Top and Bottom at Threshold:
Recent Results

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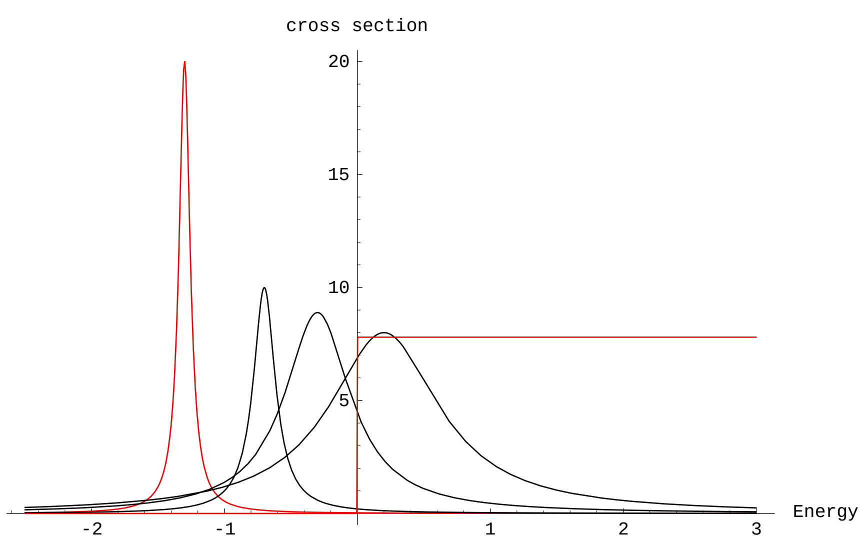
I. Motivation
II. Non-Relativistic Effective Theory
III. Non-Relativistic Renormalization Group
IV. (Some) Phenomenological Applications
V. Summary

I. Motivation

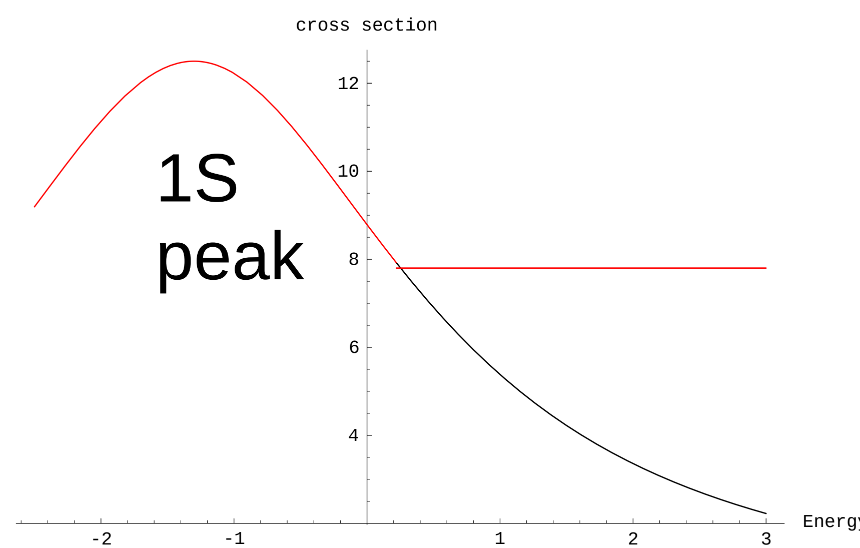
Bound states of heavy quarks

- $(Q\bar{Q})$ bound state; $J/\Psi(1^3S_1) \sim (c\bar{c}) \quad \eta_c(1^1S_0) \sim (c\bar{c})$
  $\Upsilon(1^3S_1) \sim (b\bar{b}) \quad \eta_b(1^1S_0) \sim (b\bar{b})$
- Energy levels ➪ quark mass
- Decay rate ➪ α_s
- $(t\bar{t})$: too heavy; no bound state; $e^+e^- \to Q\bar{Q}$

" $e^+e^- \to b\bar{b}$ "



" $e^+e^- \to t\bar{t}$ "



Why theoretically interesting?

- based on first principles of QCD
- highly non-trivial multi-scale dynamics
  m_Q, $|\boldsymbol{p}| \sim m_Q v$, $E \sim m_Q v^2$ [v: velocity of quark Q]
- high-order results available

Why theoretically interesting?

- based on first principles of QCD
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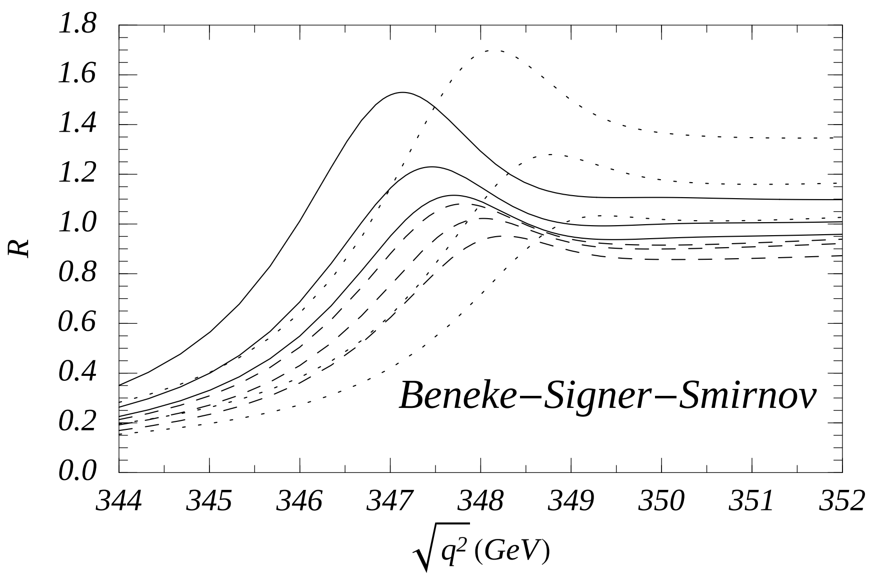
NNLO complete; example: $(t\bar{t})$ production @ threshold:



δE_n : [A. Pineda, F.J. Yndurain'98]

$\delta\psi_n$:[A. Penin, A.A. Pivovarov'98; K. Melnikov, Yelkhovsky'98]

[A.H. Hoang, M. Beneke, K. Melnikov, T. Nagano, A. Ota, A.A. Penin, A.A. Pivovarov, A. Signer, V.A. Smirnov, Y. Sumino, T. Teubner, O. Yakovlev, A. Yelkhovsky'00]

slow convergence of perturbation theory

⇨ needed: N^{3}LO analysis; NNLL (resummation)

II. Non-Relativistic Quanten Chromodynamics

Degrees of freedom

Analogy to H atom: find theory where
equation of motion $\sim$ Schrödinger equation

➪ Degrees of freedom

"potential" quarks $E \sim m_q v^2; \quad |\vec{p}| \sim m_q v$

"ultrasoft" gluons $E \sim m_q v^2; \quad |\vec{p}| \sim m_q v^2$

Degrees of freedom

Analogy to H atom: find theory where
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➪ Degrees of freedom

- "potential" quarks $E \sim m_q v^2; \quad |\vec{p}| \sim m_q v$
- "ultrasoft" gluons $E \sim m_q v^2; \quad |\vec{p}| \sim m_q v^2$

Degrees of freedom present in QCD
but not in Schrödinger-like theory:

- "hard" modes $E \sim m_q; \quad |\vec{p}| \sim m_q$
- "soft" modes $E \sim m_q v; \quad |\vec{p}| \sim m_q v$

Aim: Construct theory with dynamical potential quarks and
ultrasoft gluons as degrees of freedom
hard and soft modes at most in coefficient functions

Construction of pNRQCD

scales: mass, m: hard $\gg$ momentum, mv: soft $\gg$ energy, mv^2: ultrasoft $\gg$ $\Lambda_{\rm QCD}$

potential quarks: $\begin{cases} E_{\vec{p}} \sim mv^2 \\ |\vec{p}\,| \sim mv \end{cases}$ $\quad \frac{1}{E_{\vec{p}} - \frac{\vec{p}^{\,2}}{2m}}$

ultrasoft gluons: $\begin{cases} E_{\vec{k}} \sim mv^2 \\ |\vec{k}\,| \sim mv^2 \end{cases}$ $\quad \frac{1}{E_{\vec{k}}^2 - \vec{k}^{\,2}}$

QCD $\longrightarrow$ NRQCD $\longrightarrow$ pNRQCD (potential non-relativistic QCD)

$\uparrow$ integrate out hard scale "m" from QCD

[Caswell,Lepage'86; Bodwin,Braaten,Lepage'95]

$\uparrow$ integrate out all scales from NRQCD except potential quarks and ultrasoft gluons

[Pineda,Soto'98; Brambilla,Pineda,Soto,Vairo'00]

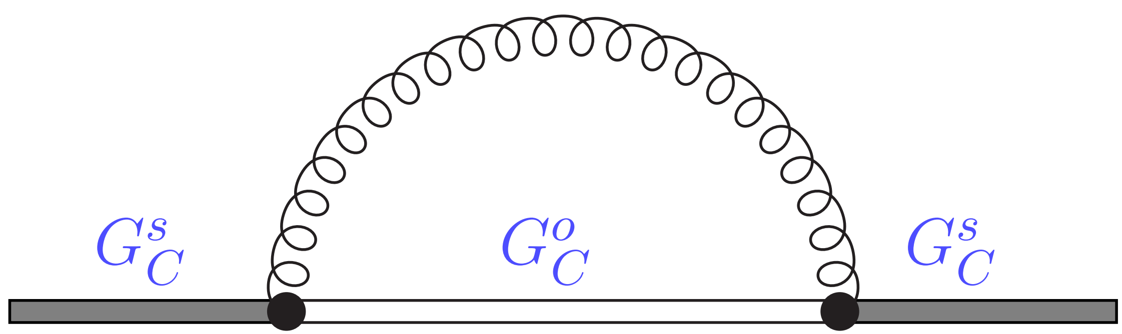
pNRQCD: $\mathcal{L}_{\rm pNRQCD} \sim \phi^\dagger \left(i\partial_0 - V(\vec{r}\,)\right)\phi + \ldots$ ➪ $i\partial_0\phi = \left(\frac{\vec{p}^{\,2}}{m} + V(\vec{r}\,)\right)\phi$ ➪ Schrödinger equation

Rad. and absorption of ultrasoft gluons

- pNRQCD still contains dynamical degrees of freedom: potential quarks and ultrasoft gluons
- propagation of quark-antiquark pair:
  - colour-singlet, colour-octet
  - non-relativistic Green function of Schrödinger equation: $(-\frac{\Delta_r}{m_q} + V_C^{s,o} - E)G_C^{s,o}(\vec{r}, \vec{r}\,', E) = \delta(\vec{r} - \vec{r}\,')$

$$V_C^s = -C_F \frac{\alpha_s}{r}, \quad V_C^o = (C_A/2 - C_F)\frac{\alpha_s}{r}$$

- quark-antiquark–us-gluon-vertex: $g_s \vec{r} \cdot \vec{E}$
-



Potentials/effective Hamiltonian

- Potentials $\leftrightarrow$ effective Hamiltonian known to N^{3}LO

  except 1 constant: a_3, 3-loop static potential

- Energy level: $E_n = E_n^C + \delta E_n^{(1)} + \delta E_n^{(2)} + \delta E_n^{(3)} + \ldots$

  ➪ bottom quark mass: $\delta m_b = 70$ MeV [Penin,Steinhauser'02]
  top quark mass: $\delta m_t = 80$ MeV

  $\delta E_n^{(3)}$: [Penin,Smirnov,MS'05;Beneke,Kiyo,Schuller'05]

- Wave function:

$$|\psi_n(0)|^2 = |\psi_n^C(0)|^2 \left(1 + \delta\psi_n^{(1)} + \delta\psi_n^{(2)} + \delta\psi_n^{(3)} + \ldots\right)$$

$$\delta\psi_n^{(3)} = K_2 \ln^2 \alpha_s + K_1 \ln \alpha_s + K_0\Big|_{\beta_0^3} + K_0\Big|_{\rm rem}$$

K_2 : [Kniehl,Penin'00; Manohar,Stewart'01]

K_1 : [Kniehl,Penin,Smirnov,MS'02;Hoang'04]

$K_0\Big|_{\beta_0^3}$: [Penin,Smirnov,MS'05;Beneke,Kiyo,Schuller'05]

- Production: large-β_0 N^{3}LO corrections $\longrightarrow$ see Y. Kiyo's talk

III. Non-Relativistic Renormalization Group

Simple Example

$$R = \frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)} \stackrel{m_q=0}{=} 3\left[1 + \frac{\alpha_s(\mu)}{\pi} r_{10} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(r_{20} + r_{21}\ln\frac{\mu^2}{s}\right)\right.$$
$$\left. + \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 \left(r_{30} + r_{31}\ln\frac{\mu^2}{s} + r_{32}\ln^2\frac{\mu^2}{s}\right) + \ldots\right]$$

- consider: $\sqrt{s} \approx m_\tau \approx 1.8$ GeV
- $\mu^2 = s$ ⇨ no large logarithms
- Use RGE
  $$\mu^2 \frac{\mathrm{d}\alpha_s(\mu)}{\mathrm{d}\mu^2} = -\frac{\alpha_s^2(\mu)}{\pi}\left[\beta_0 + \frac{\alpha_s(\mu)}{\pi}\beta_1 + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \beta_2 + \ldots\right]$$
  to compute $\alpha_s(\sqrt{s})$ starting from $\alpha_s(M_Z) = 0.118$
- Large logarithms are resummed in coupling
- Note: β_0 from divergences of 1-loop diagrams

Nonrelativistic Renormalization Group (NRG)

Consider spin-dependent part of N^3LO eff. Hamiltonian:

$$\delta\mathcal{H}_{\rm spin} = D_{S^2} \underbrace{\frac{4C_F\pi}{3m_q^2}\boldsymbol{S}^2}_{\text{NNLO}}, \qquad \boldsymbol{S} = \frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2},$$

NLO result:

$$D_{S^2} = \alpha_s \left[1 + \left(-\frac{5}{9}T_F n_l + \frac{3}{2}(1-\ln 2)T_F + \frac{11C_A - 9C_F}{18} + \frac{7}{4}C_A \ln\frac{\mu}{m_q}\right)\frac{\alpha_s}{\pi}\right]$$

[Gupta,Radford,Repko'82; Pantalone,Tye,Ng'86; Buchmüller,Ng,Tye'94]

Aim: resum $\ln\frac{\mu}{m_q}$ terms ⇨ NLL

1) Derive RGE for coupling ($\equiv D_{S^2}$)

2) solve RGE

Nonrelativistic Renormalization Group

$$\delta\mathcal{H}_{\rm spin} = D_{S^2}\frac{4C_F\pi}{3m_q^2}\boldsymbol{S}^2, \qquad \boldsymbol{S} = \frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2},$$

[Pineda'02]

- Consider soft, potential and ultrasoft running in effective theory,
  i.e.: consider corresponding UV divergences
- LL: only soft running: $\mu_s \frac{d}{d\mu_s} D_{S^2} = \alpha_s c_F^2 \gamma_s$
- NLL: in addition potential and ultrasoft contributions
- analytical calculation
- NLL result for D_{S^2} ⇨ NLL for energy level: $E_{\rm hfs}^{\rm NLL}$

Nonrelativistic Renormalization Group

$$
\begin{aligned}
E_{\rm hfs}^{\rm NLL} \quad = \quad & \frac{C_F^4 \alpha_s^4(\mu) m_b}{3} \left\{ \frac{27}{14} y^{-1} - \frac{13}{14} y^{-\frac{18}{25}} + \frac{\alpha_s(m_b)}{\pi} \left[\left(\frac{1037}{224} + \frac{405086361761\,\pi^2}{25617160800} - \frac{3}{4}\ln 2 \right) y^{-1} \right.\right. \\
& - \frac{1024\,\pi^2}{143} y^{-\frac{39}{50}} - \left(\frac{102973}{26250} + \frac{184336\,\pi^2}{25725} \right) y^{-\frac{18}{25}} + \frac{1024\,\pi^2}{675} y^{-\frac{1}{2}} + \frac{671\,\pi^2}{1029} y^{-\frac{11}{25}} - \frac{3\,\pi^2}{23} y^{-\frac{2}{25}} \\
& + \left(-\frac{13427921}{1260000} + \frac{88057\,\pi^2}{151200} \right) y^{\frac{7}{25}} + \frac{4\,\pi^2}{41} y^{\frac{16}{25}} + \frac{1377}{56} - \frac{1253587\,\pi^2}{227500} - \frac{629\,\pi^2}{7500} y \\
& - \frac{2873\,\pi^2}{7182} y^{\frac{32}{25}} {}_2F_1\left(\frac{57}{25}, 1; \frac{82}{25}; \frac{y}{2} \right) + \frac{2873\,\pi^2}{3591} y^{-1} {}_2F_1\left(1, 1; \frac{82}{25}; -1 \right) + \left(\frac{675}{28} - \frac{533}{42} y^{\frac{7}{25}} \right) \\
& \times \ln\left(\frac{\mu}{\bar{\mu}} \right) + \frac{85248\,\pi^2}{30625} y^{-1} \ln y + \left(-\frac{45834}{4375} y^{-1} \right. \\
& \left.\left.\left. + \frac{21216}{4375} - \frac{2873}{1575} y^{\frac{7}{25}} + \frac{243}{1250} y \right) \pi^2 \ln(2-y) \right] \right\}
\end{aligned}
$$

$$y = \alpha_s(\mu)/\alpha_s(m_b), n = 1, N_c = 3, n_l = 4$$

[Kniehl,Penin,Pineda,Smirnov,MS'04]

Nonrelativistic Renormalization Group

$$E_{\rm hfs}^{\rm NLL} = \frac{C_F^4 \alpha_s^4 m_b}{3} \Big[1 + \frac{\alpha_s}{\pi} \left(c_1^{(1)} \ln \alpha_s \quad + c_0^{(1)} \right)$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^2 \left(c_2^{(2)} \ln^2 \alpha_s + c_1^{(2)} \ln \alpha_s + c_0^{(2)} \right)$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3 \left(c_3^{(3)} \ln^3 \alpha_s + c_2^{(3)} \ln^2 \alpha_s + c_1^{(3)} \ln \alpha_s + c_0^{(3)} \right) + \ldots \Big]$$

LL — “Leading Log“

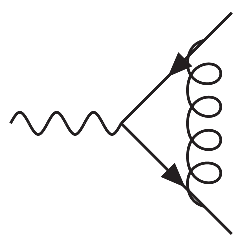
NLL — “Next-to-Leading Log“

Next step: use NLL result for $\delta\mathcal{H}_{\rm spin}$ to resum spin-dependent part of production and annihilation processes

NRG: production and annihilation

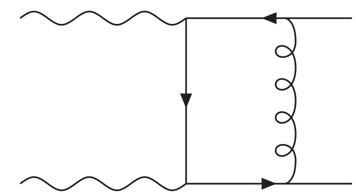
1-photon process

$$\vec{j} = c_v(\mu)\psi^\dagger\vec{\sigma}\chi + \frac{d_v(\mu)}{6m_q^2}\psi^\dagger\vec{\sigma}\vec{D}^2\chi + \ldots$$



2-photon process

$$O_{\gamma\gamma} = c_{\gamma\gamma}(\mu)\psi^\dagger\chi + \frac{d_{\gamma\gamma}(\mu)}{6m_q^2}\psi^\dagger\vec{D}^2\chi + \ldots$$



$c_v(\ln\mu/m_q)$ and $c_{\gamma\gamma}(\ln\mu/m_q)$ known to 2-loop order

[Czarnecki,Melnikov'98; Beneke,Signer,Smirnov'98; Czarnecki,Melnikov'01]

$$\mathcal{R}_q = \frac{\sigma(e^+e^-\to\mathcal{Q}(n^3S_1))}{\sigma(\gamma\gamma\to\mathcal{Q}(n^1S_0))} = \frac{\Gamma(\mathcal{Q}(n^3S_1)\to e^+e^-)}{\Gamma(\mathcal{Q}(n^1S_0)\to\gamma\gamma)} = \frac{c_s^2(\mu)}{3Q_q^2}\frac{|\psi_n^v(0)|^2}{|\psi_n^p(0)|^2} + \mathcal{O}(\alpha_s v^2)$$

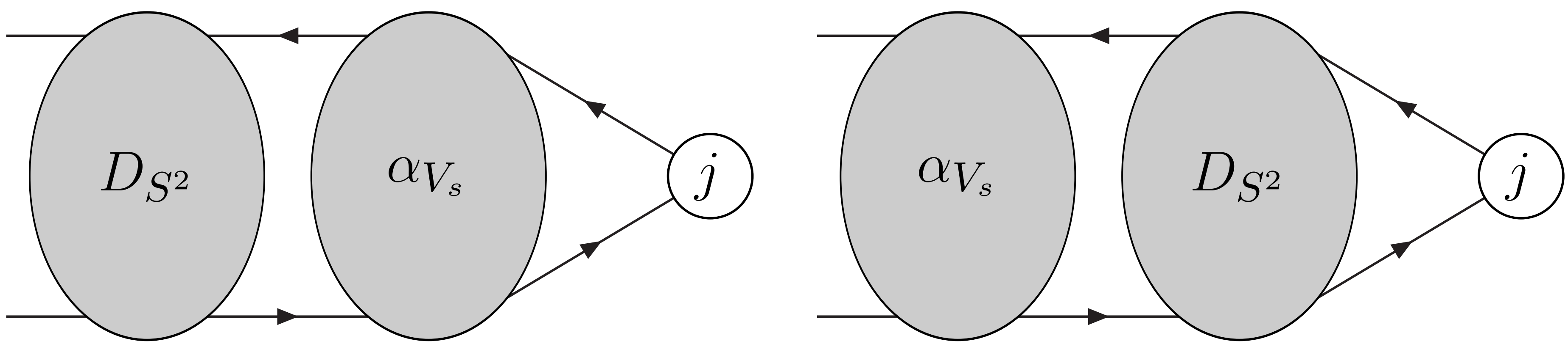
$\mu \approx \mu_{\text{soft}}$ ⇨ large logarithms in $c_s(\mu)$: $\ln\frac{\mu}{m_q}$ ⇨ resum!

⇨ $c_s(\mu)$ to NNLL

NRG: production and annihilation

$$\mathcal{R}_q = \frac{c_s^2(\mu)}{3Q_q^2}\frac{|\psi_n^v(0)|^2}{|\psi_n^p(0)|^2} + \ldots \qquad \delta\mathcal{H}_{\rm spin} = D_{S^2}\frac{4C_F\pi}{3m_q^2}\boldsymbol{S}^2$$

RGE: $\displaystyle \frac{d\ln c_s(\mu)}{d\ln\mu} = \frac{C_F^2}{3}\left[2\alpha_{V_s}(\mu)D_{S^2}(\mu) + \alpha_s(\mu)\frac{dD_{S^2}(\mu)}{d\ln\mu}\right]$



NRG: production and annihilation

$$\mathcal{R}_q = \frac{c_s^2(\mu)}{3Q_q^2} \frac{|\psi_n^v(0)|^2}{|\psi_n^p(0)|^2} + \ldots \qquad \delta\mathcal{H}_{\rm spin} = D_{S^2} \frac{4C_F\pi}{3m_q^2} \boldsymbol{S}^2$$

RGE: $$\frac{d\ln c_s(\mu)}{d\ln\mu} = \underbrace{\frac{C_F^2}{3}\left[2\alpha_{V_s}(\mu) D_{S^2}(\mu)\right.}\left. + \alpha_s(\mu)\frac{dD_{S^2}(\mu)}{d\ln\mu}\right]$$

- LL: ⇨ $\frac{2}{3}C_F^2\alpha_s^2$
- agrees with known 2-loop result

NRG: production and annihilation

$$\mathcal{R}_q = \frac{c_s^2(\mu)}{3Q_q^2} \frac{|\psi_n^v(0)|^2}{|\psi_n^p(0)|^2} + \ldots \qquad \delta\mathcal{H}_{\rm spin} = D_{S^2} \frac{4C_F\pi}{3m_q^2} \boldsymbol{S}^2$$

RGE: $\frac{d\ln c_s(\mu)}{d\ln\mu} = \frac{C_F^2}{3}\left[2\alpha_{V_s}(\mu) D_{S^2}(\mu) + \alpha_s(\mu)\frac{dD_{S^2}(\mu)}{d\ln\mu}\right]$

$D_{S^2}(\mu)$ needed to NLL $\qquad V_C = \frac{4\pi C_F \alpha_{V_s}}{\vec{k}^{\,2}} \qquad \alpha_{V_s} = \alpha_s(\mu) + \ldots$

Solution: $c_s(\mu) = c_s(m_q) e^{\alpha_s(m_q)\Gamma_{c_s}^{\rm NLL}(\mu) + \alpha_s^2(m_q)\Gamma_{c_s}^{\rm NNLL}(\mu) + \cdots}$

$$\Gamma_{c_s}^{\rm NLL}(\mu) = \frac{2\pi C_F^2(2\beta_0 - 7C_A)}{3(\beta_0 - 2C_A)^2}\left(1 - z^{\beta_0 - 2C_A}\right) - \frac{2\pi C_F^2 C_A}{\beta_0(\beta_0 - 2C_A)} \ln(z^{\beta_0})$$

$$\Gamma_{c_s}^{\rm NNLL}(\mu) = \pi^2 \sum_{i=1}^{19} A_i f_i(z) \qquad z = \left(\alpha_s(\mu)/\alpha_s(m_q)\right)^{1/\beta_0}$$

[Penin,Pineda,Smirnov,MS'04]

Result for $\Gamma_{c_s}^{\rm NNLL}$

$\Gamma_{c_s}^{\rm NNLL}(\mu) = \pi^2 \sum_{i=1}^{19} A_i f_i(z)$ $\qquad z = (\alpha_s(\mu)/\alpha_s(m_q))^{1/\beta_0}$

$$f_1(z) = z^{3\beta_0-2C_A}\,{}_3F_2\left(1, 3-\frac{2C_A}{\beta_0}, 3-\frac{2C_A}{\beta_0}, 4-\frac{2C_A}{\beta_0}, 4-\frac{2C_A}{\beta_0}, \frac{z^{\beta_0}}{2}\right)\,,$$

$$f_2(z) = z^{3\beta_0-2C_A}\,{}_2F_1\left(3-\frac{2C_A}{\beta_0}, 1, 4-\frac{2C_A}{\beta_0}, \frac{z^{\beta_0}}{2}\right)\,, \quad f_3(z) = z^{2\beta_0-(25C_A)/6}\,,$$

$$f_4(z) = z^{2\beta_0-4C_A}\,, \quad f_5(z) = z^{2\beta_0-3C_A}\,, \quad f_6(z) = z^{2\beta_0-2C_A}\,, \quad f_7(z) = z^{2\beta_0-C_A}\,,$$

$$f_8(z) = z^{\beta_0-(13C_A)/6}\,, \quad f_9(z) = z^{\beta_0-2C_A}\,, \quad f_{10}(z) = z^{\beta_0}\,, \quad f_{11}(z) = \ln(z)\,,$$

$$f_{12}(z) = \ln^2(z)\,, \quad f_{13}(z) = \ln\left(2-z^{\beta_0}\right)\,, \quad f_{14}(z) = z^{2\beta_0-2C_A}\ln\left(2-z^{\beta_0}\right)\,,$$

$$f_{15}(z) = z^{\beta_0}\ln\left(2-z^{\beta_0}\right)\,, \quad f_{16}(z) = z^{2\beta_0}\ln\left(2-z^{\beta_0}\right)\,, \quad f_{17}(z) = 1\,,$$

$$f_{18}(z) = z^{2\beta_0}\,, \quad f_{19}(z) = \mathsf{Li}_2\left(z_0^{\beta}/2\right)\,.$$

Result for $\Gamma_{c_s}^{\rm NNLL}$

$\Gamma_{c_s}^{\rm NNLL}(\mu) = \pi^2 \sum_{i=1}^{19} A_i f_i(z)$ $\qquad z = (\alpha_s(\mu)/\alpha_s(m_q))^{1/\beta_0}$

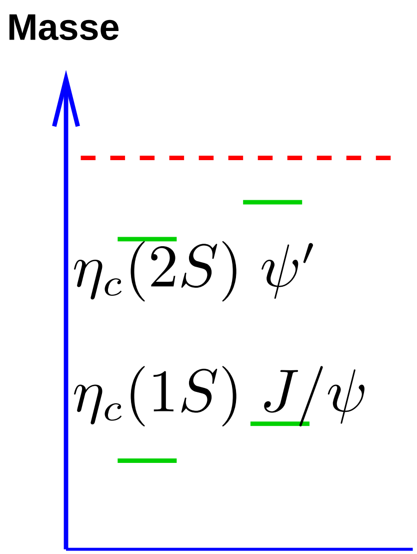
$$
\begin{aligned}
A_1 &= \frac{C_F^3(-2C_A^2 - 6C_AC_F - 4C_F^2)(C_A - 8n_lT_F)}{3(5C_A - 4n_lT_F)(9C_A - 4n_lT_F)^2(2C_A - n_lT_F)}\,,\\
A_2 &= -\frac{C_F^3(C_A^2 + 3C_AC_F + 2C_F^2)(C_A - 8n_lT_F)}{4(5C_A - 4n_lT_F)(9C_A - 4n_lT_F)(2C_A - n_lT_F)^2}\,,\\
\dots
\end{aligned}
$$

- analytical result
- reproduces known $\alpha_s^3 \ln\alpha_s$ result for $|\psi(0)|^2$

IV. Phenomenological applications: Top, Bottom, Charm

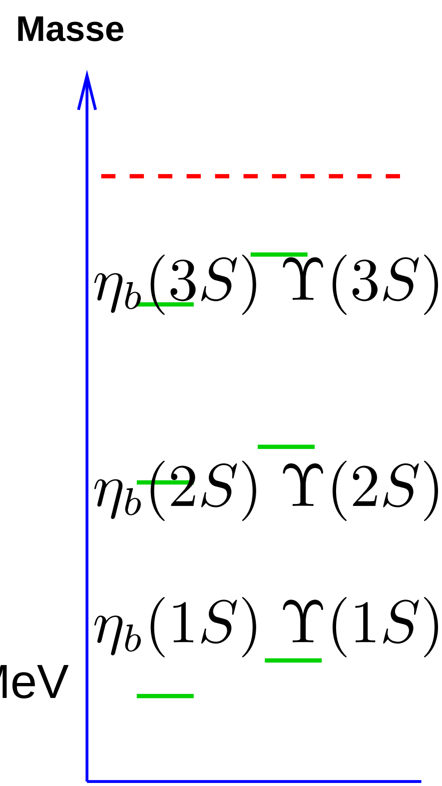
Hyperfinesplitting

Charmonium



$M(\eta_c(1S)) = 2979.2(1.3)$ MeV
$M(J/\psi) = 3096.87(4)$ MeV

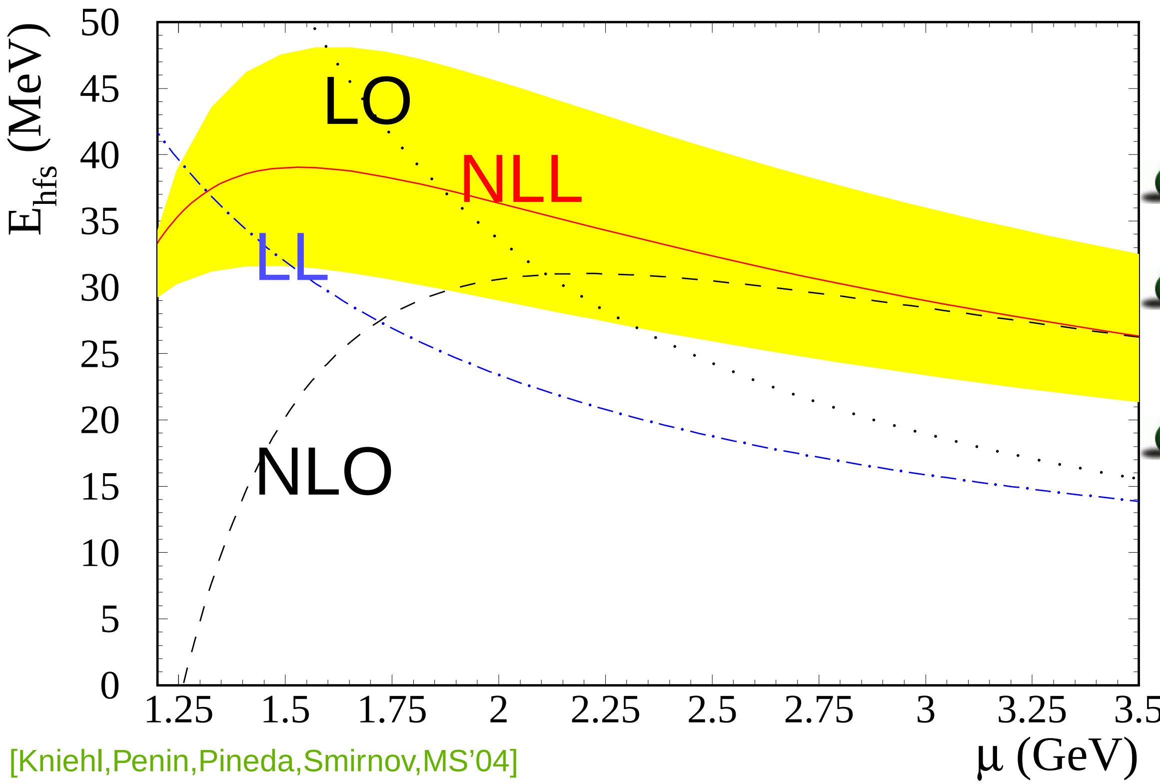
Bottomonium



$M(\Upsilon(1S)) = 9460.30(26)$ MeV
$\eta_b(1S)$ not yet observed

use $M(\Upsilon(1S))$ and $E_{\rm hfs}^{\rm NLL}$ to predict $M(\eta_b(1S))$

Prediction of M_{η_b}



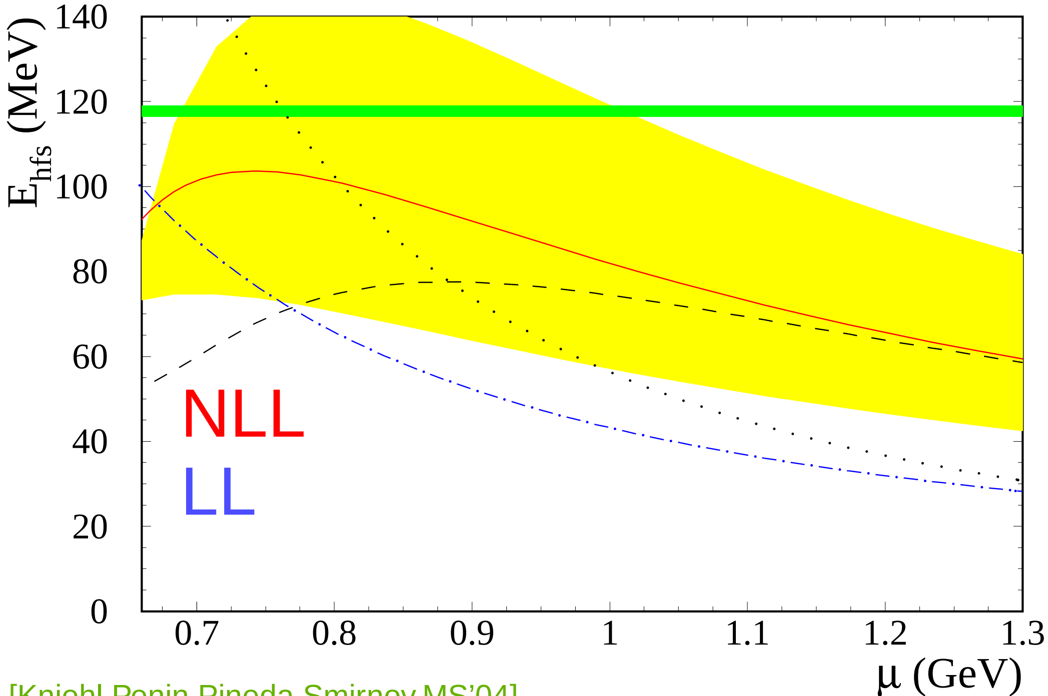
[Kniehl,Penin,Pineda,Smirnov,MS'04]

- $\alpha_s(M_Z) = 0.118 \pm 0.003$
- significance reduction of μ dependence
- Comparison with lattice:

SESAM	33.4 MeV	[N. Eicker et al.'98]
CP-PACS	33.2 MeV	[T. Manke et al.'00]

⇨ $M(\eta_b) = 9420 \pm 10(\mathrm{th})^{+9}_{-8}(\alpha_s)$ MeV

E_{hfs}: charm system

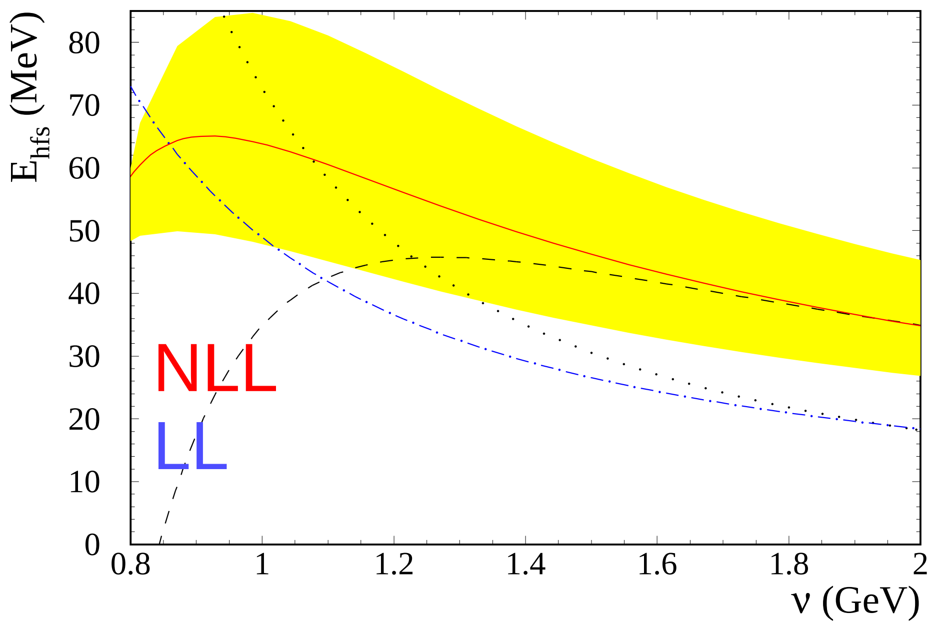


[Kniehl,Penin,Pineda,Smirnov,MS'04]

➪ $M(J/\psi) - M(\eta_c) \approx 104$ MeV

Experiment: 117.7 ± 1.3 MeV

$E_{\rm hfs}$: B_c system

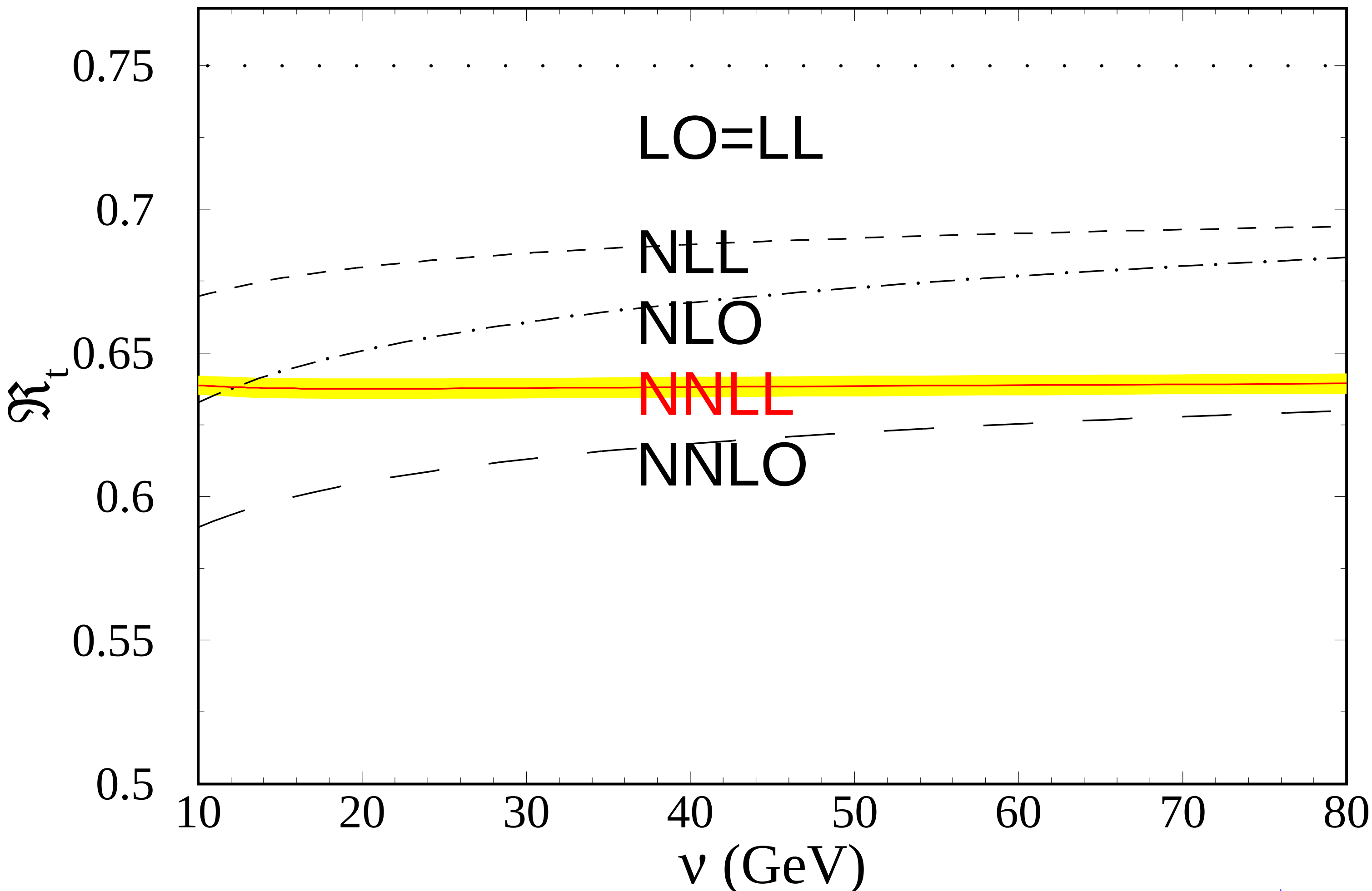


[Penin,Pineda,Smirnov,MS'04]

- generalize to non-equal mass case
- [CDF'98] $\approx$ 20 events via $B_c \to J/\psi l \nu$
  ➪ $M(B_c) = 6.40 \pm 0.39 \pm 0.13$ GeV

➪ $$M(B_c^*) - M(B_c) = 65 \pm 24\,(\mathrm{th})\,{}^{+19}_{-16}\,(\delta\alpha_s)\ \mathrm{MeV}$$

Spin-dep. at NNLL: Top

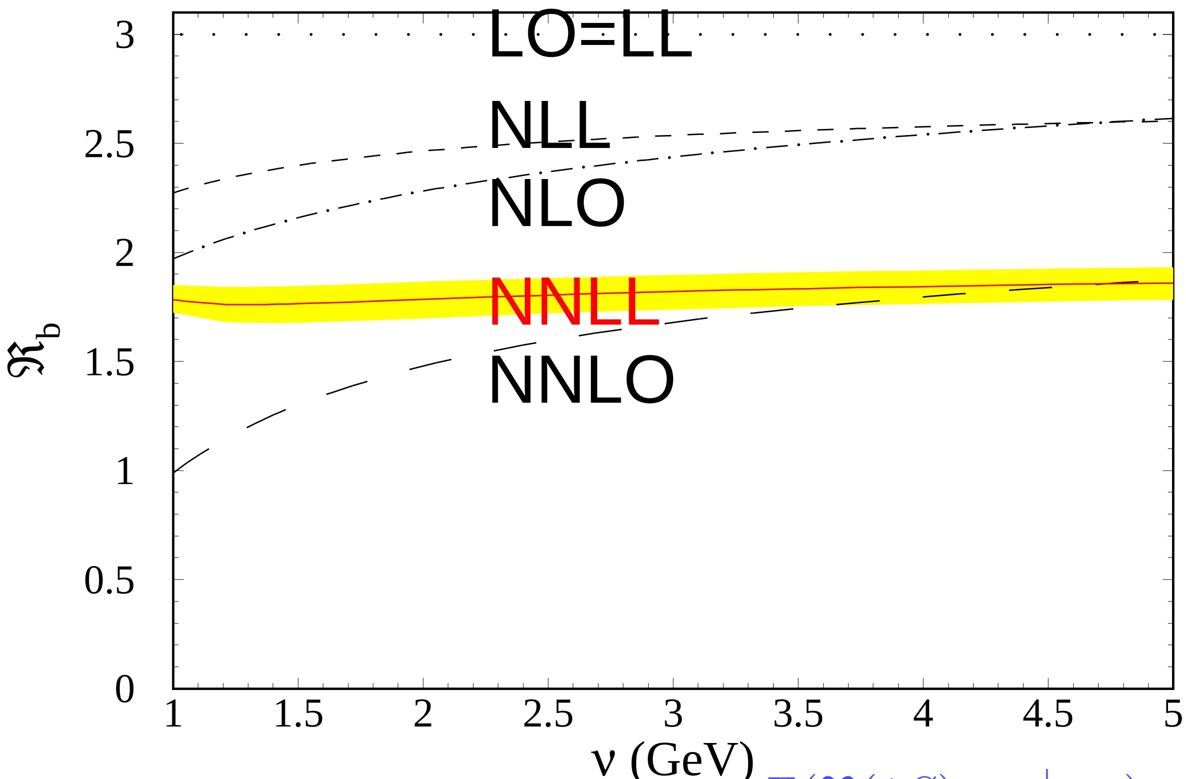


[Penin,Pineda,Smirnov,MS'04]

$$\mathcal{R}_t = \frac{\sigma(e^+e^-\to t\bar{t})}{\sigma(\gamma\gamma\to t\bar{t})}$$

Significant reduction of renormalization scale dependence

Spin-dep. at NNLL: Bottom



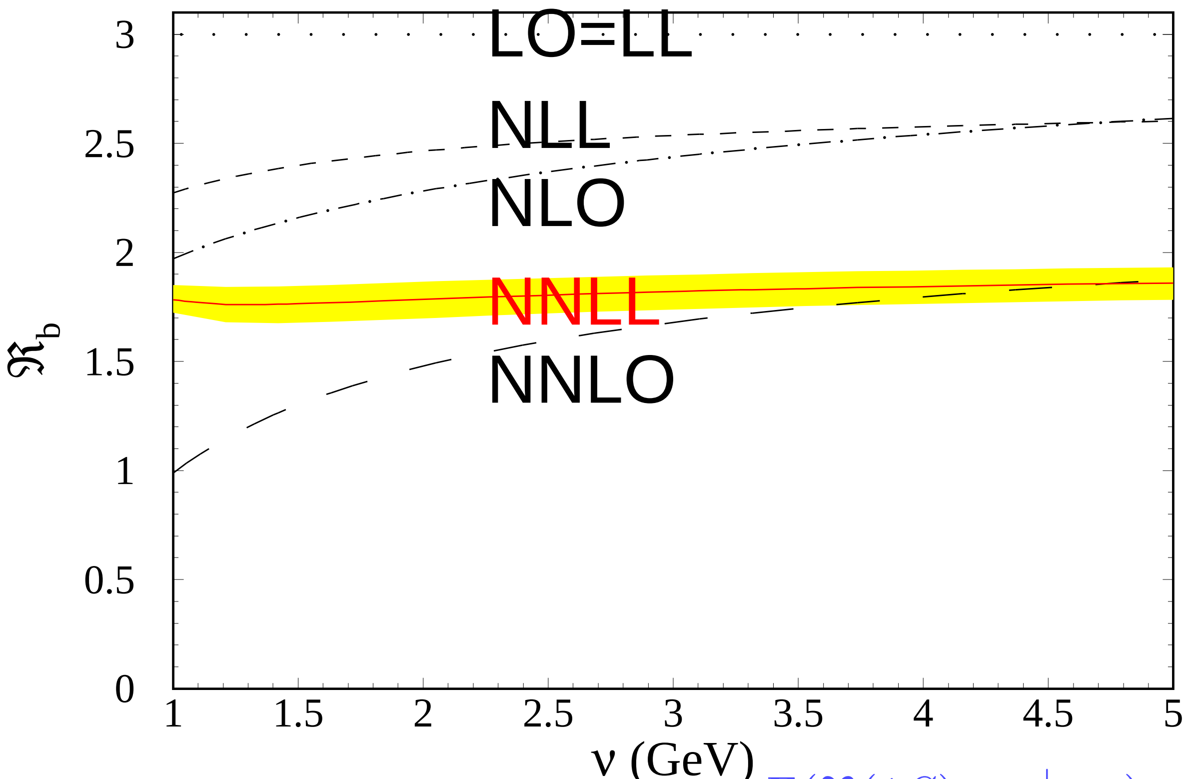
[Penin,Pineda,Smirnov,MS'04]

$$\mathcal{R}_b = \frac{\Gamma(\Upsilon(1S)\to e^+e^-)}{\Gamma(\eta_b(1S)\to\gamma\gamma)}$$

- Significant reduction of renormalization scale dependence
- "moderate" convergence
- Prediction:

$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.659 \pm 0.089(\mathrm{th.})^{+0.019}_{-0.018}(\delta\alpha_s) \pm 0.015(\mathrm{exp.})\ \mathrm{keV}$$

Spin-dep. at NNLL: Bottom



[Penin,Pineda,Smirnov,MS'04]

$$\mathcal{R}_b = \frac{\Gamma(\Upsilon(1S)\to e^+e^-)}{\Gamma(\eta_b(1S)\to\gamma\gamma)}$$

WARNING

Determination of α_s from

$$\frac{\Gamma(\Upsilon\to\text{light hadrons})}{\Gamma(\Upsilon\to\mu^+\mu^-)}$$

➪ $\alpha_s(m_b) = 0.177 \pm 0.01$ [PDG]

Uncertainty from scale dependence of NLO result

➪ $\alpha_s(M_Z) = 0.106 \pm 0.003$

compare: $\alpha_s(M_Z) = 0.118 \pm 0.003$

➪ underestimation of higher order corrections

➪ increase error!

- Significant reduction of renormalization scale dependence
- "moderate" convergence
- Prediction:

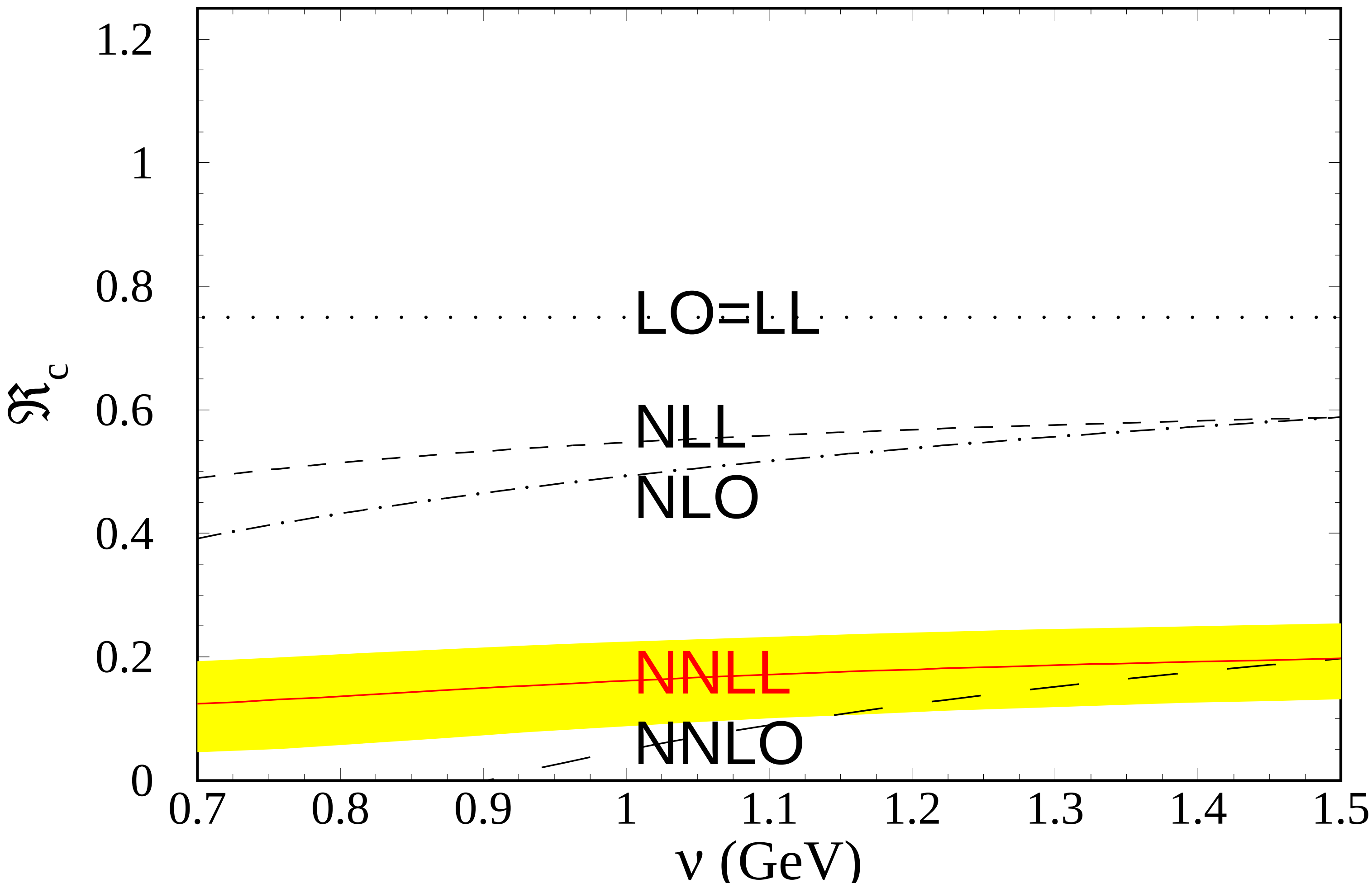
$$\Gamma(\eta_b(1S)\to\gamma\gamma) = 0.659 \pm 0.089(\text{th.})^{+0.019}_{-0.018}(\delta\alpha_s) \pm 0.015(\text{exp.})\ \text{keV}$$

Summary

- $t\bar{t}$ @ threshold, $M(\Upsilon(1S))$, $\Gamma(\Upsilon(1S) \to l^+l^-)$, ...
  NNLO complete; slow convergence
- N^{3}LO on the way:
  δE_n complete; $\delta\psi_n$: $\ln^2\alpha_s$, $\ln\alpha_s$
- resummation of logarithms:
  $\delta\mathcal{H}_{\text{spin}}$: NLL; c_v: NNLL
- HFS: $(b\bar{b})$, $(b\bar{c})$
- $\frac{\sigma(e^+e^-\to\mathcal{Q}(n^3S_1))}{\sigma(\gamma\gamma\to\mathcal{Q}(n^1S_0))}$; $\frac{\Gamma(\mathcal{Q}(n^3S_1)\to e^+e^-)}{\Gamma(\mathcal{Q}(n^1S_0)\to\gamma\gamma)}$

Backup Slides

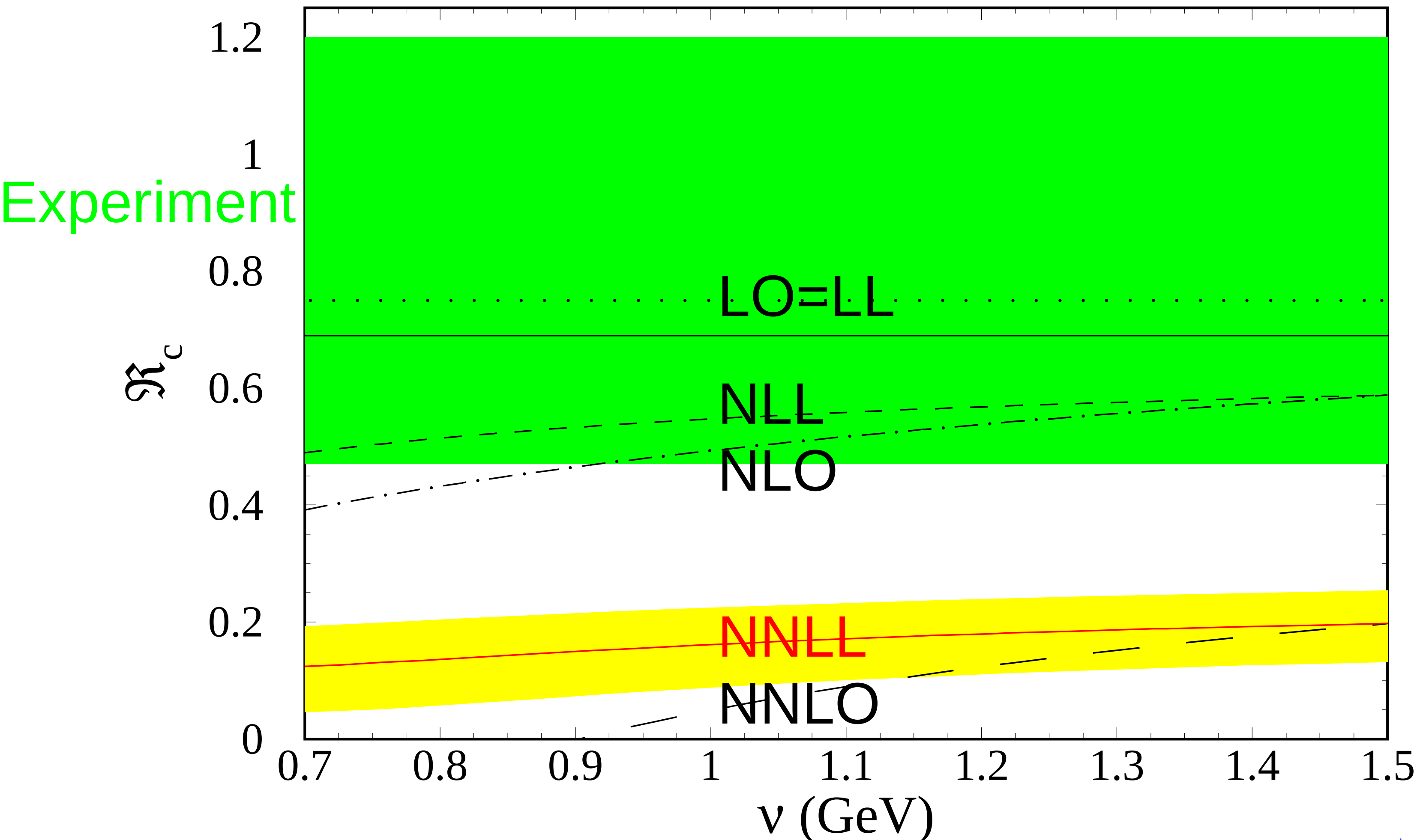
Spin-dep. at NNLL: Charm



[Penin,Pineda,Smirnov,MS'04]

$$\mathcal{R}_c = \frac{\Gamma(J/\Psi \to e^+ e^-)}{\Gamma(\eta_c(1S) \to \gamma\gamma)}$$

Spin-dep. at NNLL: Charm



[Penin,Pineda,Smirnov,MS'04]

$$\mathcal{R}_c = \frac{\Gamma(J/\Psi \to e^+e^-)}{\Gamma(\eta_c(1S) \to \gamma\gamma)}$$