geq x$. Not so trivial!

Solution by the multibranching method:

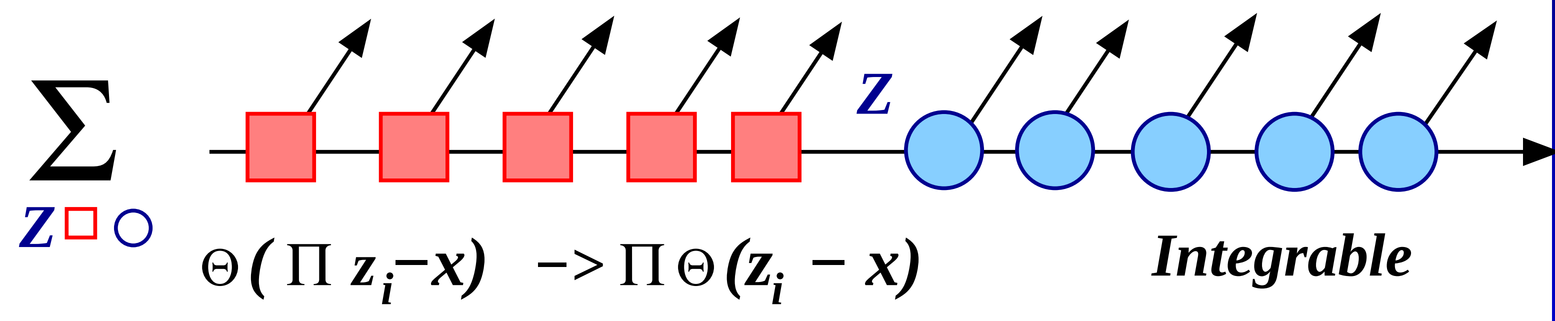


Multibranching in IIB

Each sum



Can be rearranged:



Contributions $1/z$ and $1/(1-z)$ are combined and resummed separately.

k_T-dependent PDFs

Use the CCFM equation in "1-loop approximation"

$$f(x,Q_t,q_0) = f_0(x,Q_t)$$

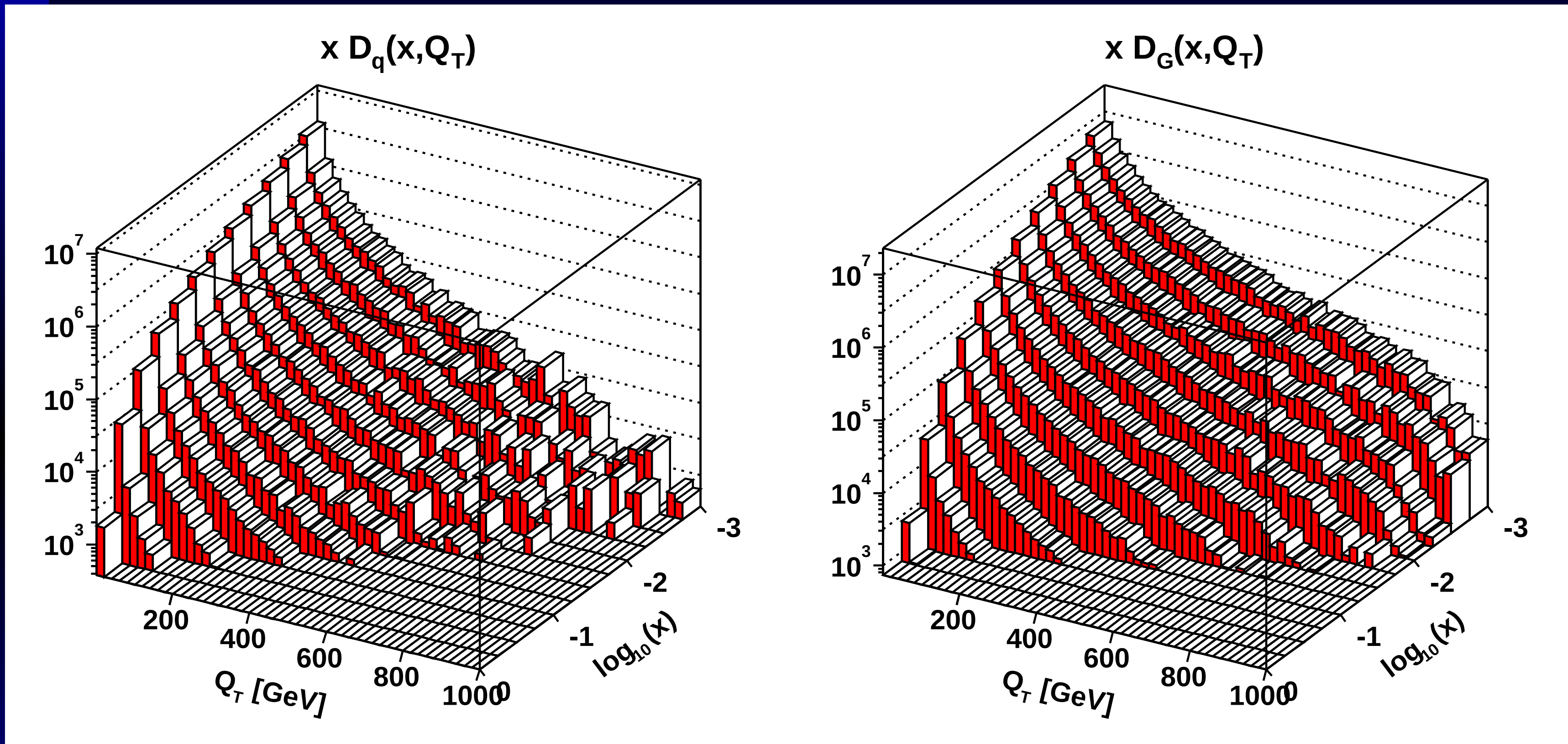
$$+ \int_{q_{min}}^{q_0} \frac{d^2\vec{q}}{\pi q^2}\frac{\alpha_S(q^2)}{2\pi} \int_x^1 \frac{dz}{z} zP(z) f\left(\frac{x}{z}, |\vec{Q}_t + (1-z)\vec{q}|, q\right)$$

$$= f_0(x,Q_t) + \sum_{n=1} \int_0^1 dz_0 \delta\left(x - \prod_{i=0}^{n} z_i\right)$$

$$\times \left[\prod_{i=1}^{n} \int_{q_{min}}^{q_{i-1}} \frac{d^2\vec{q_i}}{\pi q_i^2}\frac{\alpha_S(q_i^2)}{2\pi} \int_0^1 dz_i z_i P(z_i)\right] f_0\left(z_0, |\vec{Q}_t + \sum_{i=1}^{n}(1-z_i)\vec{q_i}|\right)$$

Integrated over d^2Q_t this equation turns into ordinary DGLAP with $xD(x,q_0) \equiv \int d^2\vec{Q}_t f(x,Q_t,q_0)$

k_T-dependent PDFs



$\vec{Q}_t = -\sum_{i=1}^{n}(1-z_i)\vec{q}_i$, the "CCFM in 1-loop approx."

Problem with solution IIB

- Efficiency (ratio of accepted to rejected events) of the order of 10^{-3} – would like higher!
- We have constructed another class of solutions – of the type I

Dilatation trick

consider $L = \delta(K - \sum k_i) \prod dk_i/k_i \theta(k_i - k_{i-1})$
introduce $1 = \delta(\lambda - k_n/K) d\lambda, \lambda \leq 1$
shift all variables $k'_i = k_i/\lambda$

$$L = d\lambda \delta(\lambda - \lambda \frac{k'_n}{K}) \delta(K - \lambda \sum k'_i) \prod \frac{dk'_i}{k'_i} \theta(k'_i - k'_{i-1})$$

$$= d\lambda \delta(\lambda - \lambda_0) \delta(K - k'_n) \prod \frac{dk'_i}{k'_i} \theta(k'_i - k'_{i-1}),$$

$$\lambda_0 = \frac{K}{\sum k'_i}$$

L. Van Hove, NPB9 (1969), S. Jadach CPC9 (1975)

Pure bremsstrahlung from $k = G, q, \bar{q}$ line

Iterative solution of the *QCD evolution eqs* from $t_0 \to t$,($t = \ln Q$):

$$x\mathcal{D}_{kk}(t,t_0;x) = e^{-\Phi_k(t,t_0)}\Bigg\{\delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int_{t_0}^{t} dt_i \int_0^1 dz_i \, \mathcal{P}^{\Theta}_{kk}(t_i, z_i)\, \delta_{x=\prod_{i=1}^{n} z_i} \Bigg\},$$

- $\theta_{x>0} = 1$ for $x > y$ and $= 0$ otherwise; $\quad \delta_{x=y} \equiv \delta(x-y)$.
- $\mathcal{P}_{kk}(t,z) \equiv \frac{\alpha(t,z)}{\pi} z P_{kk}(t) = -\mathcal{P}^{\delta}_{kk}(t)\delta_{z=1} + \mathcal{P}^{\Theta}_{kk}(t,z)$.
- $\mathcal{P}^{\Theta}_{kk}(t,z) = \mathcal{P}_{kk}(t,z)\theta_{1-z>\varepsilon(t)}$, the same as in LL DGLAP.
- $\mathcal{P}^{\delta}_{kk}(t) = \int\limits_0^{1-\varepsilon(t)} dz \mathcal{P}^{\Theta}_{kk}(z,t)$, energy sum rule, valid up to NLL.
- Sudakov formfactor: $\Phi_k(t,t_0) = \int_{t_0}^{t} dt' \, \mathcal{P}^{\delta}_{kk}(t')$.
- IR cut $\varepsilon(t) = Q_0/Q$; it is not anymore $<< 1$, as in DGLAP.

HERWIG-evolution – single step

$$\int_x^{1-\varepsilon(t)} dz_i \int_{t_0}^{t} dt_i \, \mathcal{P}^{\Theta}_{kk}(t_i, z_i) = h_k \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \int_0^1 ds_i, i = 1, 2, ..., n$$

$$z_i(y_i) = 1 - \exp(\rho^{-1}(y_i)); \quad \hat{t}_i(s_i) = \hat{t}_0 \left(\frac{\hat{t} + \ln(1 - z_i)}{\hat{t}_0} \right)^{s_i} - \ln(1 - z_i)$$

where

$$\rho(v) \equiv (\hat{t} + v) \ln(\hat{t} + v) - v - v \ln \hat{t}_0 - \hat{t} \ln \hat{t}, \quad \hat{t} \equiv t - t_\Lambda = \ln Q - \ln \Lambda_0$$

IMPORTANT: ρ^{-1} is not analytical! Inversion has to be done numerically. ρ^{-1} will enter the constraint function $\prod z_i$!

The above mapping leads to:

$$x\mathcal{D}_{kk}(t, t_0, x) = e^{-\Phi_k(t,t_0)} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \frac{h_k^n}{n!} \prod_{i=1}^{n} \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \delta_{x=\prod_{i=1}^n z_i(y_i)} \int_0^1 ds_i \right\}$$

The energy constraint

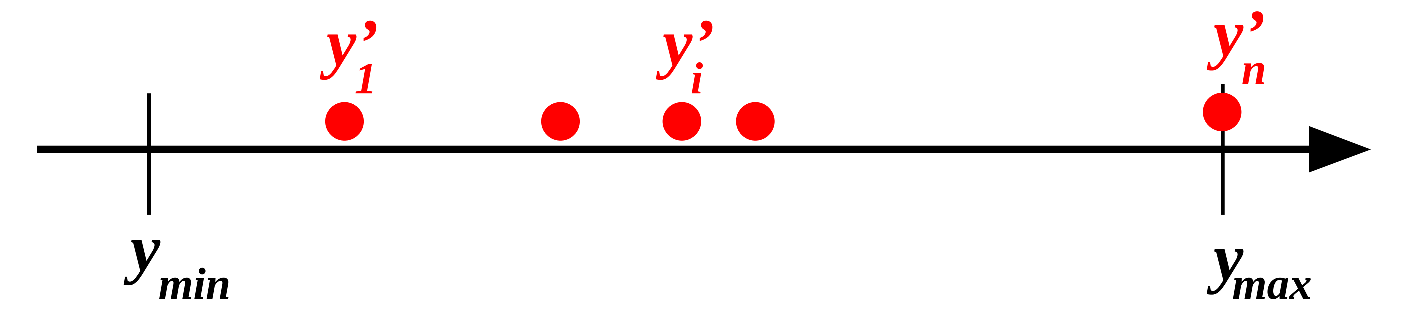
Using symmetry of the integrand we finally trade the ordering in evolution time variables t_i into ordering in the energy variables y_i $(y_0 \equiv 0)$:

$$x\mathcal{D}_{kk}(t,t_0,x) = e^{-\Phi_k(t,t_0)}\Bigg\{\delta_{x=1}+$$

$$+\,x^{-1}\sum_{n=1}^{\infty} h_k^n \prod_{i=1}^{n}\int\limits_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i\;\theta_{y_i>y_{i-1}}\,\delta\left(\ln\frac{1}{x}-\sum_j f(y_j)\right)\int\limits_0^1 ds_i\Bigg\}$$

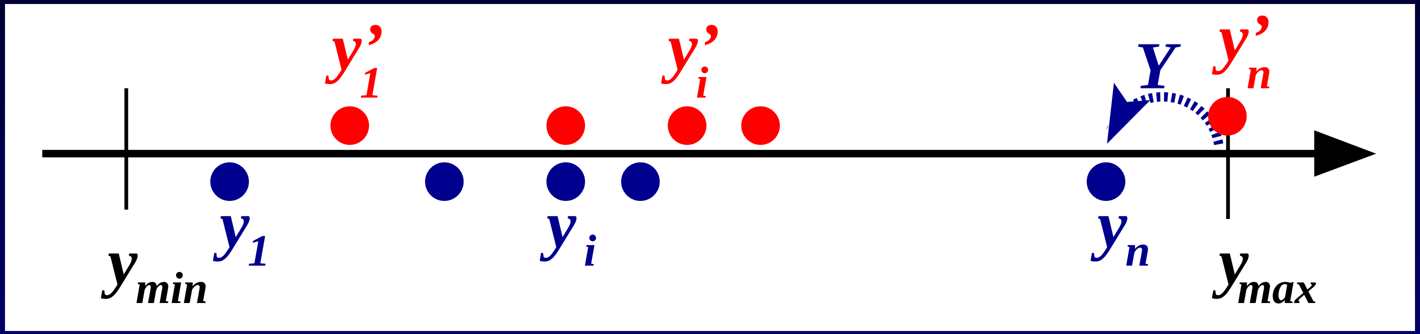
and we are ready to perform the dilatation trick on y_j variables

Linear shift: $y'_i \to y_i = y'_i - Y$



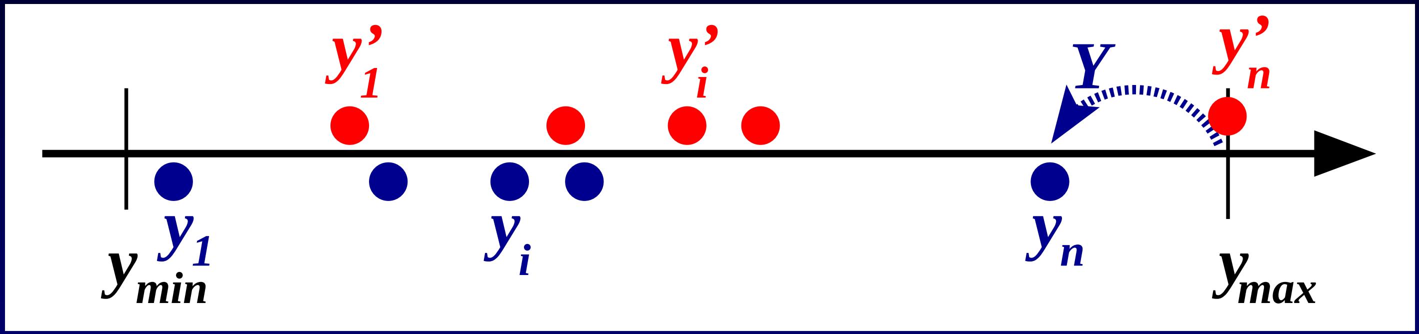
- Begin with y'_i such that one of them $y'_n \equiv y_{\max}$

Linear shift: $y'_i \to y_i = y'_i - Y$



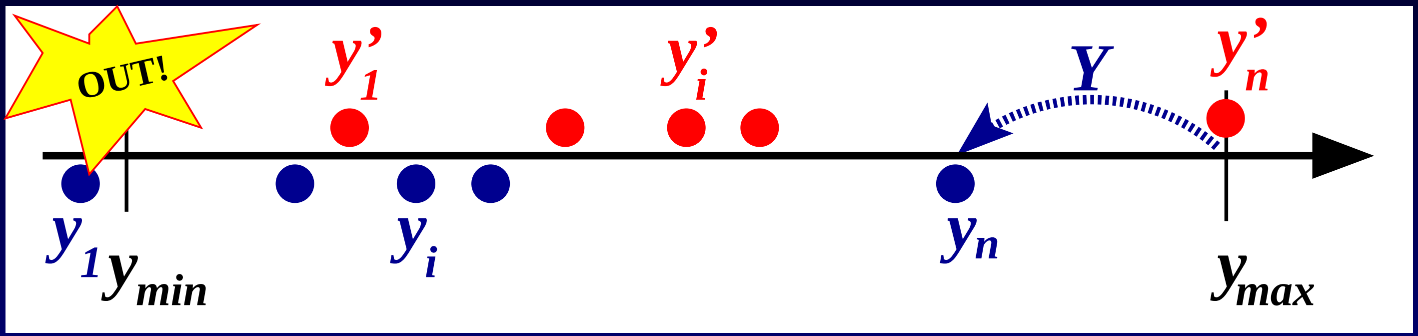
- Begin with y'_i such that one of them $y'_n \equiv y_{\max}$
- Shift $y'_i \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$

Linear shift: $y'_i \to y_i = y'_i - Y(y'_1, y'_2, ..., y'_n)$



- Begin with y'_i such that one of them $y'_n \equiv y_{\max}$
- Shift $y'_i \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y'_i

Linear shift: $y'_i \to y_i = y'_i - Y(y'_1, y'_2, ..., y'_n)$



- Begin with y'_i such that one of them $y'_n \equiv y_{\max}$
- Shift $y'_i \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y'_i
- Sometimes the smallest y'_i is shifted OUT of the phase space, below IR the limit $y_{\min}$. Such an event gets MC weight $w = 0$

Master formula for the bremsstrahlung CMC

$$x\mathcal{D}_{kk}(\tau,\tau_0;x) = e^{(\tau-\tau_0)a_k}\sum_{n=0}^{\infty}\Bigg\{e^{b_k\mathcal{R}(\varepsilon)}\delta_{n=0}\delta_{x=1} + \delta_{n>0}\theta_{1-x>\varepsilon}e^{b_k\mathcal{R}(1-x)}$$
$$\times\frac{b_k x^{\omega_k-1}}{xg(x)}P_n\left(b_k[\mathcal{R}(1-x)-\mathcal{R}(\varepsilon)]\right)\prod_{i=1}^{n}\int_0^1 dr_i\frac{\delta(1-\max r_j)}{n}\int_0^1 ds_i w^{\#}\Bigg\}$$

- Mapping $z_i(y_i) = 1-\exp(\rho^{-1}(y_i))$.
- Mapping $\hat{t}_i(s_i) = \hat{t}_0\left(\frac{\hat{t}+\ln(1-z_i)}{\hat{t}_0}\right)^{s_i} - \ln(1-z_i)$.
- Poisson distribution: $P_n(\lambda) = e^{-\lambda}\lambda^n/n!, \quad \lambda =< n >$.
- $\mathcal{R}(1-z) \equiv \rho(\ln(1-z))$, (implicitly depends on t and t_0).
- MC weight: $\boxed{w^{\#} = w_P\,\frac{xg(x)}{|\partial_Y \ln F(Y_0)|}\,\theta_{y_1'-Y_0>y_{\min}}}$,
- where $g(x) = |\partial_y \ln z(y)|_{z=x} = \frac{1-x}{x}$ is to stabilize the MC weight.
- Ordering of y_i' is here relaxed (to get explicit $1/(n-1)!$ of Poisson).

Prototype Monte Carlo

- The efficiency of the algorithm is very high – about 25% !!
- Last point to be adressed – inclusion of the quark-gluon transitions. This is done by means of hierarchic reorganization, i.e. two-level organization:
  - outer-level: transitions $q \to g \to q \to \dots$
  - inner-level: bremsstrahlung multi-emissions from single q or g

Hierarchical reorganization

Based on well known mathematics of e.g. quantum mechanical interaction picture:

If the evolution kernel can be divided into two parts

$P(t,z) = A(t,z) + B(t,z)$

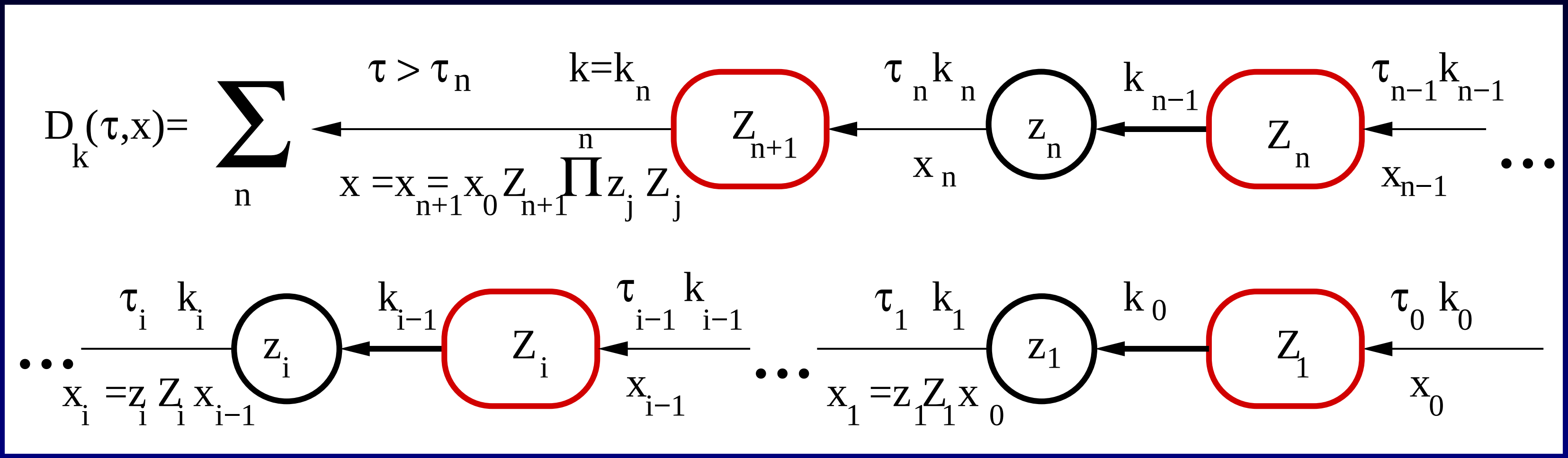
then the solution of evolution equation obeys

$$D(t) = G_A(t,t_0) \exp\left(\int_{t_0}^{t} \tilde{B}(t')dt' \right)_T D(t_0),$$
$$\tilde{B}(t) = G_A^{-1}(t,t_0) B(t) G_A(t,t_0)$$

with G_A solving the evolution for the $A(t,z)$-kernel

$$\partial_t G_A(t,t_0) = A(t) G_A(t,t_0), \quad G_A(t_0,t_0) = 1$$

Two-level hierarchic evolution – picture



Red oval is pure bremsstrahlung segment;

Black circle is Q↔G transition.

Two-level hierarchic – formula ($\mathcal{D}_{kk}$ are also multi-integrals!)

$$D_k(\tau,x) = \int dZ\; dx_0\; \mathcal{D}_{kk}(\tau, Z|\tau_0)\; D_k(\tau_0,x_0)\delta_{x=Zx_0} +$$

$$+\sum_{n=1}^{\infty} \sum_{\substack{k_{n-1}\dots,k_1 k_0 \\ k_n\neq k_{n-1}\neq\dots\neq k_1\neq k_0}} \int_0^1 dZ_{n+1} \Big[\prod_{j=1}^{n} \int_{\tau_0}^{\tau} d\tau_j \theta_{\tau_j>\tau_{j-1}} \int_0^1 dz_j \int_0^1 dZ_j\Big] \int_0^1 dx_0$$

$$\times\, \mathcal{D}_{kk}(\tau, Z_{n+1}|\tau_n)\; \Big[\prod_{i=1}^{n} \mathbf{P}^{\Theta}_{k_i k_{i-1}}(z_i)\; \mathcal{D}_{k_{i-1}k_{i-1}}(\tau_i, Z_i|\tau_{i-1})\Big]$$

$$\times\, D_{k_0}(\tau_0,x_0)\delta\Big(x - x_0 Z_{n+1}\prod_{i=1}^{n} z_i Z_i\Big), \qquad k\equiv k_n,$$

$$\mathcal{D}_{kk}(\tau, Z|\tau_0) = \frac{e^{\Phi_k(\tau,\tau_0)}}{Z}\Big\{\delta_{Z=1}$$

$$+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\int_{\tau_0}^{\tau} d\tau_i\; \theta_{\tau_i>\tau_{i-1}} \int_0^1 dz_i\; z_i \mathbf{P}^{\Theta}_{kk}(z_i)\delta_{Z=\prod_{i=1}^{n} z_i}\Big\}$$

CMC for full DGLAP → top level integrand for FOAM

- Neglecting temporarily $w^{\#}$ inside the segments $\mathcal{D}_{kk}$, gluon bremsstrahlung sub-level, we can integrate/sum analytically over all variables of the sub-level
- The overall (energy) x-constraint δ-function is eliminated using $\int dx_0$
- We are left with $3n+1$-dim. integrals (n= No. of flavor changes) of flavor-changing super-level, the INTEGRAND FOR FOAM is:

$$D_k(\tau,x) = x^{-1}\int_x^1 dZ \int_0^{R(x)} dR_1\ Z(R_1)^{\omega_k-2} e^{a_k(\tau-\tau_0)}\ x_0 D_k(\tau_0,x_0) +$$

$$+\, x^{-1}\sum_{n=1}^{\infty} \sum_{\substack{k_{n-1}\dots,k_1 k_0 \\ k_n\neq k_{n-1}\neq\dots\neq k_0}} \left[\prod_{j=1}^{n}\int_{\tau_0}^{\tau} d\tau_j \theta_{\tau_j>\tau_{j-1}}\right] \int_0^{R(x)} dR_{n+1} Z(R_{n+1})^{\omega_k-2} e^{a_k(\tau-\tau_n)}$$

$$\times \left[\prod_{i=1}^{n}\int_{x_{i+1}}^{1} dz_i\ \mathbf{P}^{\Theta}_{k_i k_{i-1}}(z_i) \int_0^{R(x_{i+1}/z_i)} dR_i\ Z(R_i)^{\omega_{k_{i-1}}-2} e^{a_{k_{i-1}}(\tau_i-\tau_{i-1})}\right]$$

$$\times\, x_0 D_{k_0}(\tau_0,x_0),$$

FOAM 1.02M, integrated with ROOT! S.Jadach & P.Sawicki

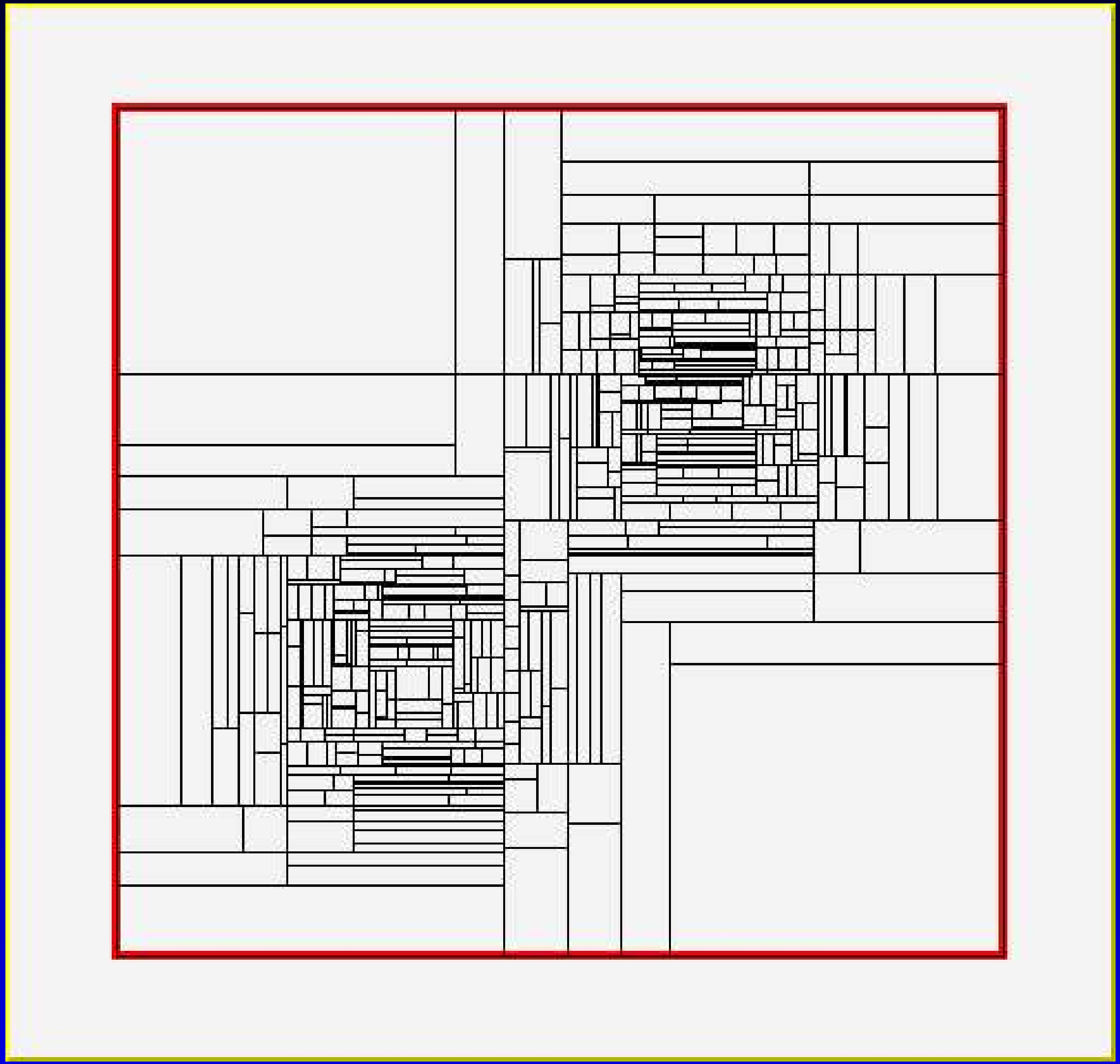
What is FOAM for?

- Suppose you want to generate randomly points (vectors) according to an arbitrary probability distribution in n dimensions. FOAM can do it for you! Even for distributions with strong peaks and discontinuous!
- FOAM generates random points with weight one or with variable weight.
- FOAM is capable to integrate using effi cient "adaptive" Monte Carlo method.

How does it work?

- It creates hyper-rectangular "foam of cells", which is more dense around its peaks. See 2-d example of 1000 cells for doubly peaked distribution:

FOAM 1.02M, example of grid



CMC algorithm of type I, full DGLAP

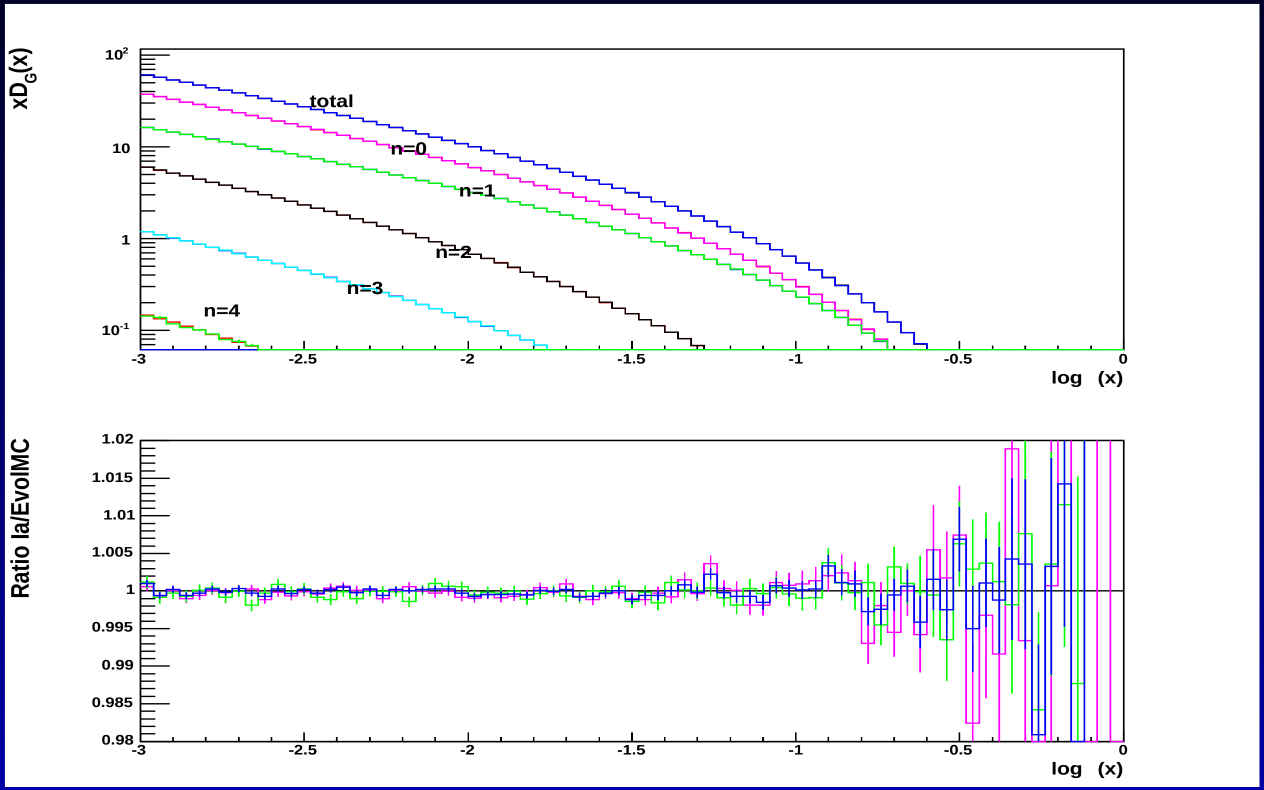
CMC algorithm description

- Generate super-level variables n, k_i, τ_i Z_i and z_i using Foam general purpose MC tool.
- Limiting no. of flavor transition ($G \to Q$ and $Q \to G$) to $n = 0, 1, 2, 3$ is enough, for the $\sim 0.2\%$ precision.
- For each pure gluon bremsstralung segment defined by Z_i and (τ_i, τ_{i-1}), $i = 1, 2, ..., n + 1$, gluon emission variable $(z_j^{(i)}, \tau_j^{(i)},\ j = 1, 2, ..., n^{(i)}$, are generated using previously described dedicated CMC.
- Weight$= 1$ events available!

Numerical tests

- In next slides we show example of numerical results from such a non-Markovian CMC `EvolCMC` for “evolution” ranging from $Q = 1$GeV to $Q = 1$TeV, $x > 10^{-3}$,
- They are compared with the results of the Markovian unconstrained evolution of our own `EvolFMC`
- `EvolFMC` was previously x-checked with `QCDnum16` and `APCHEB`
- The agreement of Nonmarkovian `EvolCMC` and Markovian `EvolFMC` is excelent, $\sim 0.25\%$.

Test of non-Markovian Constrained MC; DGLAP



$n = 0$: $G \to G$

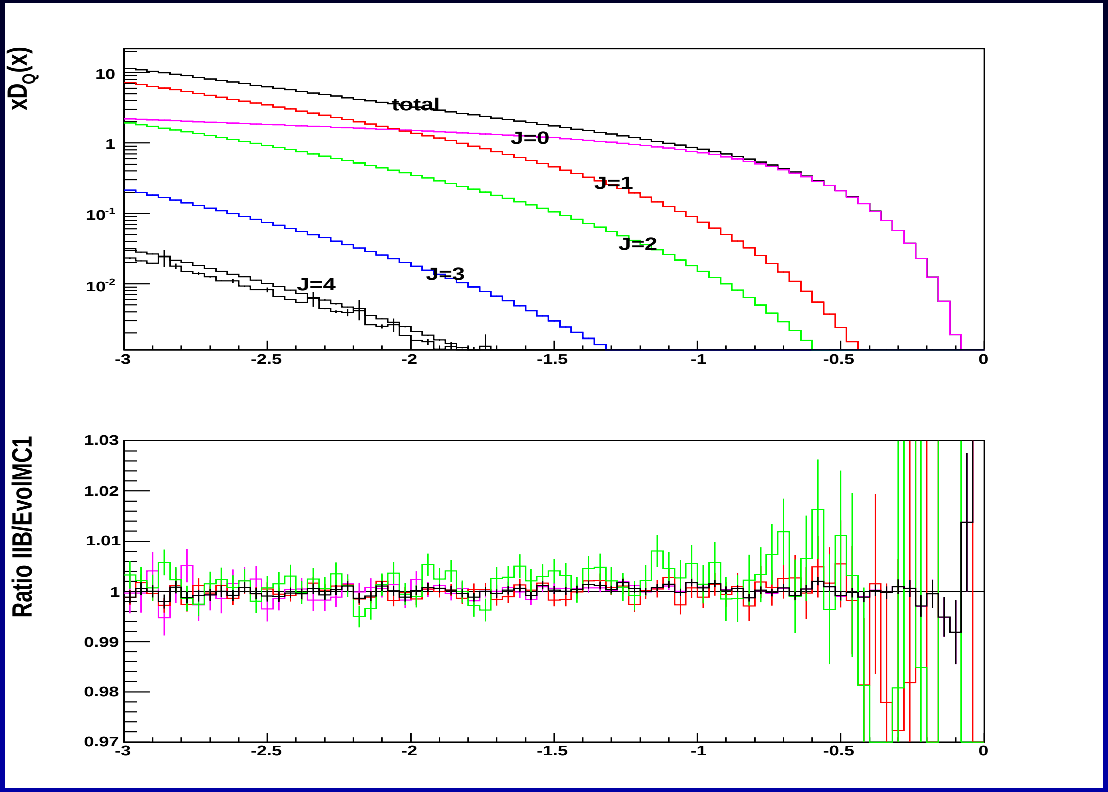
$n = 1$: $Q \to G$ and any no. of gluon emissions out of Q and G,

$n = 2$: $G \to Q \to G$, etc. $n = 3$: $Q \to G \to Q \to G$, etc.

$n = 4$: $G \to Q \to G \to Q \to G$, etc.

"Total" is the sum of $n = 0, 1, 2, 3, 4$. Evolution from proton at 1GeV to 1TeV. Non-Markovian CMC (EvolCMC) agrees with unconstrained Markovian MC (EvolFMC) to $\sim$ 0.2%!

Test of non-Markovian Constrained MC; HERWIG



$J = 0$: $Q \to Q$

$J = 1$: $G \to Q$ and any no. of gluon emissions out of Q and G,

$J = 2$: $G \to Q \to G \to Q$, etc. $J = 3$: $G \to Q \to G \to Q$, etc.

$J = 4$: $Q \to G \to Q \to G \to Q$, etc.

"Total" is the sum of $n = 0, 1, 2, 3, 4$. Evolution from proton at $Q = 1$GeV up to 1TeV. Non-Markovian CMC agrees with Markovian MC to $\sim 0.2\%$!

NEW (June 2005) and UNPUBLISHED!!!

Summary and outlook

- It is demonstrated using prototype program that the Constrained MC works in practice for the HERWIG evolution and for standard LL DGLAP with Quark−Gluon transitions.
- Still to be done soon: Including the rest of NLL corrections into CMC, mapping into full $d = 4$ phase space, and more...
- How to exploit this new technology in the construction of the full scale parton shower MC? To be seen...
- Most likely application: unifi ed approach with unintegrated PDFs (CCFM style) and parton shower MC, up to NLL.