Atmospheric Neutrinos and Proton Decay with Hyper-Kamiokande

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Atmospheric Neutrino Generation



• Cosmic rays strike air nuclei and the decay of the out-going hadrons gives neutrinos

$$P + A \rightarrow N + \pi + + x$$

$$\mu^{+} + \nu_{\mu} \rightarrow e^{+} + \nu_{e} + \overline{\nu_{\mu}}$$

- Primary cosmic rays Isotropic about Earth
- vs travel 10 10,000 km before detection
- Both neutrinos and antineutrinos in the flux
 - ~ 30% of final analysis samples are antineutrinos
- Flux spans many decades in energy ~100 MeV - 100TeV+
- Excellent tool for broad studies of neutrino oscillations
 - Access to sub-leading effects with high statistics 2

Hyper-Kamiokande: Introduction



- Present studies are performed assuming
 - 560 kton fiducial volume
 - Equivalent detector performance for SK
- No additional improvements relative to Super-K analyses
 - Ie, expected improvements in event reconstruction with fiTQun are not included
 - exception: "finer" binning studies exist
- Similarly no extrapolation of flux and cross section systematics

Atm v	Hyper-K
σ _{mom} e/μ	5.6% /3.6%
$\sigma_{_{dir}}$ e / μ	3.0° / 1.8°
ν CC Purity :	
FC e-like	94.2 %
FC µ-like	95.7 %
PC µ-like	98.7 %

Super-K Atmospheric v Analysis Samples

Fully Contained (FC)



Partially Contained (PC)



Upward-going Muons (Up-





- In total 19 analysis samples: multi-GeV e-like samples are divided into v-like and v-like subsamples
- Dominated by $v_{\mu} \rightarrow v_{\tau}$ oscillations
- Interested in subdominant contributions to this picture
- le three-flavor effects, Sterile Neutrinos, LIV, etc.

Comparison to Current Super-K Exposure

	Hyper-K	SK-IV
Fiducial Vol.	560 kton	22.5 kton
Eff. Area	22,000 m ²	1500 m ²
Protons	1.8×10^{35}	$7.5 imes 10^{33}$
Neutrons	1.4×10^{35}	$6.0 imes 10^{33}$
Fully Contained μ -/e-like	740,200	41,000
Partially Contained μ -like	64,400	3,100
Upward-Going μ	83,400	7,400

- Event rates are a comparison between 10 years of Hyper-K and 12.8 years of SK
 - Compare: HK beam events 42,000 v_{μ} and 7,000 v_{e}
- Analyses exposures have been adjusted to account for difference in fiducial volume and effective area between Hyper-K and Super-K

Introduction

- Some Introductory Material
- Atmospheric neutrinos as signal
 - Standard PMNS Oscillations
 - Sensitivity to Earth's chemical composition
 - Search for $\Delta m_{g} \sim eV^{2}$ scale sterile neutrinos
 - Search for Lorentz invariance violation
 - Comments on leptonic unitarity studies
 - Atmospheric neutrinos as **background**
 - Search for WIMP-induced neutrinos from the galactic center
 - Search for WIMP-induced neutrinos from the sun
 - Proton decay searches

Summary

Atmospheric Neutrinos As Signal

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Searching for Three-Flavor Effects: Oscillation probabilities ~100 km **Cosine Zenith Angle Cosine Zenith Angle** $P(v_{\mu} \rightarrow v_{\mu})$ $P(v_{u} \rightarrow v_{e})$ 0.9 0.6 0.8 0.5 0.5 0.5 0.7 0.6 0.4 0 0.5 0.3 0.4 0.3 0.2 -0.5 -0.5 0.2 0.1 0.1 10² 10² 10 ~10,000 km¹ 10 Energy [GeV] Energy [GeV] "Sub-GeV" "Multi-Ge

- Key Points
- No $v_{\mu} \rightarrow v_{e}$ Appearance above ~20 GeV,
- Resonant oscillations between 2-10 GeV (for v or \overline{v} depending upon MH)
- No oscillations above 200 GeV
- No oscillations from downward-going neutrinos above ~5 GeV
- Expect effects in most analysis samples, largest in upward-going v_{a}



Hyper-K Sensitivity 10 Years



- Expect better than ~3σ sensitivity to the mass hierarchy using atmospheric neutrinos alone
- 3σ Octant determination possible if $\sin^2 2\theta_{23} < 0.99$

CP Violation Sensitivity



- Limited sensitivity to CP-violation with atmospheric v alone
- Hyper-K can constrain only about 50% of δ_{cp} space at 3 σ , so one of the CP-conserving points is allowed at that C.L.
- Sensitivity from SubGeV e-like samples becomes limited due to flux and cross section systematics
 - Reconstruction, systematic, and analysis improvements possible and expected to help considerably

Oscillation-induced v_{-} measurements



- Super-K has demonstrated the ability to identify v_{r} events in the atmospheric neutrino data (3.8σ)
- After 10 years Hyper-K will have O(2,000) $v\tau$ events that can be used to study
 - CC v cross section, leptonic universality, etc.

LBNE

28.5

44.8

~8

Geophysics: Chemical Composition of Earth's Outer Core



Density profile of the Earth is well known from seismic measurements

 Outer core is thought to be liquid iron+Ni and another light element (Unmeasured!)

Z/A ratio is important to understanding formation of Earth and its magnetic field
 With 10 years of data Hyper-K can open the field of Earth Spectroscopy

- First Z/A measurement, can exclude lead-based and water-based outer core
- Longer exposures more useful (want to discriminate iron from pyrolite)

Hyper-K's sensitivity to Sterile Neutrino Mixing

- Searches for sterile neutrinos with the atmospheric neutrinos are independent of the sterile Δm^2 and the number sterile neutrinos
 - For $\Delta m_s^2 \sim 1 \text{ eV}^2$ oscillations appear fast
- $| U_{\mu 4} |^2$ Induces a decrease in event rate of μ -like data of all energies and zenith angles
- **U**_{$\tau 4} |² Shape distortion of angular distribution of higher energy <math>\mu$ -like data</sub>
- Sensitivity gains are limited by
 - flux and cross section errors
 - Better knowledge during actual hyper-K running can improve these constraints

	Hyper-K	SK-IV
$ U_{\mu 4} ^2$	0.029	0.038
U ₁₄ ²	0.066	0.164



Lorentz-invariance violating oscillations

$$H = UMU^{\dagger} + V_e + H_{LV}$$

$$H_{LV} = \begin{pmatrix} 0 & a_{e\mu}^T & a_{e\tau}^T \\ (a_{e\mu}^T)^* & 0 & a_{\mu\tau}^T \\ (a_{e\tau}^T)^* & (a_{\mu\tau}^T)^* & 0 \end{pmatrix} - \frac{4E}{3} \begin{pmatrix} 0 & c_{e\mu}^{TT} & c_{e\tau}^{TT} \\ (c_{e\mu}^{TT})^* & 0 & c_{\mu\tau}^T \\ (c_{e\tau}^T)^* & (c_{\mu\tau}^T)^* & 0 \end{pmatrix}$$

- Lorentz invariance violating effects can be probed using atmopsheric neutrinos
- Analysis using the Standard Model Extension (SME)
- Effects of LIV controlled by two sets of complex parameters
 - $a_{\alpha\beta}^{T}$ dim = 3 induces oscillation effects ~ L
 - $\mathbf{c}_{\alpha\beta}^{\mathsf{TT}}$ dim = 4 induces oscillation effects ~ $\mathbf{L} \times \mathbf{E}$
- Hyper-K Sensitivity will be ~ 3× better than Super-K



	еµ	ετ	μτ		еµ	ετ	μτ
$\mathfrak{R}(a^T)$	4×10 ⁻²⁰ MiniBooNE	8×10 ⁻²⁰ Double Chooz	-	$\mathfrak{R}(c^{TT})$	1×10 ⁻¹⁹ MiniBooNE	1×10 ⁻¹⁷ Double Chooz	-
SK:	2×10 ⁻²³	4×10 ⁻²³	6×10 ⁻²⁴		2×10 ⁻²⁶	1×10 ⁻²⁴	5×10 ⁻²⁷
нк:	7×10 ⁻²⁴	2×10 ⁻²³	2×10 ⁻²⁴		6×10 ⁻²⁷	7×10 ⁻²⁵	2×10 ⁻²⁷

Combination of Beam and Atmospheric Neutrinos

Beam and atmospheric neutrino data provide largely complimentary sensitivity with several common systematic error sources (cross section, detector)

Hyper-K Hierarchy Sensitivity With Beam Inputs



- Plots for true inverted hierarchy are similar
- Large benefit of precise determination of θ_{23} and Δm_{23}^2 from the beam
 - Example of benefit of combination with beam (neutrino mode only)
 - 1 Year of running: 1.5×10^{21} POT, with 560 kton FV



Combination With Beam Neutrinos

- Though atmospheric neutrinos have limited sensitivity to CP-violation relative to the beam measurement, the sensitivity is largely complementary
- Multiple baselines and matter effects give weaker degeneracies
- Addition of atmospheric neutrino data to the beam measurement can improve the δ_{cp} measurement, particularly in regions of limited sensitivity for the beam

Comment on Leptonic Unitarity

If the PMNS matrix is unitary we expect these relations (for I = e, μ , τ)

Normalization
$$N_l \equiv \sum_{i=1,2,3} |U_{li}|^2 = 1$$

Triangle

$$T_{lm} \equiv \sum_{i=1,2,3} U_{li} U_{mi}^* = 0$$

flavor states

- The pieces of the matrix that can be probed depend on L and E of neutrino source
- Hyper-K will have both "fixed" L/E (beam) and "varying" L/E (atmospheric v)
- Computations assume that the U_{pmns} is unitary, but this can be tested

Eigenstates

Models of new physics (SeeSaw, SUSY) predict U_{pmns} is piece of a larger matrix

For LBL vµ disappearance:
$$|U_{\mu3}|^2 (1 - |U_{\mu3}|) \rightarrow \frac{|U_{\mu3}|^2 (|U_{\mu1}|^2 + |U_{\mu2}|^2|)}{\sum_i |U_{\mu i}|^2|}$$

Hyper-Kamiokande can probe many elements of this matrix by itself with combined beam and atmospheric neutrino measurements

Comment on Leptonic Unitarity

If Unitarity is NOT assumed, then to first order

$$\begin{cases} \text{LBL } \nu_{\mu} \rightarrow \nu_{\mu} \\ \text{LBL } \nu_{\mu} \rightarrow \nu_{e} \\ \text{ATM } \nu_{\mu} \rightarrow \nu_{\tau} \\ \text{ATM Reson } \nu_{\mu} \rightarrow \nu_{e} \\ \text{ATM Sub-GeV } \nu_{\mu} \rightarrow \nu_{\mu} \end{cases}$$

$$\frac{|U_{\mu3}|^2 (|U_{\mu1}|^2 + |U_{\mu2}|^2|)}{\mathbb{R} \{U_{\mu3}U_{e3}^* (U_{\mu1}U_{e2}^* + U_{\mu3}U_{e2}^*)\}} \\ \mathbb{R} \{U_{\mu3}U_{\tau3}^* (U_{\mu1}U_{\tau2}^* + U_{\mu3}U_{\tau2}^*)\} \\ (r |U_{\mu3}|^2 - 1) \\ |U_{\mu1}|^2 |U_{\mu2}|^2$$

- Typically single oscillation channels are sensitive to multiple parts of the mixing matrix
 - true for any experiment
- However atmospheric neutrino measurements have sufficient breadth in L/E to have some sensitivity to both "1-2" and "2-3" columns of the mixing matrix (in principle)
 - separating $U_{\mu 1}$ and $U_{\mu 2}$ with (1.0~3.0 GeV data)
- To really make progress improvements in detector performance and systematic errors (flux, cross-section) will be essential

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Global Study of Leptonic Unitarity



- Hyper-K Beam + Atmospheric measurements:
- Contribute to normalizations
- $\alpha = \mu$ (red line)
- $\alpha = \tau$ (orange line)
- i = 3 (brown line)

- Contribute to closure of triangles
- α,β = e, μ (cyan line)
- $\alpha,\beta = \mu,\tau$ (orange line)
- i,j = 2,3 (brown line)
- Hyper-K can provide high statistics measurements with full systematic correlations to improve (overconstrain) our understanding of these relations

Atmospheric Neutrinos As Background

Indirect Dark Matter Searches

Annihilation in the Galactic Center



Search for WIMP Annihilations in the Galactic Center



- Data and MC are binned in momentum and direction to the galactic center
- Signal for a given WIMP mass appears in only some of analysis samples, but is peaked towards the galactic center
 - Remaining analysis samples help control background and its uncertainty
- Hyper-K's sensitivity should exceed Super-K's limits by a factor of 4~5

Search for WIMP Annihilations in the Sun

*Solar composition, VDF, Nuclear form factor



Red lines are Hyper-K 5.6 Mton year sensitivity

- Similarly the data can be binned in the direction to the sun
- Hyper-K limits are expected to be a factor of 3~5 stronger than Super-K in the absence of a signal
 - Strongest limits on SD interactions at low WIMP masses
 - Possible to exclude hints for SI interactions with hardest channel ($\tau^+\tau^-$)

Nucleon Decay Physics Potential

5			+_0
ρ	\rightarrow	e	π

- 2 or 3 e-like rings
- No decay-e
- Hyper-K Selection
- 85<Mπ⁰<185 MeV/c² (3ring)
- 800<M_p<1050 MeV/c²
- Ptot<250MeV/c

	Hyper-K
Signal ε	45%
BG / Mton yr	1.6
10yr. Sens. 90%	1.3 x 10 ³⁵ yr
SK Limit 90%	1.7 x 10 ³⁴ yr

- Hyper-K is the only effective way to probe this decay beyond existing limits
- Efficiency is limited by pion nuclear effects in water
 - Prospects for gains with improved reconstruction
- Surviving atmospheric neutrino background
 - CC 1-pion processes, Deep inelastic scattering
 - Often accompanied by neutrons ¬ BKG reduction

Hyper-K's Sensitivity to $p \rightarrow e^+ \pi^0$

- Baseline Analysis
- Improved Analysis cuts
- BKG Reduced by 50% (n tagging)
- BKG Reduced by 70% (n-tagging)
- Super-Kamiokande has demonstrated neutron tagging via
- $n + p \rightarrow d + \gamma$ (2.2 MeV)
- Hyper-K's tagging depends on detector configuration, Photocoverage, Gd doping etc.
- If no signal is observedlifetime limits $\tau/B > 10^{35}$ years possible with
 - 3.6 Mtonxyear (red, default)
 - 3.0 Mtonxyear (green)
 - 2.4 Mtonxyear (blue)
- Background reduction is an essential component of the Hyper-K nucleon decay program
 - potential for large sensitivity gains exists

Hyper-K's Sensitivity to $p \rightarrow e^+ \pi^0$

Proton Decay : p $\rightarrow \overline{v}$ K+

Hyper-K's Sensitivity to : $p \rightarrow v K+$

	Hyper-K
Signal ε	7.6 - 37%
BG / Mton yr	1.8 - 2556
10yr. Sens. 90%	3.2 x 10 ³⁴
SK Limit 90%	7.8 x 10 ³³ preliminary

- Backgrounds from atmospheric neutrino kaon production
- $\nu + p \rightarrow \nu K^{+}\Lambda + \gamma$ (poorly measured)
- $\nu + p \rightarrow \mu p + \gamma$
- Signal efficiency gains possible (likely):
 - Improve γ (faster PMTs)
 - improve π^+ tagging (fitter. improvement)
 - Background reduction with n tagging

Hyper-K's Sensitivity to	Other Decay M	1odes
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Mode	Sensitivity (90% CL) [years]	Current limit [years]
$p \rightarrow e^+ \pi^0$	13.0×10^{34}	1.7×10^{34}
$p \to \overline{\nu} K^+$	3.2×10^{34}	$0.78{ imes}10^{34}$
$p \to \mu^+ \pi^0$	9.0×10^{34}	1.1×10^{34}
$p \rightarrow e^+ \eta^0$	5.0×10^{34}	0.42×10^{34}
$p \rightarrow \mu^+ \eta^0$	3.0×10^{34}	0.13×10^{34}
$p \rightarrow e^+ \rho^0$	1.0×10^{34}	0.07×10^{34}
$p \to \mu^+ \rho^0$	0.37×10^{34}	0.02×10^{34}
$p ightarrow e^+ \omega^0$	0.84×10^{34}	0.03×10^{34}
$p o \mu^+ \omega^0$	0.88×10^{34}	0.08×10^{34}
$n \rightarrow e^+ \pi^-$	3.8×10^{34}	0.20×10^{34}
$n \rightarrow \mu^+ \pi^-$	2.9×10^{34}	0.10×10^{34}

- Generally speaking, Hyper-K is expected to have an order of magnitude better sensitivity than Super-K to many decay channels
- For background dominated modes, like $p \rightarrow e^+X$, $\mu^+\nu\nu$, $\nu\pi^+$ etc., the improvement is roughly a factor of 4 or 5

Other Physics at Hyper-K

- Atmospheric neutrino flux measurements
- Tau neutrino studies (oscillation-induced, cross section)
- Non-standard Neutrino Interactions in atmospheric neutrinos
- Search for WIMP annihilation at the center of the Earth
- Various nucleon decay modes
 - $p \rightarrow \nu \pi^{\scriptscriptstyle +}$, $n \rightarrow \nu \pi^{\scriptscriptstyle 0}$
 - $p \rightarrow l^+M^0$ (other antilepton + meson modes)
 - $n \rightarrow l^{-} M^{+}$ (Recent theoretical interest)
 - B+L modes
 - dinucleon decay modes
- \blacksquare n \leftrightarrow n oscillations
- Astrophysical neutrino source search
- The statistical uncertainty at Super-K on many of the analyses above is large so generically we can expect improvements at Hyper-K

Summary

- Atmospheric neutrino physics at Hyper-K is expected to be expansive and precise
- 3σ+ mass hierarchy and octant determination
- Improved sensitivity to exotic oscillation scenarios
- New studies of v_{τ} physics and lepton unitarity
- First measurements of Earth core's chemical composition
- In combination with the beam neutrino data further precision is expected
- Nucleon decay physics potential is equally promising
 - Sensitivity to $p \rightarrow e^+\pi^0$ at $\tau/B > 10^{35}$ years (only with Hyper-K!)
 - Sensitivity to $p \rightarrow \nu K^+$ at $\tau/B > 10^{34}$ years and beyond
 - Order of magnitude increase in sensitivity in many other modes
- The future of non-accelerator measurements at Hyper-K is bright

Supplements

		_	⊦ 0
n	\rightarrow	ρ	π
Μ			

Hyper-K Selection 2 or 3 e-like rings

- No decay-e
 85<Mπ⁰<185 MeV/c² (3ring)
 800<M_p<1050 MeV/c²
 Ptot<250MeV/c

	Hyper-K	LBNF
Signal ε	45%	45%
BG / Mton yr	1.6	~1
10yr. Sens. 90%	1.3 x 10 ³⁵ yr	~10 ³⁴

$$p \rightarrow e^{+}\pi^{0}$$

$$MC$$

$$(p \rightarrow e^{+}\pi^{0})$$

$$(p \rightarrow e^{$$

- Efficiency and background rates for this mode are similar for Hyper-K and LBNE
- This is basically true for other lepton + pion modes
- Smaller size of LBNE makes it less competitive, generally nuclear effects are expected to be larger
- Hyper-K is the only effective way to probe this decay beyond existing limits

3.2 x 10³⁴ ↔ 3.3 x10³⁴

producits 10yr. Sens. 90%

LBNE exhibits good sensitivity to decay modes with a Kaon present

- Significant advantage over water Cherenkov detectors
- Most complementary physics
- If a signal is present, it should be discernible at both HK and LBNE
- LBNE in-situ measurements of BG kaon processes will help HK

Proton Decay Sensitivity Summary

Evidence for v_{τ} Appearance at Super-K

- Search for events consistent with hadronic decays of au leptons
- Multi-ring e-like events, mostly DIS interactions
- Negligible primary v_{τ} flux so v_{τ} must be oscillation-induced : **upward-going**

Event selection performed by neural networkTotal efficiency of 60%

$$\beta = 0 : \text{no } v_{\tau}$$

Data =
$$\alpha(\gamma) \times bkg + \beta(\gamma) \times signal$$

Result	Background	DIS (γ)	Signal
SK-I+II+III	0.94 ± 0.02	1.10 ± 0.05	1.42 ± 0.35

This corresponds to 180.1 ± 44.3 (stat) +17.8-15.2 (sys) events, a

3.8 σ excess (Expected 2.7 σ significance)

- Upgraded detector electronics in SK-IV store all PMT hits in a 500 µsec window after a physics trigger
- Search for the 2.2 MeV gamma from $p(n,\gamma)d$
- Search is performed using a neural network built from 16 variables
 - Data and MC show good agreement on atmospheric neutrino sample
- Future: Implement neutron tagging to help distinguish v/v interactions and to reduce proton decay backgrounds

2.2 MeV γ Selection	
Efficiency	20.5%
Background / Event	0.018

Reconstruction Related

	Hyper-K	LBNE
σ _{mom} e/μ	5.6% /3.6%	2.4% / 3%
$\sigma_{_{ m dir}}$ e/ μ	3.0° / 1.8°	1° / 1°
$\sigma_{_{\rm E}}$ Had. Sys.	***	30/ √E %
$\sigma_{_{dir}}$ Had Sys.	***	10°
v CC Purity :		
FC e-like	94.2 %	97.8 %
FC µ-like	95.7 %	99.7 %
PC μ-like	98.7 %	99.6 %

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- LBNE can tag protons as well as Kaons, both of which are (mostly) invisible at HK
- LBNE has very good PID to separate hadrons from leptons and create very pure analysis samples

Sterile Oscillations Results PRD.91.052019 (2015)

- Turning off sterile matter effects while preserving standard three-flavor oscillations provides a pure measurement of | U₁₄ |²
- Using sterile matter effects, but decoupling v_{e} oscillations provides a joint measurement of $|U_{\mu4}|^2$ and $|U_{\tau4}|^2$, with a slightly biased estimate of the former
- Using SK-I+II+III+IV data (4438 days) $| \bigcup_{\mu 4} |^2 < 0.041$ at 90% C.L. $| \bigcup_{\tau 4} |^2 < 0.18$ at 90% C.L.

Tests of Lorentz Invariance

 $H = UMU^{\dagger} + V_e + H_{LV}$

$$\pm \begin{pmatrix} 0 & a_{e\mu}^{T} & a_{e\tau}^{T} \\ (a_{e\mu}^{T})^{*} & 0 & a_{\mu\tau}^{T} \\ (a_{e\tau}^{T})^{*} & (a_{\mu\tau}^{T})^{*} & 0 \end{pmatrix} - E \begin{pmatrix} 0 & c_{e\mu}^{TT} & c_{e\tau}^{TT} \\ (c_{e\mu}^{TT})^{*} & 0 & c_{\mu\tau}^{TT} \\ (c_{e\tau}^{TT})^{*} & (c_{\mu\tau}^{TT})^{*} & 0 \end{pmatrix}$$

- Lorentz invariance violating effects can be probed using atmopsheric neutrinos
 - Focus here on isotropic effects
 - (sensitive to sidereal effects as well...)
- Analysis using the Standard Model Extension (SME)
 - Not a perturbative calculation
 - Effects computed using full solutions of the Hamiltonian
- Effects of LIV controlled by two sets of complex parameters
 - dim = 3 induces oscillation effects ~ L
 - dim = 4 induces oscillation effects ~ L × E

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Constraints on Lorentz Invariance Violating Oscillations: 90% C.L.

SK-I+II+III+IV : 4438 days of data

PRD.91.052019 (2015)

- Perform separate fits on both hierarchy assumptions for each coefficient and each sector : eµ , e\tau, $\mu\tau$
- No indication of Lorentz invariance violation
 - Limits placed on the real and imaginary parts of **6 parameters** $\leq O(10^{-23})$
 - New limits on $\mu\tau$ sector, improvements by **3 to 7** orders of magnitude over existing limits

S. Antusch, et al. arxiv hep-ph/0607020v5.pdf

Non-Unitary

$$|N| = \begin{pmatrix} 0.75 - 0.89 & 0.45 - 0.65 & < 0.20 \\ 0.19 - 0.55 & 0.42 - 0.74 & 0.57 - 0.82 \\ 0.13 - 0.56 & 0.36 - 0.75 & 0.54 - 0.82 \end{pmatrix}$$

Unitary

$$|U| = \begin{pmatrix} 0.79 - 0.88 & 0.47 - 0.61 & < 0.20 \\ 0.19 - 0.52 & 0.42 - 0.73 & 0.58 - 0.82 \\ 0.20 - 0.53 & 0.44 - 0.74 & 0.56 - 0.81 \end{pmatrix}$$

Basic Source of Complementarity

Hyper-Kamiokande : Size

	Hyper-K	LBNE
Fiducial Vol.	560 kton	34 kton
Eff. Area	22,000 m ²	800 m ²

LBNE : Detailed reconstruction

	Hyper-K	LBNE
σ _{mom} e/μ	5.6% /3.6%	2.4% / 3%
$\sigma_{_{dir}}$ e / μ	3.0° / 1.8°	1° / 1°
Threshold (KE)	С	50 MeV / C

Complementary and Competitive

