Virtual Corrections to Bremsstrahlung in High-Energy Collider Physics:

LHC and e^+e^- Colliders

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Outline

We will describe radiative corrections to Bremsstrahlung and related processes – focusing on applications to luminosity, fermion pair production, and radiative return.

- BHLUMI and the Bhabha Process
  - e^+e^- luminosity monitor
- The KK MC and fermion pair production
- Radiative Return Applications /Cross-checks
- The Drell-Yan Process
  - hadronic luminosity monitor

Radiative Corrections to Bhabha Scattering

In the 1990s, S. Jadach, M. Melles, B.F.L. Ward and S.A. Yost calculated all two-photon real and virtual corrections to the small angle Bhabha scattering process used in the luminosity monitor at LEP and SLC.

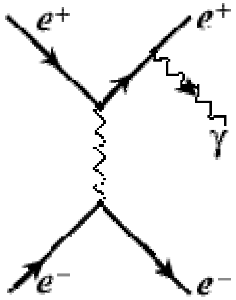
These corrections were used to bring the theoretical uncertainty in the luminosity measurement, as calculated by the BHLUMI Monte-Carlo program, to within a **0.06%** precision level.

The e^+e^- Luminosity Process

In e^+e^- scattering (SLC, LEP, ILC), the luminosity is calibrated using small angle Bhabha scattering

$$e^+e^- \longrightarrow e^+e^- + n\gamma$$

- This process has both experimental and theoretical advantages:
    - A large, clean signal
    - Almost pure QED (3% Z exchange)



- The angle cuts were 1-3 degrees at LEP1, 3-6 degrees at LEP2.

The BHLUMI Monte Carlo Program

BHLUMI was developed into an extremely precise tools for computing the Bhabha luminosity process in $e^+ e^-$ colliders. The project was begun by S. Jadach, B.F.L. Ward, E. Richter-Was, and Z. Was and continued with contributions by S. Yost, M. Melles, M. Skrzypek, W. Placzek and others.

BHLUMI exponentiates the universal soft photon part of the amplitudes.

The finite YFS residuals must be calculated to the desired order using in a double expansion in a and logarithms $L = \log(|t|/m_e^2)$

Historical Progress

	Year	Experiment	Theory
	1982	2%	2%
	1990	0.8%	1%
	1992	0.6%	0.25%
BHLUMI	1996-7	0.1 - 0.15%	<0.11%
	1999	0.05%	**<0.061%**

Progress in the Luminosity Calculation

To obtain a precision level comparable to experiment at LEP1 and LEP2 we found it necessary to eliminate the leading source of error, which was an uncalculated component of the photon radiative corrections to the scattering process.

- Exact calculation of two-photon Bremsstrahlung: all graphs radiating two photons from any fermion lines have been calculated.
- Order α correction to one-photon Bremsstrahlung: include all graphs where one real plus one virtual photon are emitted from a fermion line.
- Order α^2 correction to Bhabha scattering: two virtual photons can be included by adapting known results.

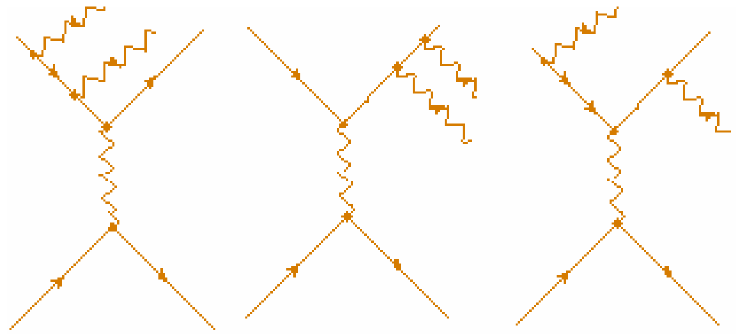
Completing these calculations reduced the uncertainty from order α^2 photon effects from 0.1% for LEP1 down to 0.027%, and from 0.2% for LEP2 down to 0.04%.

Exact 2 Photon Bremsstrahlung

Historically, we started with the two-photon Bremsstrahlung process. The exact two photon Bremsstrahlung cross section was calculated by Jadach, Ward, and Yost in 1992. This process is

$$e^+(p_1) + e^-(p_2) \rightarrow e^+(q_1) + e^-(q_2) + \gamma(k_1) + \gamma(k_2)$$

This corresponds to Feynman diagrams of the form



These diagrams were calculated exactly using helicity spinor techniques (which treat the electron as massless), adding electron masses perturbatively.

YFS Soft Photon Techniques

Precision calculations require considerable care, because the cross sections are always dominated by soft (low-energy) photons. But the contribution of soft photons is well known due to results of **Yennie, Frautschi and Suura**, who exponentiated them to all orders. The soft photon contribution generally factorizes as a soft factor *S* times a lower order cross section.

Obtaining precision results for multi-photon Bremsstrahlung requires subtracting these dominant, but known contributions and carefully calculating the "YFS residuals" left over. In the case of 2 real photons, the new result we calculated was actually β_2 in the differential cross section (at fixed order)

$$d\sigma(k_1, k_2) = \sigma_0\, S(k_1)\, S(k_2) + \beta_1(k_1)\, \mathrm{S}(k_2) + \beta_1(k_2)\, \mathrm{S}(k_1) + \beta_2(k_1, k_2).$$

2 soft photons — 1 soft, 1 hard photon — 2 hard photons

YFS Exponentiated Cross Section

These fixed-order beta functions are an ingredient in the YFS-exponentiated cross section calculated by BHLUMI, the KK MC, and related MC programs (YFS2, YFS3, BHWIDE, KORALZ, YFSWW3 and KORALW , …):

$$d\sigma_{exp} = e^{2\alpha\, Re\, B + 2\alpha \tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_j k_j)+D} \bar{\beta}_n(k_1,\ldots,k_n) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0}$$

with

$$2\alpha \tilde{B} = \int^{k \le K_{max}} \frac{d^3 k}{k_0} \tilde{S}(k)$$

YFS form factor for real photon emission

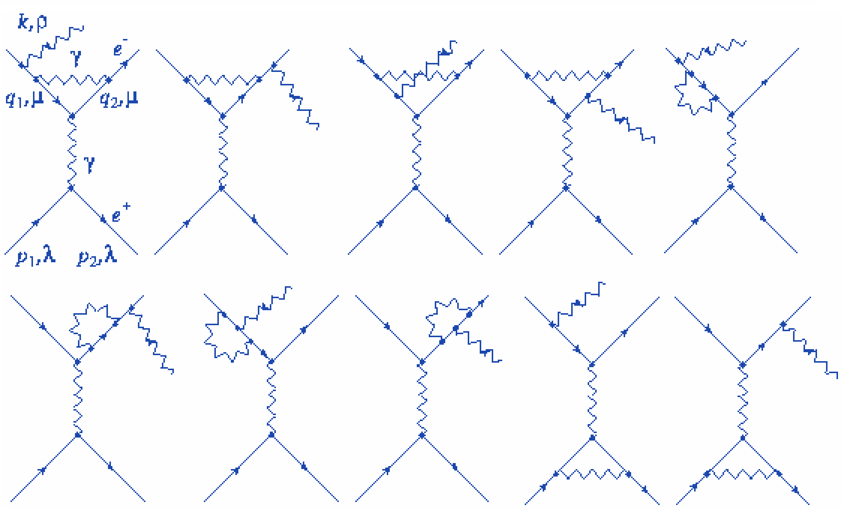
$$D = \int d^3 k \frac{\tilde{S}(k)}{k^0} \left(e^{-iy\cdot k} - \theta(K_{max} - k)\right)$$

Real + Virtual Photon Emission

The second, and considerably more difficult contribution completed was the case where one real photon is emitted, but another is virtual, present only in an internal “loop” in the graph. The complexity of the calculations increases rapidly with the addition of loops. (In many calculations, 2 loops is state of the art.) In fact, refinements of this process are still in progress, since some approximations were made.

$$e^+(p_1) + e^-(p_2) \rightarrow e^+(q_1) + e^-(q_2) + \gamma(k)$$

We need this process to order α^2. This requires including an internal photon in a loop.



Computational Method

The graphs shown were calculated by S. Jadach, M. Melles, B.F.L. Ward and S. Yost in *Phys. Rev.* **D65**, 073030 (2002), based on earlier results for the corresponding *t*-channel graphs by the same authors, *Phys. Lett.* **B377**, 168 (1996).

The results were obtained using

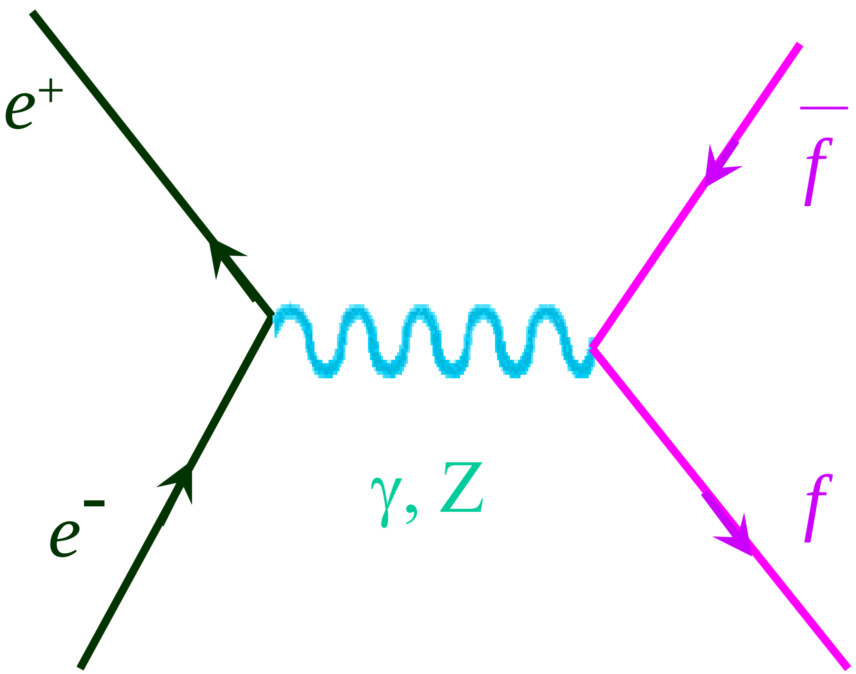
- Helicity spinor methods
- Vermaseren's algebraic manipulation program FORM
- Oldenborgh's FF package of scalar one-loop Feynman integrals (later replaced by analytic expressions)
- Mass corrections added via methods of Berends *et al* (CALCUL Collaboration), which were checked to show that all significant collinear mass corrections were included.

Fermion Pairs

Fermion pair production

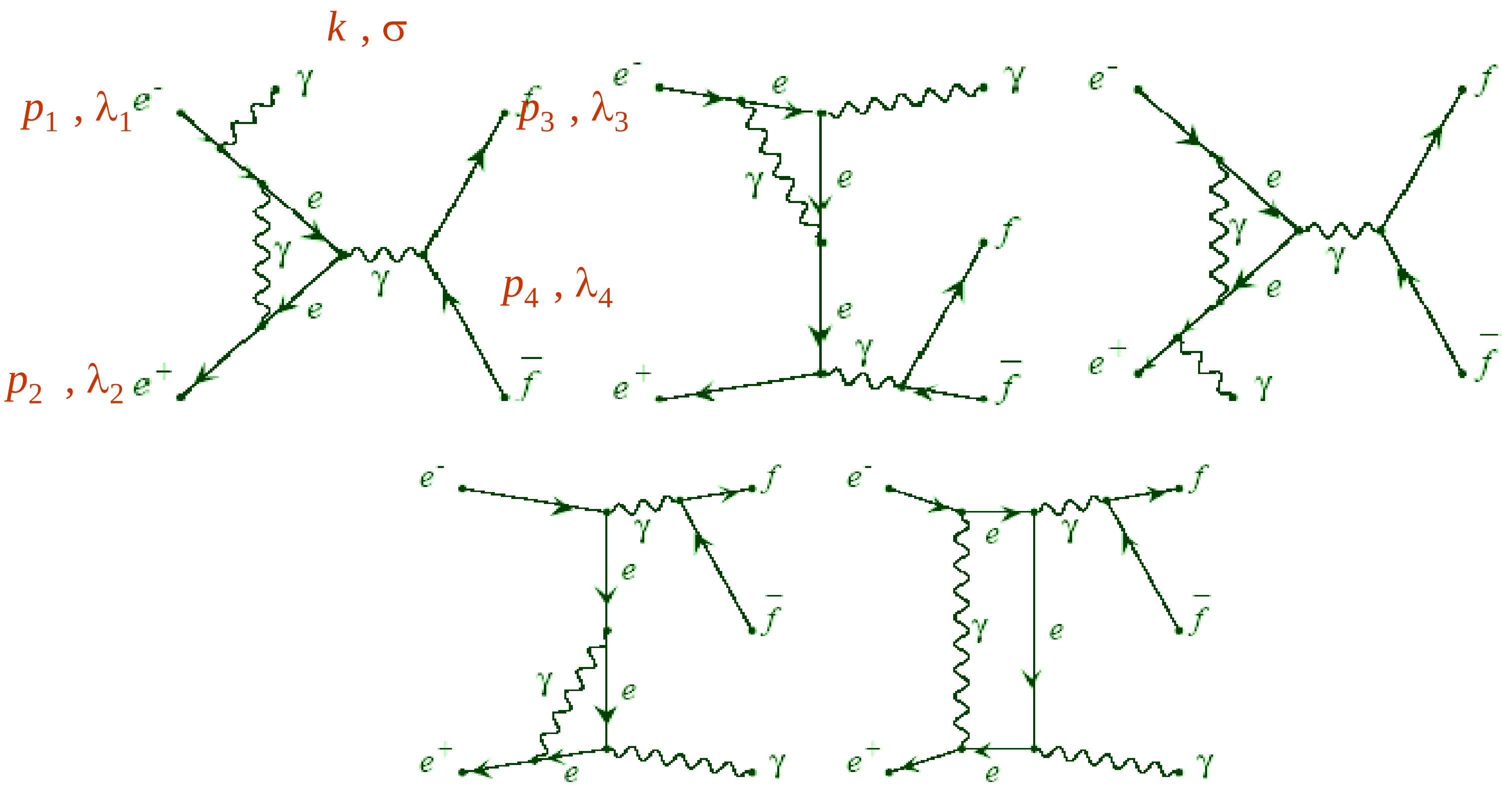
$$e^+ e^- \longrightarrow f\bar{f}$$

plays a critical role in extracting precision electroweak physics from e^+e^- colliders.



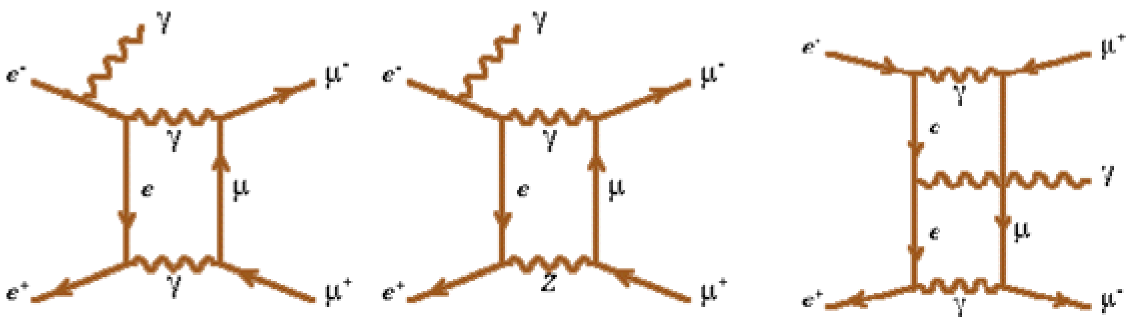
Radiative Corrections to Fermion Pair Production

For ISR, relevant Bremsstrahlung corrections include…



Assumptions Made In The Calculation

A small angle approximation was made, which suppresses initial-final state interference terms, including the box diagrams of the type shown below. The addition of such diagrams is in progress with S. Majhi,



Similar diagrams were neglected in the BHLUMI program, since they were not needed in the small-angle Bhabha luminosity process at LEP precision levels.

Such diagrams may be be needed for precision ILC physics – calculating them would settle this question.

KK MC

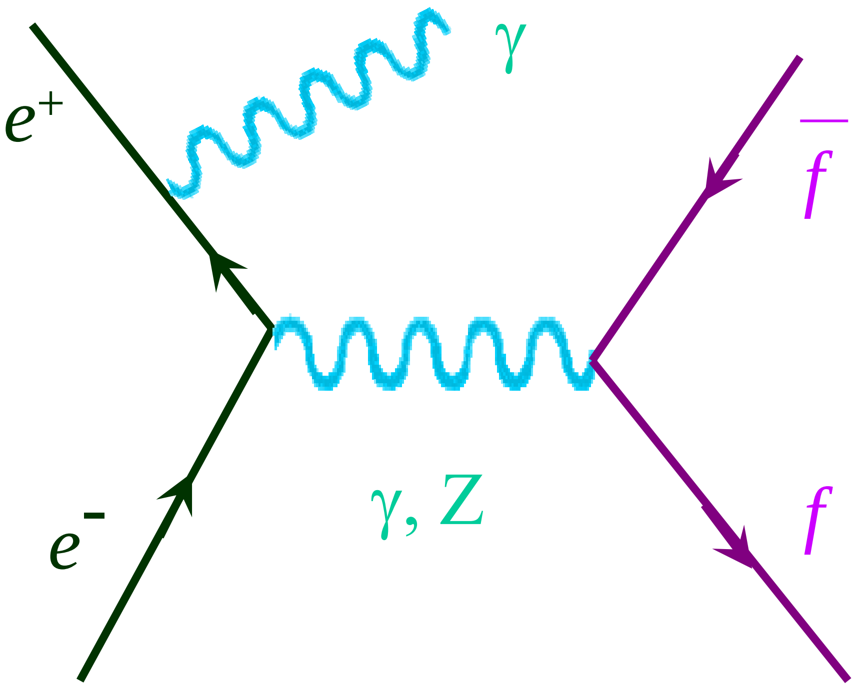
The KK MC includes initial state photon radiation, and all other two-photon (real or virtual) to fermion pair processes.

It also includes YFS exponentiation, and the effects of Z boson exchange

A Bremsstrahlung Application: Radiative Return

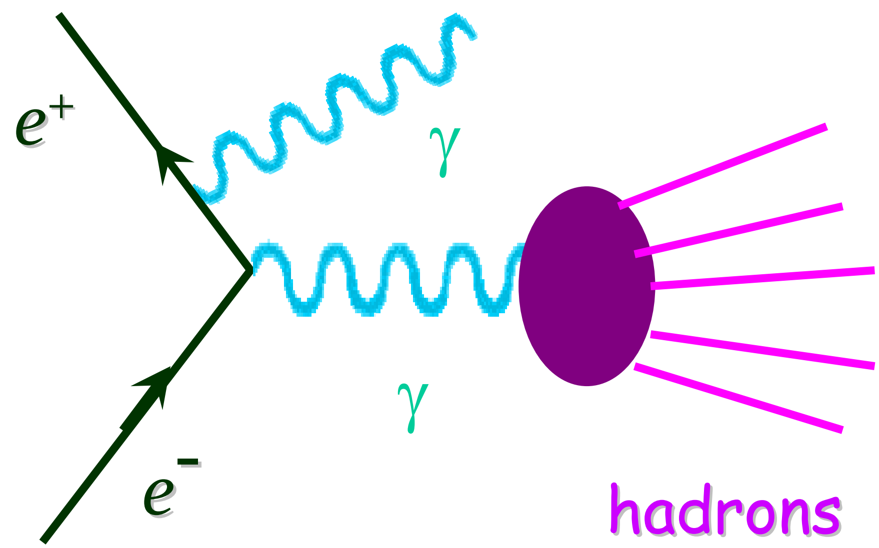
Radiating a hard photon from the initial state can be a convenient way to explore a range of energies in a fixed-energy collider.

For example, the Z resonance can be probed in detail using a collider with $s > M_Z^2$.



Radiative Return

Hadronic final states can also be studied using the radiative return method.



J. Kuhn *et al* have developed a MC called PHOKHARA to calculate this process for hadronic and leptonic (*ie* muon pair) final states.

Comparison of Radiative Corrections

PHOKHARA incorporates initial state radiative corrections via a "Leptonic Tensor".

The work of Kuhn *et al* is also an important cross-check on the radiative corrections calculated for the KK MC.

Since the initial state radiation is calculated exactly at order α^2 in both PHOKHARA and KK, and since both expressions are available analytically, it is useful to know how the expressions compare.

The Complete Result

The differential cross section for initial state radiation (ISR) can be expressed as

$$\frac{d\sigma_1^{\mathrm{ISR}(1)}}{d^2\Omega dr_1 dr_2} = \frac{1}{(4\pi)^4 s'} \sum_{\lambda_i,\sigma} \mathrm{Re}[(\mathcal{M}_1^{\mathrm{ISR}(0)})^* \mathcal{M}_1^{\mathrm{ISR}(1)}].$$

$r_i = 2\, p_i \cdot k$

and the differential cross section with virtual corrections can be expressed as

$$\frac{d\sigma_1^{\mathrm{ISR}(0)}}{d^2\Omega dr_1 dr_2} = \frac{1}{2(4\pi)^4 s'} \sum_{\mathrm{helicities}} \left|\mathcal{M}_1^{\mathrm{ISR}(0)}\right|^2$$

$$\left|\mathcal{M}_1^{\mathrm{ISR}(0)}\right|^2 = \frac{e^6}{s^2 r_1 r_2}(t_1^2 + t_2^2 + u_1^2 + u_2^2) + \text{mass corrections}$$

The Complete Result

The complete amplitude with virtual corrections can be expressed in terms of the amplitude without virtual corrections (pure hard Bremsstrahlung) as

$$\mathcal{M}_1^{\mathrm{ISR}(1)} = \frac{\alpha}{4\pi}(f_0 + f_1 I_1 + f_2 I_2)\mathcal{M}_1^{\mathrm{ISR}(0)}$$

where I_1 and I_2 are spinor factors which vanish in the collinear limits $p_i{\cdot}k = 0$:

Spinor product: $|s_\lambda(p,k)|^2 = 2p\cdot k$

$$I_1 = \sigma\lambda_3 s_{\lambda_1}(p_1,k)s_{-\lambda_1}(p_2,k) \times \frac{s_{\lambda_3}(p_4,p_2)s_{-\lambda_3}(p_2,p_3) - s_{\lambda_3}(p_4,p_1)s_{-\lambda_3}(p_1,p_3)}{s_{-\sigma}(p_1,p_2)s_{-\sigma}(p_3,p_4)s_\sigma^2(p_{21},p_{34})},$$

$$I_2 = \lambda_1\lambda_3\frac{s_{\lambda_1}(p_1,k)s_{-\lambda_1}(p_2,k)s_{\lambda_3}(p_4,k)s_{-\lambda_3}(p_3,k)}{s_{-\sigma}(p_1,p_2)s_{-\sigma}(p_3,p_4)s_\sigma^2(p_{21},p_{34})},$$

with $p_{ij} = p_i$ or p_j when $\sigma = \lambda_i$ or λ_j.

Form Factors

For helicities $\sigma = \lambda_1$. Otherwise interchange r_1 and r_2: (mass terms added later)

$$
\begin{aligned}
f_0 &= 2\left\{\ln\left(\frac{s}{m_e^2}\right) - 1 - i\pi\right\} + \frac{r_2}{1-r_2} + \frac{r_2(2+r_1)}{(1-r_1)(1-r_2)}\left\{\ln\left(\frac{r_2}{z}\right) + i\pi\right\} \\
&- \left\{3v + \frac{2r_2}{1-r_2}\right\}\mathrm{Lf}_1(-v) + \frac{v}{(1-r_2)}\,R_1(r_1,r_2) + r_2\,R_1(r_2,r_1),
\end{aligned}
$$

$$
\begin{aligned}
f_1 &= \frac{r_1-r_2}{2(1-r_1)(1-r_2)} + \frac{z(1+z)}{2(1-r_1)^2(1-r_2)}\left\{\ln\left(\frac{r_2}{z}\right) + i\pi\right\} \\
&+ \frac{z}{1-r_2}\left\{\frac{1}{2}R_1(r_1,r_2) + r_2 R_2(r_1,r_2)\right\} + \frac{v}{4}\left\{R_1(r_1,r_2)\delta_{\sigma,1} + R_1(r_2,r_1)\delta_{\sigma,-1}\right\}
\end{aligned}
$$

$$
\begin{aligned}
f_2 &= 2 - \frac{1+z}{2(1-r_1)(1-r_2)} + \frac{z(r_2-r_1)}{2(1-r_1)^2(1-r_2)}\left\{\ln\left(\frac{r_2}{z}\right) + i\pi\right\} \\
&+ 2z\,\mathrm{Lf}_2(-v) + \frac{z}{1-r_2}\left\{\frac{1}{2}R_1(r_1,r_2) + (2-r_2)R_2(r_1,r_2)\right\} \\
&+ \frac{r_1-r_2}{4}\left\{R_1(r_1,r_2)\delta_{\sigma,1} + R_1(r_2,r_1)\delta_{\sigma,-1}\right\}
\end{aligned}
$$

$r_i = 2\, p_i \cdot k$

$v = r_1 + r_2$

$z = 1 - v$

Special Functions

The form factors depend on the following special functions, designed to be stable in the collinear limits (small r_i) which are most important in the cross section.

$$
\begin{aligned}
R_1(x,y) &= \mathrm{Lf}_1(-x)\left\{\ln\left(\frac{1-x}{y^2}\right) - 2\pi i\right\} \\
&+ \frac{2(1-x-y)}{1-x}\,\mathrm{Sf}_1\left(\frac{y}{1-x}, \frac{x(1-x-y)}{1-x}\right), \\
R_2(x,y) &= z + \frac{1}{1-x}\left\{\ln\left(\frac{y}{1-x-y}\right) + i\pi\right\} \\
&+ \mathrm{Lf}_2(-x)(\ln y + i\pi) - (1-x-y)\,\mathrm{Lf}_1(-x) - \frac{1}{2}\,\mathrm{Lf}_1^2(-x) \\
&+ \frac{1-x-y}{(x+y)(1-x)}\left\{x\,\mathrm{Lf}_1\left(\frac{-y}{1-x}\right) - y\,\mathrm{Lf}_2\left(\frac{-y}{1-x}\right)\right\} \\
&+ \left(\frac{1-x-y}{1-x}\right)^2 \mathrm{Sf}_2\left(\frac{y}{1-x}, \frac{x(1-x-y)}{1-x}\right),
\end{aligned}
$$

with

$$
\mathrm{Lf}_0(x) = \ln(1-x), \qquad \mathrm{Lf}_{n+1}(x) = \frac{1}{x}\left(\mathrm{Lf}_n(x) - \mathrm{Lf}_n(0)\right),
$$

$$
\mathrm{Sf}_0(x,y) = \mathrm{Sp}(x+y), \qquad \mathrm{Sf}_{n+1}(x,y) = \frac{1}{y}\left(\mathrm{Sf}_n(x,y) - \mathrm{Sf}_n(x,0)\right)
$$

Dilogarithm (Spence function)

Mass Corrections

Mass corrections were added following Berends, *et al* (CALCUL collaboration). We checked that all significant mass corrections are obtained in this manner.

The most important corrections for a photon with momentum k radiated collinearly with each incoming fermion line p_1 and p_2 are added via the prescription

$$\left|\mathcal{M}_{1\gamma}^{(m)}\right|^2 = -\sum_i \frac{e^2 m_e^2}{p\cdot k} \left|\mathcal{M}_{\rm Born}(p_i - k)\right|^2$$

At the cross-section level, the net effect is that the spin-averaged form factor f_0 receives an additional mass term

$$\begin{aligned} \langle f_0 \rangle^m &= \frac{2m_e^2}{s}\left(\frac{r_1}{r_2}+\frac{r_2}{r_1}\right)\frac{z}{(1-r_1)^2+(1-r_2)^2} \\ &\times \left\{\langle f_0\rangle + \ln\left(\frac{s}{m_e^2}\right)(\ln z - 1) - \frac{3}{2}\ln z + \frac{1}{2}\ln^2 z + 1\right\} \end{aligned}$$

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$$\left|\mathcal{M}_{1\gamma}^{(m)}\right|^2 = -\sum_i \frac{e^2 m_e^2}{p\cdot k} \left|\mathcal{M}_{\rm Born}(p_i - k)\right|^2$$

Genuine m_e^2/s contributions are completely negligible, but this class of corrections produces finite contributions when integrated over the phase space of a radiated photon, coming from the collinear limit.

Next to Leading Log Approximation

For Monte Carlo use, it is desirable to have the shortest possible equation with sufficient accuracy. For most of the range of hard photon energies, the leading log (LL) and next to leading log (NLL) contributions suffice. These include all terms important in the collinear limits (r_i small).

- To NLL order, the spinor terms I_1 and I_2 can be dropped, since f_1 and f_2 are at most logarithmically divergent for small r_i.
- The spin-averaged collinear limit of f_0 is ...

$$
\begin{aligned}
\langle f_0 \rangle^{\rm NLL} &= 2\{L-1\} + \frac{r_1(1-r_1)}{1+(1-r_1)^2} + \frac{r_2(1-r_2)}{1+(1-r_2)^2} + 2\ln r_1 \ln(1-r_2) \\
&+ 2\ln r_2 \ln(1-r_1) - \ln^2(1-r_1) - \ln^2(1-r_2) + 3\ln(1-r_1) \\
&+ 3\ln(1-r_2) + 2\,{\rm Sp}(r_1) + 2\,{\rm Sp}(r_2) + \langle f_0 \rangle_m^{\rm NLL}
\end{aligned}
$$

big log $L = \ln\left(\dfrac{s}{m_e^2}\right)$

mass corrections

YFS Residuals

The Monte Carlo program will calculate YFS residuals, which are obtained by subtracting the YFS factors containing the infrared singularities. This amounts to subtracting a term

$$4\pi B_{\rm YFS}(s,m) = \left(4\ln\frac{m_0}{m}+1\right)\left(\ln\frac{s}{m^2}-1-i\pi\right)-\ln^2\left(\frac{s}{m^2}\right)-1+\frac{4\pi^2}{3}+i\pi\left(2\ln\frac{s}{m^2}-1\right)$$

from the form factor f_0. At NLL order, we would have

$$\overline{\beta}_1^{(2)} = \overline{\beta}_1^{(1)}\left(1+\frac{\alpha}{2\pi}\langle f_0\rangle^{\rm NLL}\right)$$

In our comparisons we will actually subtract this NLL term and look at the NNLL behavior of each expression.

Comparisons

IN Igarashi and Nakazawa, *Nucl. Phys.* **B288** (1987) 301

- spin-averaged cross section, fully differential in r_1 and r_2 , no mass corrections

BVNB Berends, Van Neerven and Burgers, *Nucl. Phys.* **B297** (1988) 429

- spin-averaged cross section, differential only in $v = r_1 + r_2$, includes mass corrections

KR Kuhn and Rodrigo, *Eur. Phys. J.* **C25** (2002) 215

- spin-averaged Leptonic tensor, fully differential in r_1 and r_2 , includes mass corrections

The **KR** comparison is new, and closest to our calculation in its assumptions.

The New Comparison

The new comparison is to the leptonic tensor of Kuhn and Rodrigo, which was constructed for radiative return in hadron production, but can be adapted to fermion pairs by changing the final state tensor. The ISR result is expressed as a leptonic tensor

$$L_0^{\mu\nu} = \frac{e^6}{ss'^2}\{a_{00}s\eta^{\mu\nu} + a_{11}p_1^\mu p_1^\nu + a_{22}p_2^\mu p_2^\nu + a_{22}(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) + i\pi a_{-1}(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu)\}$$

which can be contracted with a final state tensor to get the squared matrix element:

$$H^{\mu\nu} = e^2(p_3^\mu p_4^\nu + p_3^\nu p_4^\mu - p_3 \cdot p_4 \eta^{\mu\nu})$$

$$|\mathcal{M}^{\rm ISR}|^2 = zL_0^{\mu\nu}H_{\mu\nu}.$$

The New Comparison: Kuhn & Rodrigo

The coefficients in the leptonic tensor can be separated into a tree-level term and a one-loop correction,

$$a_{ij} = a_{ij}^{(0)} + \frac{\alpha}{\pi} a_{ij}^{(1)}$$

In the following, we will use the notation: $c_{ij} = a_{ij}^{(1)} - a_{ij}^{\mathrm{IR}}$

with

$$a_{ij}^{\mathrm{IR}} = a_{ij}^{(0)} \left[2(L-1)\ln v_{\min} + \frac{1}{2}L - 1 + \frac{\pi^2}{3} \right]$$

cutoff on $v = 2E_\gamma/\sqrt{s}$

Removing this IR function is found to be equivalent to subtracting Re B_{YFS} in our version of the calculation.

Analytical Comparison

The expression for the leptonic tensor is very different from our earlier exact result, but it is possible to show that in the massless NLL limit, they agree analytically.

The virtual correction to the YFS residual can be expressed in the NLL limit as

$$\overline{\beta}_1^{(2)} = \overline{\beta}_1^{(1)} \left(1 + \frac{\alpha}{2\pi} C_1\right) + C_2$$

with coefficients

$$\begin{aligned} C_1 &= \frac{1}{2a_{00}^{(0)}} \left(\frac{c_{11}}{z} + z c_{22} - 2c_{12} \right), \\ C_2 &= \frac{c_{11}}{4z} + \frac{z c_{22}}{4} - \frac{c_{12}}{2} - c_{00}, \end{aligned}$$

We find that

$$C_1 = \langle f_0 \rangle^{\mathrm{NLL}}$$

Comparison of Mass Corrections

Mass corrections to the KR result were added to calculate radiative return for untagged photons, which may be collinear with the initial state fermions. This was done via an expansion of the exact result in powers of m_e^2/r_i.

The mass terms in each expression are found to produce identical results through NLL order, which in the case of mass corrections requires the inclusion of a "big logarithm" $L = \log(s/m_e^2)$ in the unintegrated collinear limit of the matrix element.

The soft collinear limit of the mass corrections of both expressions was also checked, and found to agree exactly, including terms of order 1 (NNLL).

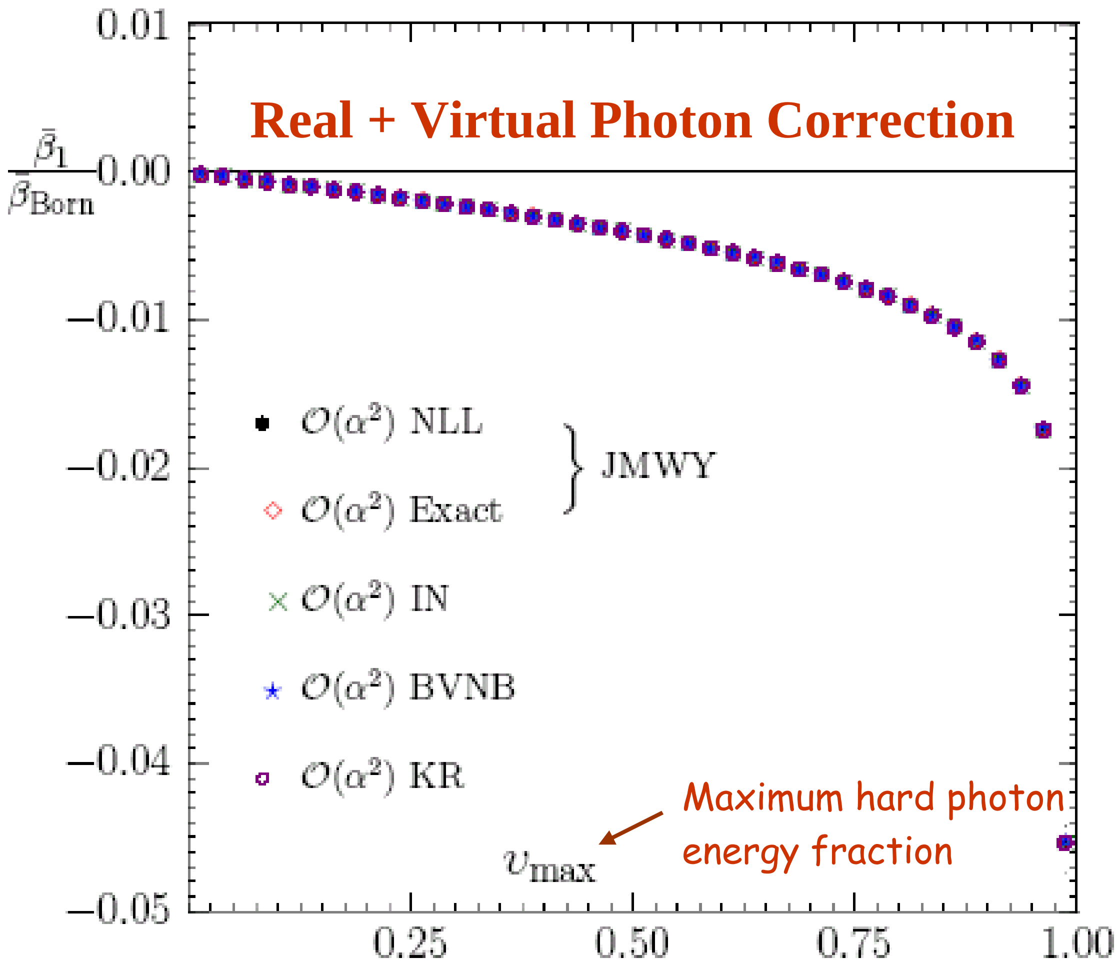
Remaining comparisons are done numerically. Care is needed since the expressions for the mass corrections to the coefficients a_{ij} contain apparent m_e^4/r_i^2 and m_e^6/r_i^3 factors in the collinear limit, all of which cancel when the numerators multiplying them are summed carefully.

Monte Carlo Results

Results of a KK Monte Carlo run with 10^8 events at E_{CMS} = 200 GeV.

This figure shows the complete real + virtual photon radiative correctionto muon pair production.

The standard YFS infrared term $4\pi B_{YFS}$ has been subtracted to create a finite result.

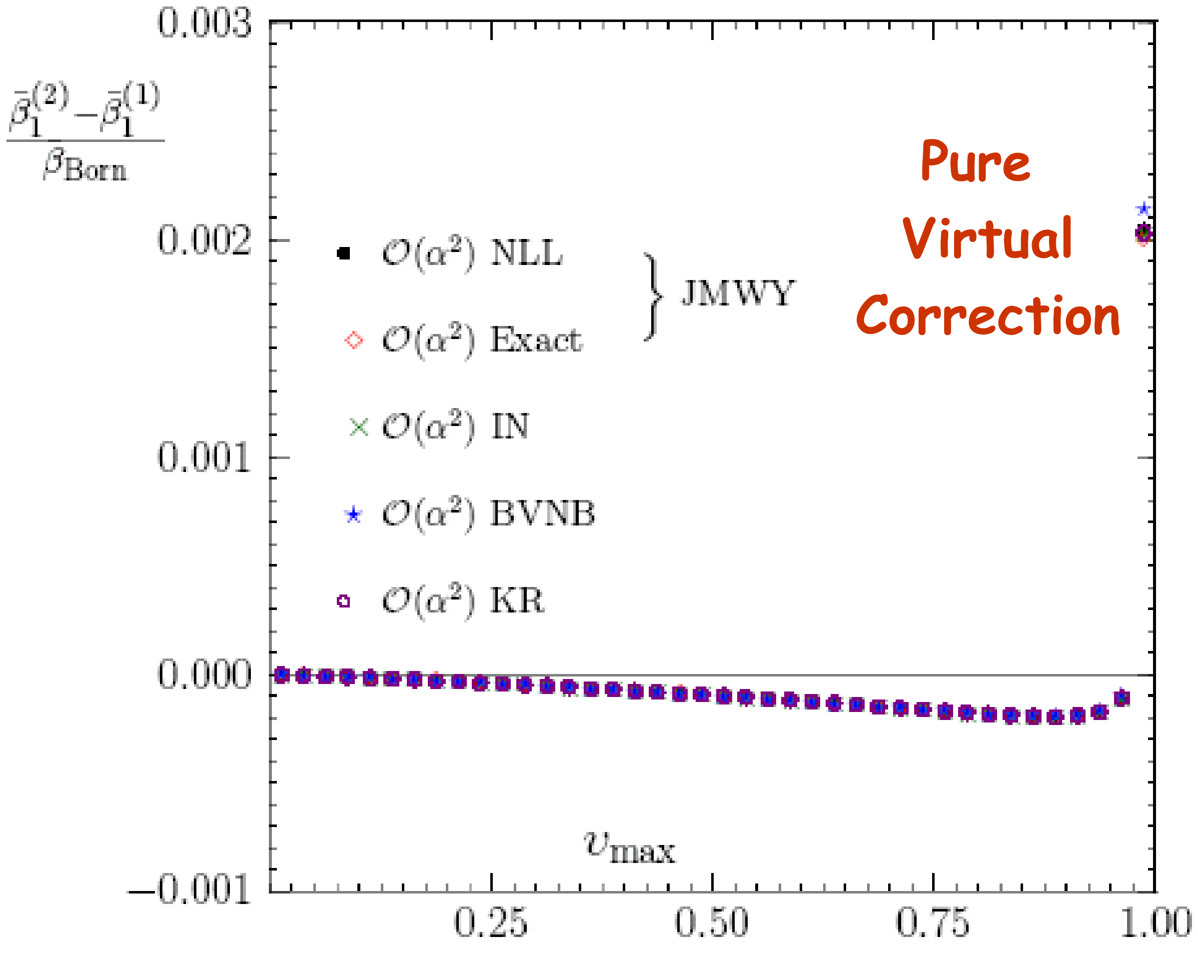


Monte Carlo Results

Results of a KK Monte Carlo run with 10^8 events at E_{CMS} = 200 GeV.

This figure shows only the pure virtual photon correction to single hard bremmstrahlung.

The standard YFS infrared term $4\pi B_{YFS}$ has been subtracted to create a finite result.

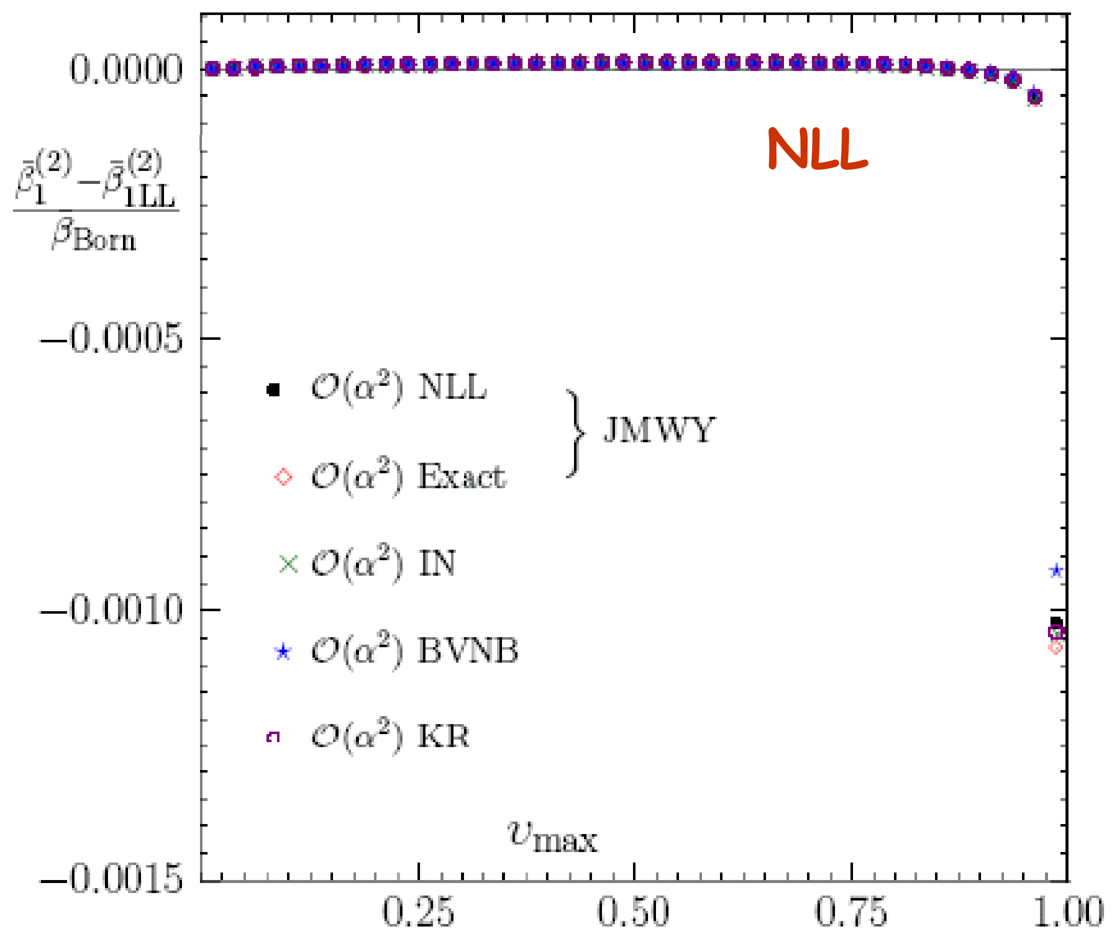


Monte Carlo Results

Results of a KK Monte Carlo run with 10^8 events at $E_{\rm CMS}$ = 200 GeV.

This figure shows the next to leading log (NLL) contribution to the real + virtual photon cross section.

The leading log (LL) contribution has been subtracted from each expression.

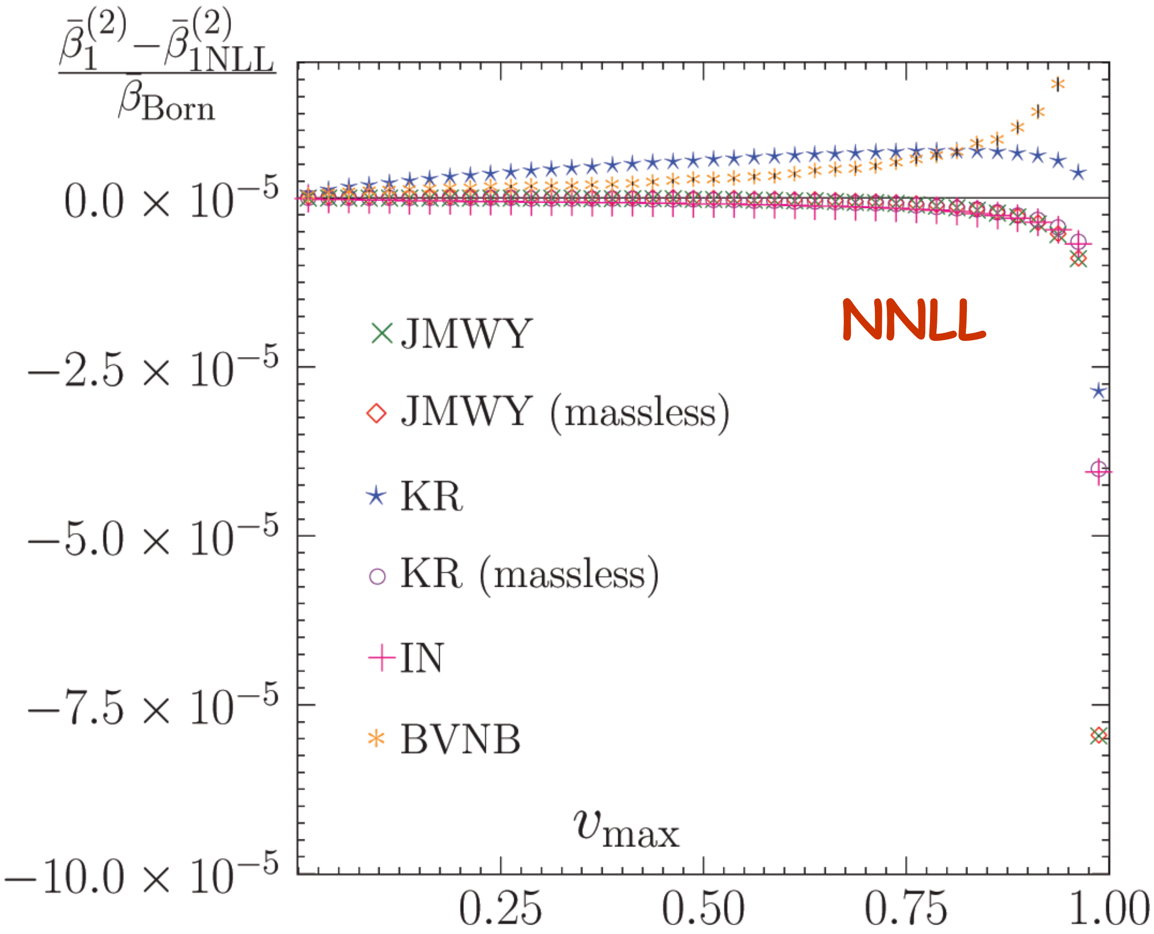


Monte Carlo Results

Results of a KK Monte Carlo run with 10^8 events at E_{CMS} = 200 GeV.

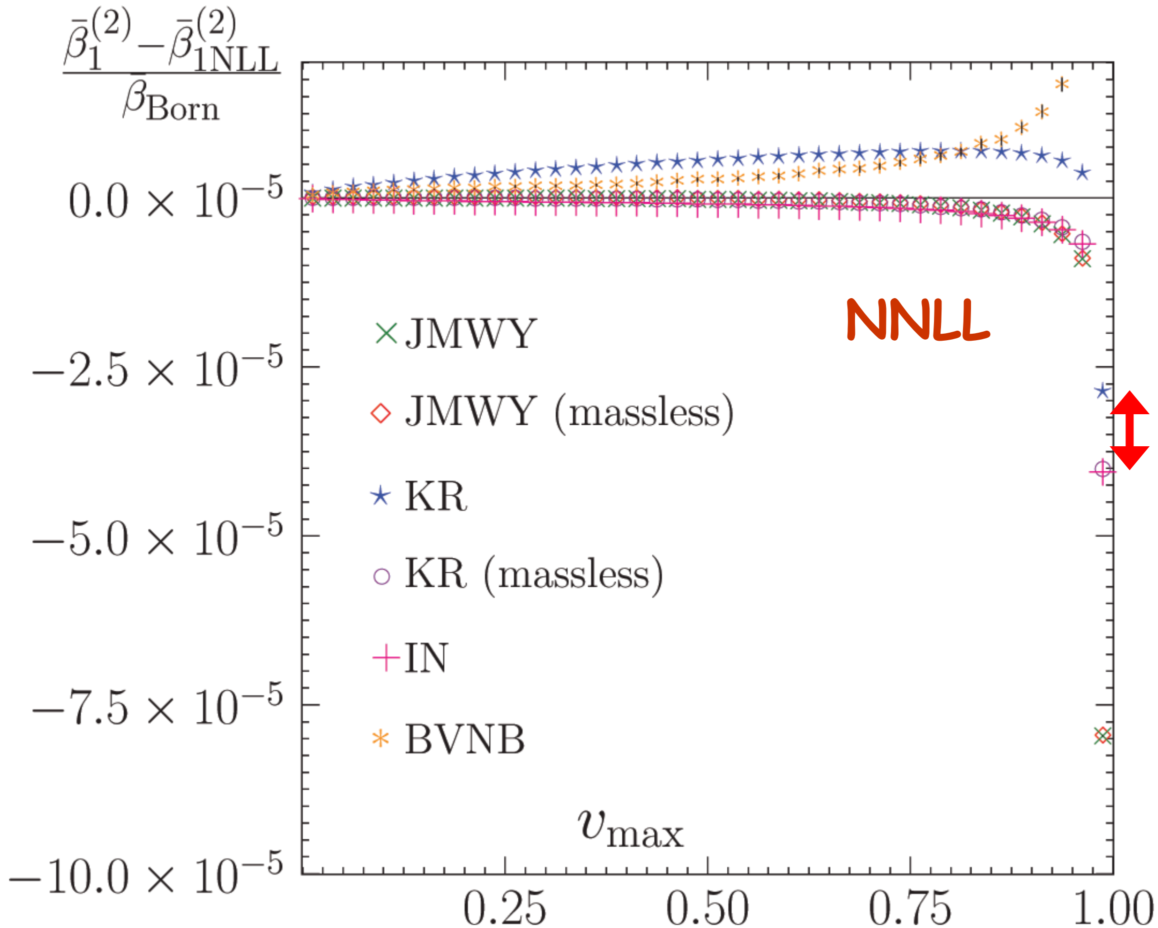
This figure shows the sub-NLL contribution to the real + virtual photon correction to muon pair production

The NLL expression of JMWY has been subtracted in each case to reveal the NNLL contributions.



Monte Carlo Results

The difference in the mass corrections is negligible over most of the range of photon energies, but reaches a maximum of about 1.2 x 10^{-5} in units of the Born cross-section.



Summary of Comparisons

- The size of the NNLL corrections for all of the compared "exact" expressions is less than 2×10^{-6} in units of the Born cross section for photon energy cut $v_{max} < 0.75$.

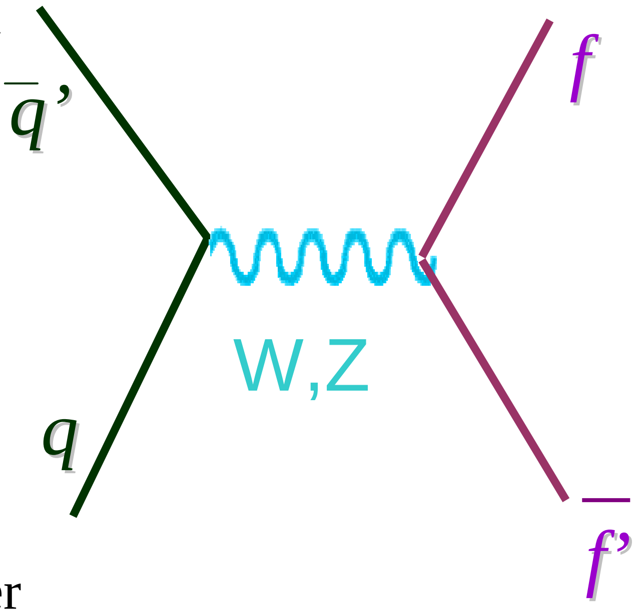
- For $v_{max} < 0.95$, all the results except BVNB agree to within 2.5×10^{-6} of the Born cross section.

- For the final data point, $v_{max} = 0.975$, the KR and JMWY results differ by 5×10^{-5} (with mass terms) or 3.5×10^{-5} (without mass terms) of the Born cross section.

- These comparisons show that we have a firm understanding of the precision tag for an important part of the order α^2 corrections to fermion pair production in precision studies of the final LEP2 data analysis, radiative return at Φ and B-factories, and future ILC physics.

The Drell-Yan Process

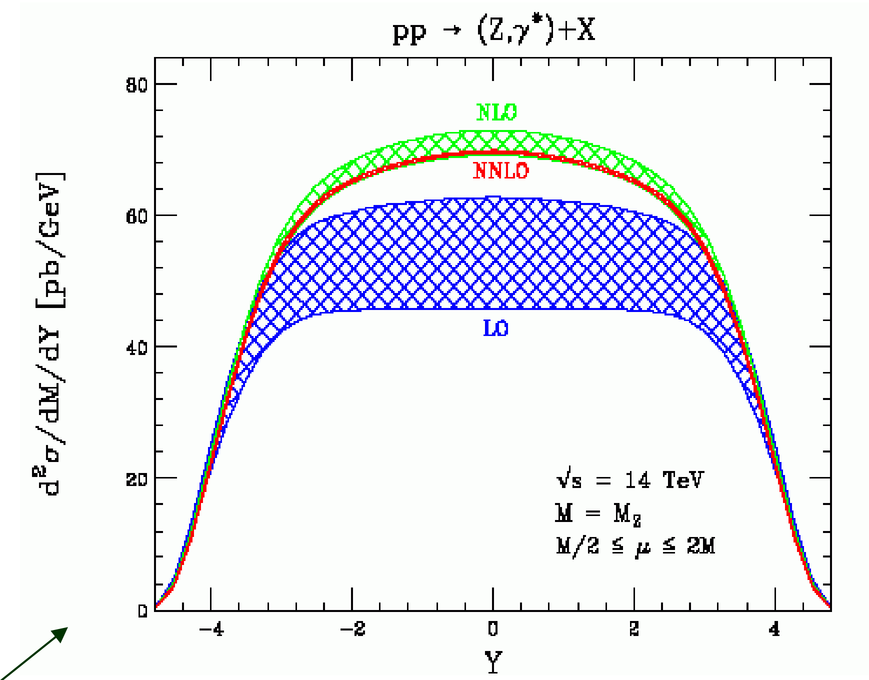
- W & Z production has been proposed as a luminosity monitor process for the LHC, and is currently in use at FNAL.
- The current parton cross section is known to 10%. It would be useful for LHC physics to improve this to the 1-2% level.
- This improvement will require calculating first order corrections in α_{ew} and second order corrections in α_s.



The Drell-Yan Process

- The α_s^2 contributions (NNLO QCD) are known for the integrated cross section (Hamburg et al., Harlander & Kilgore)

- A NNLO calculation of the Z boson rapidity distribution is also available (Anastasiou, Dixon, Melnikov & Petriello)



The Drell-Yan Process

What's missing?

- A fully exclusive calculation is needed to construct a MC event generator at NNLO.
- Also – mixed electroweak-QCD corrections must be added to reach the desired precision level.

The hard parton-level process must then be combined with PDFs in a MC program designed to generate the required distribution of partons plus mixed QCD and QED Bremsstrahlung.

This requires a careful implementation of the multi-gluon / multi-photon phase space.

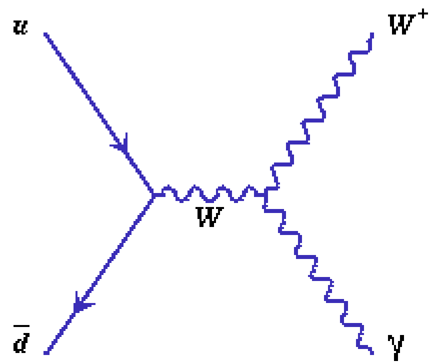
YFS Monte Carlo for Hadron Colliders

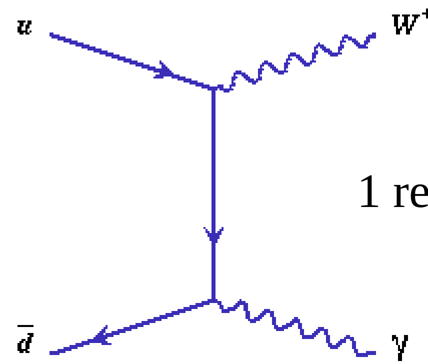
- We are building a Monte Carlo program following our previous approach in the electroweak sector.
- The goal is a YFS-exponentiated MC program for the Drell-Yan process – the proposed LHC luminosity process.
- Combined QCD and QED exponentiation must be included with due attention to the non-abelian structure .
- See talk of B.F.L. Ward later in this meeting for details on exponentiation in this context.
- The resulting hadronic Monte Carlo should have extremely precise control over the soft and collinear limits.

Electroweak Radiative Corrections to W Production

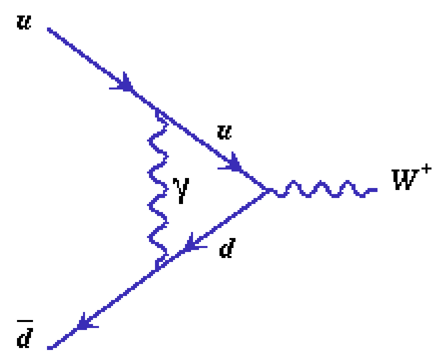
Examples of relevant O(α) electroweak corrections for initial state Bremsstrahlung are shown here.

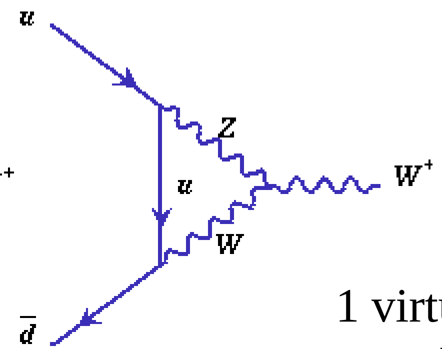
The final state fermions are not shown in these diagrams.





1 real photon





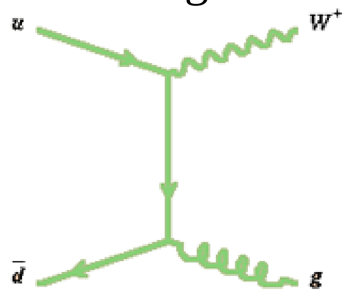
1 virtual photon or Z boson

QCD Radiative Corrections to W Production

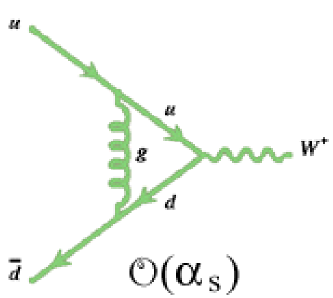
Here are some representative first and second order gluonic corrections.

NLO

1 real gluon



1 virtual gluon



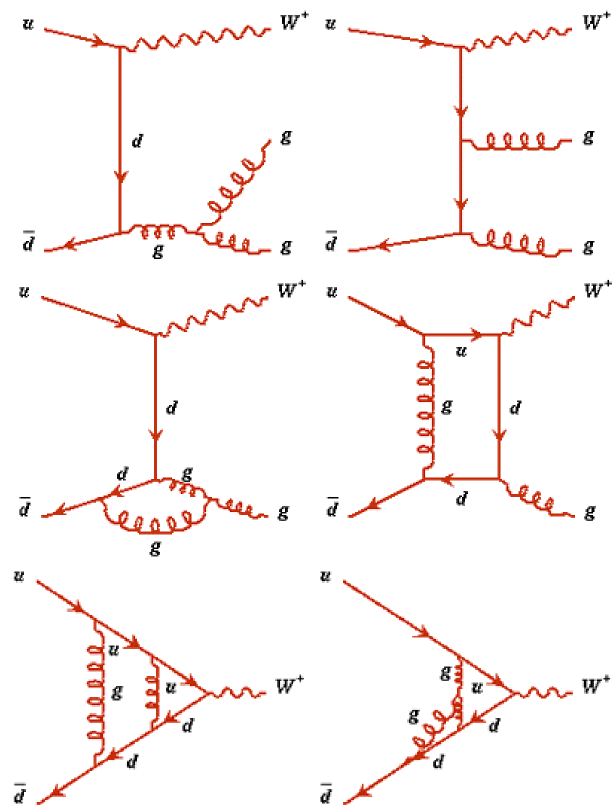
$\mathcal{O}(\alpha_s)$

NNLO

2 real gluons

Real + virtual gluon

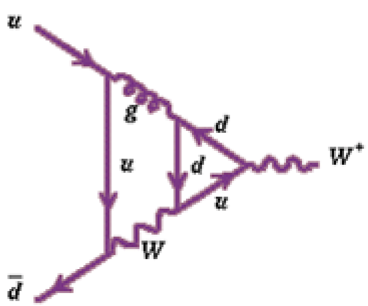
2 virtual gluons

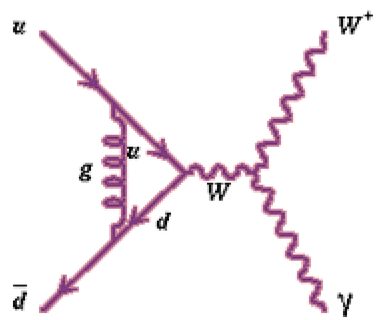


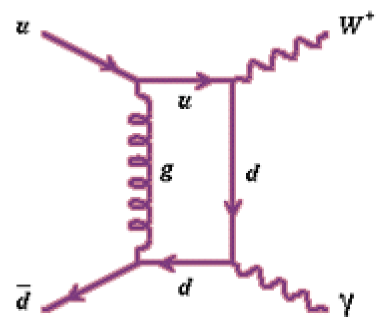
$\mathcal{O}(\alpha_s^2)$

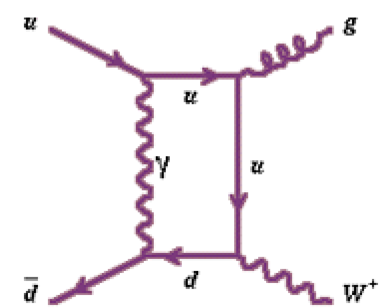
Higher Radiative Corrections to W Production

Further radiative corrections can be added as needed to obtain the cross section for *W* production to the desired accuracy. The next corrections would be mixed strong and electroweak radiative corrections, such as the representative graphs shown below. There are hundreds of graphs at this level, when all of the standard model interactions which contribute are included.









$$\mathcal{O}(\alpha_s \alpha_{ew})$$

Virtual gluon + electroweak loop

Virtual gluon + real photon

Virtual photon (or Z) + real gluon

Some Technical Challenges

- The large number of diagrams contributing to these processes create challenges for stable numerical evaluation.
- Common reduction techniques produce millions of terms, which are both slow to evaluate in a MC setting, and prone to numerical error, since adding such a large number of terms produces large round-off errors even if the individual terms are calculated precisely.
- A significant part of the problem will involve developing and testing new methods for organizing and calculating the terms in the calculation in a stable manner.

YFS-Exponentiated MC

Our intention is to incorporate the matrix elements for the parton-level processes into a "QCED – Exponentiated MC Program" to be described by B.F.L. Ward at this conference.

The exact phase space for the radiated gluons and photons may be constructed using the methods developed for the YFS-exponentiated programs BHLUMI, KK, *etc*., but extended to non-abelian gauge theory.

Soft and collinear singularities may be extracted and exponentiated – with due care to handle the genuine non-abelian singularities which arise in QCD. Cancellation of real and virtual singularities is implemented exactly.

The QCD exponentiation will be conducted at order $\alpha_s^2 L$ on an event-by-event basis in the presence of showers without double-counting of soft and collinear emission effects.

Summary and Outlook

- Careful calculation of higher order Bremsstrahlung corrections led to an extremely precise tool for Bhabha luminosity calculations (BHLUMI), and similar techniqes have led to a high-precision MC tool for fermion pair production at e^+e^- colliders (KK MC).
- A complete result on the wide angle corrections to the process $e^+ e^- \longrightarrow f\bar{f} + \gamma$ will be ready soon .
- We have outlined the construction of a YFS-exponentiated Monte Carlo program for the fully exclusive Drell-Yan process.
- The YFS Monte Carlo approach has many advantages for representing the exact multi-photon and multi-gluon phase space in a manner which can naturally extract and exponentiate large soft and collinear factors.